scientific reports

Check for updates

OPEN

Amputated life-testing based on extended Dagum percentiles for type of group inspection plans: optimal sample sizes, termination time ratios analysis

Basma Ahmed[1], G. G. Hamedani[2], Getachew Tekle Mekiso[3]✉, Yusra A. Tashkandy[4], M. E. Bakr[4], Eslam Hussam[5] & Haitham M. Yousof[6]

This paper introduces a novel approach to life-testing using Extended Dagum (EXD) percentiles within the framework of group inspection plans. The methodology focuses on optimizing sample sizes and analyzing termination time ratios to enhance the reliability of quality control procedures. By leveraging the flexibility of the EXD distribution, the proposed approach accurately models complex survival data, accommodating heavy-tailed characteristics often encountered in practice. The study presents a detailed analysis of optimal sample sizes and the critical termination time ratios that balance producer and consumer risks. Through comprehensive simulations and real applications, the paper demonstrates the effectiveness of the proposed methodology in ensuring robust and efficient life-testing strategies, offering valuable insights for practitioners in industries such as manufacturing, engineering, and reliability analysis.

Keywords Extended Dagum distribution, Amputated testing, Average group number, Average sample number, Operating characteristic function, Multiple group sampling inspection plans with three and four steps, Producer's risk

Abbreviations

AVSN	Average sample number
CDF	Cumulative distribution function
EXD	Extended Dagum
M-SGRSP	Multiple GRSP
PDF	Probability density function
S-SGRSPs	Single-stage group inspection plans
T-SGRSP	Two-stage GRSP
OCF	Operation characteristic function

The industrial processes involve highly intricate organizational frameworks. Consequently, there are typically some failures and disturbances as a result of the erratic events that affect the system. For all businesses, reducing production system failures is generally more crucial. In order to boost output and performance at the moment, process control takes precedence over quality control. Another method for guaranteeing a particular standard of quality in a good or service is quality control. It could include any steps a company determines are necessary to ensure the validation and control of a feature of a product or service. Determining whether the monitored process or product will display these attributes is the primary goal of the quality control. This ensures that the provided goods, services, or processes are dependable, satisfactory, and adhere to stringent standards. If there are

[1]Department of Information System, Higher Institute for Specific Studies, Giza, Egypt. [2]Department of Mathematical and Statistical Sciences, Marquette University, Milwaukee, USA. [3]Department of Statistics, College of Natural and Computational Science, Wachemo University, Hossana, Ethiopia. [4]Department of Statistics and Operations Research, College of Science, King Saud University, P.O. Box 2455, 11451 Riyadh, Saudi Arabia. [5]Department of Mathematics, Faculty of Science, Helwan University, Cairo, Egypt. [6]Department of Statistics, Mathematics and Insurance, Benha University, Benha 13511, Egypt. ✉email: getachewtekle@wcu.edu.et

a lot of test units, we cannot test each one. We apply statistical quality control methods and destructive testing to tackle this problem. The acceptance sampling plan is an essential inspection tool used by quality assurance managers to decide whether to accept or reject a product based on established quality parameters. Typically, while using an acceptance sampling technique, only one item is inspected; however, in order to save time and money, multiple items may be reviewed simultaneously. This process is known as group acceptance sampling. Based on the item categories, we will refer to the acceptance sampling plan as the group sampling inspection plan (GRSP). Determining the number of groups in this type of test is the same as calculating the sample size. Testing for unexpected death usually involves the use of a sudden death tester. This method, popular among manufacturers, aims to reduce testing times significantly. The concept was initially proposed by several scholars, such as Vlcek et al.[1], Balasooriya[2], Pascual and Meeker[3]. The most recent advancements in the sudden death testing were introduced by Jun et al.[4], based on the assumption that the lifespans of the objects conform to the Weibull distribution with specific shape parameters. In the sudden death tests designed by Jun et al., they introduced both single and double sampling variable schemes. These schemes serve as efficient methodologies for assessing the reliability and longevity of objects, contributing to a more streamlined and effective testing process. The incorporation of Weibull distribution parameters adds a statistical perspective to the evaluation, enhancing the overall precision of the sudden death testing approach.

Because the GRSP may evaluate more objects in a given test time, it may occasionally be preferred over a single sample technique. The number of groups and the number of objects to be inspected are both defined in the group inspection plan. The examination limit and the group size determination are the parameters in this case. According to the research conducted by Aslam et al.[5], the process of selecting the specimen size N involves its division into $\mathcal{H}$ groups, with each group having a designated group size denoted by $\mathcal{G}$. The items within each group undergo simultaneous testing on different testers for a predetermined duration. The total specimen size N is thus expressed as the product of the $\mathcal{H}$ and $\mathcal{G}$, making $\mathsf{N} = \mathcal{H}\mathcal{G}$. The experiment is terminated if any group experiences more failures than the predetermined examination limit $\mathcal{L}_{\rfloor}$ during the specified experiment time.

This methodology has gained substantial recognition and has been widely embraced by numerous researchers across various studies. For instance,[6,7] made early contributions to this area, followed by further advancements from Rao in a series of works[8–10]. The methodology has also been applied and extended in studies by[11,12], Mughal and Ismail[13], and Mughal et al.[14], demonstrating its versatility across different contexts. Rao et al.[15,16] and Rosaiah et al.[17] have further enriched the literature, offering insights that have been critical in the development of this field. More recently, Yiğiter et al.[18] and Almarashi[19] have contributed to the evolving understanding of the methodology, particularly in specialized applications. The work of Aziz et al.[20] and Ali et al.[21] has provided additional evidence of the methodology's robustness, with Saber et al.[22] offering contemporary perspectives and applications. A significant body of research has been produced by Ahmed and Yousof[23], as well as Tashkandy et al.[24], further underscoring the methodology's importance in the current academic landscape. Of particular note are the contributions of Mohamed et al.[25–27], who have provided valuable applications of this methodology within the realms of insurance outcomes and actuarial work. Their research has not only solidified the methodology's standing but also opened new avenues for its application in risk management and financial analysis.

In the domain of acceptance sampling and life-testing plans, recent research has introduced a range of innovative methods and approaches. This literature review discusses several key studies that contribute to the development of sampling plans, focusing on their methodologies, results, and potential areas for further exploration. Liaqat et al.[28] in their study proposed a novel sampling plan using the Kumaraswamy generalized power Weibull distribution. Their approach, which employs a minimum angle method, aims to enhance sampling efficiency and reliability for lifetime data analysis. This contribution is significant as it provides a new perspective on handling lifetime data with a specific distribution. However, the model's complexity may pose challenges in practical application, and further research could focus on simplifying the approach and evaluating its practical feasibility in real-world scenarios. Teh et al.[29] explored and introduces the NGCHSP-1 plan, which leverages the minimum angle method to improve acceptance sampling efficiency for the generalized exponential distribution. This study adds value by presenting a new technique that balances precision and efficiency. Future studies could benefit from comparing NGCHSP-1 with existing sampling plans across various distributions and sample sizes to assess its robustness and generalizability. Rha et al.[30] addressed the optimization of sampling plans within the context of functional regression models. Their approach integrates functional data analysis with sampling plan design, offering a novel method to improve sampling strategies. This advancement is noteworthy,however, the practical application of these methods could be enhanced by validating them with real-world functional data and exploring their effectiveness in diverse settings. Ahmed and Yousof[23] proposed a group acceptance sampling plan utilizing percentiles tailored for the Weibull Fréchet model. This method aims to refine acceptance sampling by leveraging percentile-based criteria. While the approach offers a useful addition to sampling strategies, further research could investigate its performance compared to other percentile-based methods and its applicability in different contexts. Wu et al.[31] presented a stage-independent sampling plan using variables inspection and the Cpk index, enhancing lot determination and quality control. This approach is valuable for its practical focus on quality assessment. Future research could explore the method's application in various industrial settings and its performance across different process capabilities. Fayomi and Khan[32] developed a sampling plan tailored for truncated lifetimes within the context of a generalized transmuted-exponential distribution. Their work addresses the challenge of truncated data and contributes to the precision of acceptance sampling plans. Further investigation into the method's performance with various truncation levels and distributions could provide additional insights into its robustness and applicability. Marques et al.[33] focus on integrating mixed repetitive sampling to enhance quality control. Hussain et al.[34] proposed a new sampling plan for a specific distribution, focusing on quality control data. This approach offers a fresh perspective on acceptance sampling tailored to the

odd exponential-logarithmic Fréchet distribution. Future research could evaluate the method's performance in various quality control scenarios and compare it with other sampling plans for similar distributions.

On the other hand, Narayan et al.[35] presented a framework for uncertainty quantification in computational models used in biomedicine and bioengineering. The authors focus on addressing the challenges of uncertainty in computational predictions and propose methods to improve the reliability of biomedical simulations. The framework aims to enhance model accuracy and robustness by quantifying and managing uncertainties effectively. Loizidou et al.[36] reviewed explores computer-aided detection and classification of breast cancer in mammography. The authors evaluate various algorithms and techniques used for improving diagnostic accuracy in mammographic images. The review highlights advancements in image processing and machine learning that have contributed to more accurate and efficient breast cancer screening and diagnosis. Tripathi and Saha[37] introduced a modified chain group sampling inspection plan specifically designed for scenarios involving item failures under a time-truncated scheme. The authors propose adjustments to traditional sampling methods to better handle items that fail within a specified time frame, aiming to improve inspection accuracy and efficiency. Liu and Wu[38] focused on an efficient partial sampling inspection method for lot sentencing based on process yield. The authors develop a new approach to lot inspection that optimizes sampling strategies based on the yield of the manufacturing process. This method seeks to enhance decision-making processes and improve overall inspection effectiveness. Hou et al.[39] provided a comparative analysis of various variable selection methods through numerical simulations and empirical data. The authors examine the performance of different methods in selecting relevant variables, aiming to identify the most effective techniques for improving model accuracy and interpretability. Zhao & Yu[40] discussed a multi-view computable online learner modeling approach using heterogeneous networks. The authors explore AI-enabled methods for modeling and learning from diverse data sources in real-time. This approach aims to enhance the adaptability and performance of online learning systems through advanced AI techniques. However, many other distributions can be used and compared with our new model, see Yousof et al.[41,42], Mansour et al.[43–48], Korkmaz et al.[49–51], Korkmaz et al.,[52], Elgohari and Yousof[53], (2024), Rasekhi et al.[54,55], Hamed et al.,[56], Shehata and Yousof[57], Shehata et al.[58–61], Alizadeh et al.[62–65], Alizadeh et al.[66,67] and Alizadeh et al.[68], Hashempour et al.,[69], Elbatal et al.,[70], Aljadani et al.[71] and Alizadeh et al.[72].

These studies delve into the development of statistical sampling plans with specific parameters and their utilization in monitoring and controlling the quality of the different products. The focus lies on the producer–consumer relationship, examining predetermined production quantities against pre-defined criteria to assess whether a batch meets the acceptable quality level. Such analyses contribute to enhancing the overall understanding of quality control processes and their applications in industrial settings.

Single-sample plans are fundamental in acceptance sampling, especially when the quality of a batch needs to be assessed quickly and efficiently. The simplest form of a single-sample plan is "single sampling by attributes," which is particularly effective in situations where inspection outcomes are dichotomous, meaning they can be classified into two distinct categories, such as "acceptable" or "defective". In this approach, a random sample is drawn from the lot, and each item in the sample is inspected. Based on the inspection results, the entire lot is either accepted or rejected. The decision criteria typically involve comparing the number of defective items found in the sample against a pre-defined acceptance number. If the number of defects is less than or equal to this acceptance number, the lot is accepted; otherwise, it is rejected. Schilling and Neubauer[73] provide an in-depth discussion of this method, highlighting its practical application in quality control processes where quick decisions are needed, and detailed inspection of every item is impractical or too costly. The single-sample plan is especially useful in industries where the cost of passing defective items must be balanced against the cost of inspecting every single unit.

Plans for double and triple sampling take into account the tendency of many experienced inspectors to give a questionable lot another go. Therefore, in the event of two-fold sampling, a second sample is taken, and the lot's disposition is determined thereafter, if the results of the first sample are unable to definitively indicate whether the lot should be accepted or rejected. The mathematical aspects of the strategy and the empirical outcomes support the rationale of this approach. Firstly, compared to a single-sampling plan with the same protection, the average sample number (AVSN) for a double-sampling plan is frequently set lower. A logical extension of double sampling is to permit the collecting of additional samples to obtain even more distinction in a lot's disposition. These methods are called multiple sampling plans when the last sample is constructed to force a decision at that point, as in the case of double sampling.

The goal here is to present a new extended model for comparing the different group sample plans for quality inspection under single, two, and multiple steps due to the amputation lifetime tests where a product's lifetime follows the EXD model. The significance of this research rests in the fact that in order to satisfy consumer wants and prosper in both domestic and international markets, premium items must be offered at competitive pricing. This study is noteworthy because it directly applies statistical sampling techniques to the final commodities, raw materials, and partially processed materials that are produced by factories and businesses. Further research and analysis are also necessary for a number of flexible probability distributions, especially in light of acceptance plans and related ideas.

The paper introduces EXD percentiles as a potential tool for group sampling inspection plans. This contributes to the advancement of statistical methodologies for quality control, offering new techniques to improve efficiency and accuracy in inspection processes. By proposing EXD percentiles, the paper provides statisticians and quality control professionals with additional tools for making informed decisions about sampling strategies. This can lead to more effective quality control measures, ultimately improving product reliability and customer satisfaction. Life-testing experiments are crucial for assessing the reliability and durability of the products over time. By applying EXD percentiles in life-testing experiments, the paper demonstrates their practical utility in real scenarios, offering insights into their effectiveness and reliability. The paper includes empirical studies to validate the proposed methodology. By conducting experiments and applying EXD percentiles in practical

settings, the paper provides empirical evidence of their efficacy and applicability, enhancing their credibility and adoption by practitioners. Introducing EXD percentiles expands the theoretical framework of statistical quality control. By exploring their properties and applications, the paper contributes to the broader understanding of statistical methodologies, paving the way for further research and innovation in the field.

The structure of this work is carefully organized to guide the reader through the development and application of group inspection plans based on the severed life-test for the EXD model. The following sections are laid out as follows: Section "The amputated life-test for the EXD model" presents a comprehensive overview of the severed life-test specifically designed for the EXD model. This section delves into the theoretical underpinnings of the test, explaining its relevance and applicability to the modeling of life data with heavy tails. Section "The S-SGRSPs" describes the process of developing S-SGRSPs. The methodology and criteria for determining optimal sample sizes and decision thresholds are discussed, providing a foundation for implementing effective quality control measures. Section "Design of the two-stage GRSP" introduces the creation of a Two-Phase Group Sample Inspection Strategy. This section outlines the step-by-step procedure for extending the single-stage plan into a more robust two-phase approach, offering greater flexibility and accuracy in decision-making. Section "Design of the multiple GRSP (M-SGRSP)" proposes a Multiple Group Sample Inspection Technique. Here, the paper expands on the concepts introduced in previous sections to design a multi-stage inspection process that can be tailored to complex industrial scenarios requiring multiple inspection points. Section "Explanation and examples of the application of the tables" provides real-world examples from a range of industries to demonstrate the practical application of the recommended group sampling inspection plans. Case studies are used to illustrate the effectiveness of these plans in different contexts, highlighting their versatility and impact. Section "Comparisons of one-stage and iterative GRSP" presents a detailed comparison between iterative and one-stage GRSPs. This section critically evaluates the performance of both approaches, offering insights into their relative strengths and weaknesses and guiding the selection of the most appropriate strategy for specific industrial needs. By structuring the work in this manner, the paper ensures a logical progression from theoretical foundations to practical applications, facilitating a comprehensive understanding of group inspection plans in the context of life-testing.

The amputated life-test for the EXD model

Several scholars have proposed an abbreviated life-test method, wherein acceptance sampling plans are developed under the assumption that the product's lifespan adheres to specific lifetime distributions, such as exponential, gamma, or Weibull distributions. This method views an item's lifetime, represented as $\mathcal{V}_{\nabla}$, as a quality feature. The product is considered nonconforming if $\mathcal{V}_{\nabla} \geq \mathcal{V}_{\nabla}^{0}$; otherwise, it satisfies the standard. However, because to time limits, it may not be feasible to undertake extensive life-testing until every item fails. Because of this, researchers frequently choose to count the number of nonconforming items and end the test at a predetermined period[74]. This abbreviated life-test technique is often called an amputated life-test.

Amputated life tests look at a number of important areas, such as figuring out the right distribution for the item's lifespan, creating acceptable sample plans, and calculating the item's mean, median, or quantile lifespan. Acceptance sampling schemes based on amputation life-tests with single-item groups and different statistical lifetime distributions have been used by numerous researchers. investigations by Goode and Kao[75], Fertig and Mann[76], Gupta and Grel[77], Kantiam and Rosaeah[78], Balakrishnan et al.[79], Tsai and Wu[80], Lio et al.[81], Gui and Aslam[82], and Ahmed et al.[83] are a few examples of these kinds of investigations.

The EXD model, which Gomes-Silva et al. (2017) presented in their paper, has shown to be a flexible tool for analysing different kinds of data, including lifespan data, as well as in engineering, medicine, and other academic domains. By providing a thorough framework for simulating a wide range of events, the EXD model enables researchers to better understand intricate systems and processes. The cumulative distribution function (CDF) and probability density function (PDF) of the EXD model, characterized by a parameter vector $\underset{-}{\mathbf{V}} = (\mathcal{B}, \gamma, \Omega, \tau, \psi)$ are as follows:

$$F\left(\ddagger; \underset{-}{\mathbf{V}}\right) |\ddagger > 0 = \left\{1 - [1 - T(z)]^{\Omega}\right\}^{\psi}, \tag{1}$$

and

$$f\left(\ddagger; \underset{-}{\mathbf{V}}\right) |\ddagger > 0 = \frac{1}{\tau}\Omega\psi\gamma\mathcal{B}\ddagger^{-(1+\mathcal{B})}\left(1 + \frac{1}{\tau}\ddagger^{-\mathcal{B}}\right)^{-\gamma-1}[1 - T(z)]^{\Omega-1}\left\{1 - [1 - T(z)]^{\Omega}\right\}^{\psi-1}, \tag{2}$$

respectively, where $T(z) = \left(1 + \frac{1}{\tau}\ddagger^{-\mathcal{B}}\right)^{-\gamma}$ $\tau > 0$ refers to the scale parameter of the model, and $\mathcal{B}, \gamma, \psi$ and $\Omega > 0$ are the additional shape parameters. Hence, the ED distributed item has the following percentile time:

$$\mathcal{V}_{\nabla}|(\mathcal{B}, \gamma, \Omega, \tau, \psi > 0) = \left[\tau\left\{[1 - h(\psi, \Omega)]^{-\frac{1}{\gamma}} - 1\right\}\right]^{-\frac{1}{\mathcal{B}}}, \tag{3}$$

where $h(\psi, \Omega) = \left(1 - \nabla^{\frac{1}{\psi}}\right)^{\frac{1}{\Omega}}$.

According to (3), the median of the EXD model $\mathcal{V}_{0.5}$ is obtained when $\nabla = 0.5$. In inspection plans, the characteristic τ sometimes known as the quality parameter, and the shape characteristics are presumed to be known. It is difficult to locate or create inspection plans if it is unknown, and they can be calculated using the

quality control engineers' accumulated experience or failure data. Since the EXD model is skewed, it is advisable to establish inspection strategies based on the median or ∇th percentile lifetime rather than the mean life. As a result, unless otherwise specified, we regard $\mathcal{V}_\nabla$ as the quality parameter. From Eq. (3) it is evident that for fixed $\mathcal{B} = \mathcal{B}_0, \psi \geq \psi_0, \Omega = \Omega_0, \gamma = \gamma_0, \mathcal{V}_\nabla \geq \mathcal{V}_\nabla^0 \iff \tau \geq \tau_0$, where

$$\tau_0 = \frac{1}{\mathcal{Q}\left(\psi, \Omega, \gamma | \nabla\right)} \left(\mathcal{V}_\nabla^0\right)^{-\mathcal{B}}, \tag{4}$$

and

$$\mathcal{Q}\left(\psi, \Omega, \gamma | \nabla\right) = \left[1 - h\left(\psi, \Omega\right)\right]^{-\frac{1}{\gamma}} - 1.$$

It is worth noting that τ_0 is dependent on $\mathcal{B}_0$, ψ_0, Ω_0 and γ_0. In order to design the GRSPs for the EXD model to guarantee that the ∇th percentile time under consideration, it is determined that $\mathcal{V}_\nabla \geq \mathcal{V}_\nabla^0$ equivalently $\tau \geq \tau_0$.

In this study, the development of GRSPs for the EXD model is undertaken with the primary objective of ensuring that the product under scrutiny meets a predetermined quality standard, denoted as $\mathcal{V}_\nabla$. This quality metric is established by consumer expectations and represents the ∇th percentile lifetime of the product. The aim is to ensure that this lifetime exceeds a specified threshold, denoted as $\mathcal{V}_\nabla^0$, with a predetermined level of confidence denoted as ξ^*.

It is deemed advantageous to establish GRSPs in such a manner that the product's ∇th percentile lifespan is at least $\mathcal{V}_\nabla^0$. If there exists adequate evidence to demonstrate that $\mathcal{V}_\nabla \geq \mathcal{V}_\nabla^0$ with a certain level of customer confidence, then the batch of products is considered acceptable. Conversely, if this condition is not met, the batch is rejected. For the sake of simplicity, the termination time of the test for each item, denoted as $\ddagger_0$, is assumed to be a coefficient $\mathcal{U}_\nabla$ times the stated value of the lifespan ∇th percentile $\mathcal{V}_\nabla^0$. Mathematically, this relationship is expressed as $\ddagger_0 = \mathcal{U}_\nabla \mathcal{V}_\nabla^0$, where $\mathcal{U}_\nabla$ represents the coefficient factor.

To illustrate, suppose the product's lifetime ∇th percentile exceeds 3000 h, and a coefficient $\mathcal{U}_\nabla = 0.5$ is chosen. In this scenario, the test for each item will cease after 1500 h from the commencement of the test.

Given the specified value of the test termination time $\ddagger_0$, the probability of failure for each item, denoted as Π , is equivalent to the CDF of the EXD model at $\ddagger_0$. Thus, the value of Π can be expressed as follows:

$$\Pi = \Pr\left(\mathcal{Z} < \ddagger_0\right) = \left\{1 - \left[1 - \left(1 + \mathcal{Q}\left(\psi, \Omega, \gamma | \nabla\right) \frac{\mathcal{U}_\nabla^{-\mathcal{B}}}{\varrho^{-\mathcal{B}}}\right)^{-\gamma}\right]^{\Omega}\right\}^{\psi}, \tag{5}$$

and $\varrho = \mathcal{V}_\nabla / \mathcal{V}_\nabla^0$, when the above-mentioned shape parameters, coefficient factor $\mathcal{U}_\nabla$, and ratio ϱ are determined with a certain vales, the proportion Π can be evaluated. This formulation allows for the quantification of the probability of failure for each item based on the chosen termination time and the characteristics of the EXD model.

The S-SGRSPs

Here, we develop an S-SGRSP with the goal of making sure that the $\mathcal{Z}_\nabla$ percentiles life of an item in a batch is more than the given threshold, $\mathcal{V}_\nabla \geq \mathcal{V}_\nabla^0$. This plan functions on the presumption that each item's lifespan conforms to the EXD model, with defined shape parameters. Finding out if the batch of items satisfies the necessary criteria is the goal; a batch is considered to be in compliance if the actual percentiles life, $\mathcal{V}_\nabla$, is more than the designated threshold, $\mathcal{V}_\nabla^0$. In practical terms, if $\mathcal{V}_\nabla \geq \mathcal{V}_\nabla^0$ for a given batch at a certain level of consumer's risk, the batch will be accepted; otherwise, it will be rejected. This approach ensures that the quality of the batch is assessed based on the statistical criteria, balancing the needs for efficiency in production with the requirement for maintaining quality standards.

The following algorithm constitutes the core methodology employed within the group inspection plan:

Step 1: Sampling: A random sample, consisting of N items, where N equals the product of the chosen $\mathcal{H}$ and $\mathcal{G}$, is extracted from the provided batch. Subsequently, the items within each group are assembled for testing purposes.

Step 2: Determination of Limits: The assay limit ($\mathcal{L}_\rfloor$) and the experiment period ($\mathcal{Z}_0$) are established.

Step 3: Life-test Execution: Conduct life-tests on $\mathcal{H}$ groups until the designated experiment duration, $\mathcal{Z}_0$, is reached. During this period, record the number of failures ($\rfloor$) in each group before the experiment's conclusion.

Step 4: Decision Making: Based on the recorded failures, make decisions regarding the acceptance or rejection of the batch as follows:

- Accept the batch if the $\rfloor$ in any group is less than the assay limit ($\mathcal{L}_\rfloor$).
- Reject the batch if the $\rfloor$ in any group equals or exceeds the predetermined rejection threshold ($\mathcal{R}_e$).

Consequently, the S-SGRSP can be characterized by the parameters $\left(\mathcal{H}, \mathcal{G}, \mathcal{L}_\rfloor, \mathcal{U}_\nabla\right)$, which govern the inspection strategy for ensuring batch quality and reliability.

It is crucial to mention that the suggested plan is a generalization of the familiar single-sample approach, it is crucial to mention. When $\mathcal{G} = 1$, this plan is the standard single sample plan. Assay limit refers to the upper or lower boundary within which a particular characteristic or measurement must fall for a sample to be considered acceptable. It is a predefined criterion that determines whether an item passes or fails an inspection based on its measured attributes. In essence, the assay limit is about the range of values that are deemed acceptable for a given

characteristic. On the other hand, the rejection threshold is the point beyond which a lot or batch is considered nonconforming and is therefore rejected. This threshold is typically used in the context of determining the acceptability of an entire batch of items, based on the proportion of items within the batch that exceed the assay limit. While the assay limit applies to individual items, assessing whether each one meets the required specifications, the rejection threshold is concerned with the overall quality of a batch. If too many items in a batch exceed the assay limit, the batch as a whole may be rejected based on the rejection threshold. In other words, the assay limit focuses on individual conformity, while the rejection threshold assesses batch conformity, taking into account the proportion of items that meet or fail the assay limits.

Accepting a batch is comparable to accepting the hypothesis $H_0 = \mathcal{V}_\nabla \geq \mathcal{V}_\nabla^0$. The item's quality is expressed as meeting at least one criterion by the null hypothesis. As in hypothesis testing, there are two kinds of errors (or risks). Consumer's error ξ and producer's error δ are described as the probability that an inferior batch will be accepted, and an outstanding batch will be rejected, respectively. The level of consumer confidence determines the risk to the consumer. Consumer risk will be $\xi = 1 - \xi^*$ if the confidence level of the consumer is ξ^*. The consumer's error is fixed, that is, it cannot surpass $1 - \xi^*$ and $\mathcal{V}_\nabla < \mathcal{V}_\nabla^0$. We must now establish the proper number of groups $\mathcal{H}$ so that

$$\left[\sum_{|=0}^{\mathcal{L}_\rfloor} \binom{\mathcal{G}}{|} \Pi^{|}(1-\Pi)^{\mathcal{G}-|}\right]^{\mathcal{H}} \leq 1 - \xi^*, \tag{6}$$

where Π represents the proportion of product quality within the group before the termination time $\mathcal{Z}_0$, assuming that the veritable percentile life $\mathcal{V}_\nabla$ equals the fixed percentile life $\mathcal{V}_\nabla = \mathcal{V}_\nabla^0$ and

$$\Pi = \Pr\left(\mathcal{Z} < \ddagger_0\right) = \left\{1 - \left[1 - \left(1 + \mathcal{U}_\nabla^{-\mathcal{B}}.\mathcal{Q}\left(\psi, \Omega, \gamma|\nabla\right)\right)^{-\gamma}\right]^{\Omega}\right\}^{\psi}, \tag{7}$$

The reliance of Π on the ratio $\mathcal{U}_\nabla = \frac{\mathcal{Z}_0}{\mathcal{V}_\nabla^0}$ is clear, as this ratio directly influences the termination time $\mathcal{Z}_0$ relative to the predetermined quality threshold $\mathcal{V}_\nabla^0$. Moreover, Π exhibits a monotonically increasing behavior with respect to this ratio. As the ratio $\mathcal{U}_\nabla$ increases, indicating a longer termination time relative to $\mathcal{V}_\nabla^0$, the probability of failure Π also increases. This is intuitively sensible, as prolonging the test duration allows more opportunities for failure to occur. Conversely, reducing the ratio $\mathcal{U}_\nabla$ by selecting a shorter termination time decreases Π, reflecting a lower likelihood of failure within the specified timeframe. In essence, the relationship between Π and $\mathcal{U}_\nabla$ underscores the crucial balance between test duration and the desired level of reliability. Longer testing periods may offer greater confidence in product reliability but could also entail increased costs and time. On the other hand, shorter testing durations may expedite product development and time-to-market but could potentially compromise reliability. Thus, understanding and appropriately managing the $\mathcal{U}_\nabla$ ratio is essential for achieving the desired balance between reliability, cost, and time in product testing and development processes.

Hence, specifying the ratio $\mathcal{U}_\nabla$ becomes a pivotal step in the experimental design process. By setting this ratio according to the requirements of the study or the expectations of the consumer base, researchers can effectively control the duration of the testing phase relative to the predetermined quality threshold $\mathcal{V}_\nabla^0$.

Equation (6) provides a critical tool for evaluating the experiment's outcomes. If the observed number of defectives does not exceed $\mathcal{L}_\rfloor$, then, according to Eq. (6), it is possible to establish, with a predefined level of confidence denoted by ξ^*, that the cumulative distribution function $F\left(\ddagger \mid \mathcal{B}, \gamma, \Omega, \tau, \psi\right)$ is less than or equal to the target cumulative distribution function $F_0(\ddagger|\mathcal{B}, \gamma, \Omega, \psi, \tau_0)$. This relationship implies that the parameter τ (or equivalently, $\mathcal{V}_\nabla$) exceeds or equals τ_0 (or $\mathcal{V}_\nabla^0$) with the desired level of certainty.

In simpler terms, if the observed number of defects remains within the specified limit $\mathcal{L}_\rfloor$, then it can be inferred, with the chosen level of confidence, that the product's reliability parameter τ surpasses or meets the predetermined threshold τ_0. Alternatively, in terms of the product's lifespan, it indicates that $\mathcal{V}_\nabla$ is greater than or equal to $\mathcal{V}_\nabla^0$ with the desired level of certainty.

This inference is crucial for decision-making regarding the acceptance or rejection of the batch under examination. If the observed number of defects falls within the acceptable range $\mathcal{L}_\rfloor$, it provides confidence that the product meets or exceeds the specified quality standards. Conversely, if the number of defects exceeds $\mathcal{L}_\rfloor$, it may suggest shortcomings in product reliability, warranting further investigation or potential rejection of the batch. Thus, Eq. (6) serves as a valuable tool for assessing product reliability and making informed decisions in the context of quality control and assurance. The optimal $\mathcal{H}$ satisfying (6) for the S-SGRSP of the EXD model with $\mathcal{B}$ = 4,γ = 3, $\psi = {}^1/_8$ and $\Omega = {}^1/_4$ according to $\mathcal{L}_\rfloor$= 0 and 1 ; ξ^*= 75%, 90%, 95% 99%; $\mathcal{U}_\nabla = 0.539$, 0.955, 2.255, 3.835, 5.420 at $\nabla = 0.15$ when $\mathcal{G}$ =3 and 5 are presented in Table 1. In view of Table 1, the values of $\mathcal{H}$ decrease as the $\mathcal{G}$ and $\mathcal{U}_\nabla$ increase.

For example:

I. for $\mathcal{U}_{0.15} = 0.539|(\xi^* = 75\%, \mathcal{G} = 3; \mathcal{L}_\rfloor = 0)$, we have $\mathcal{H} = 12$. However, for $\mathcal{U}_{0.15} = 5.420|(\xi^* = 75\%, \mathcal{G} = 3; \mathcal{L}_\rfloor = 0)$, we have $\mathcal{H} = 1$ and the Average group number (AVGN) decreased from 34.6 to 3.4.
II. for $\mathcal{U}_{0.15} = 0.539|(\xi^* = 75\%, \mathcal{G} = 3; \mathcal{L}_\rfloor = 1)$, we have $\mathcal{H} = 135$. However, for $\mathcal{U}_{0.15} = 5.420|(\xi^* = 75\%, \mathcal{G} = 3; \mathcal{L}_\rfloor = 1)$, we have $\mathcal{H} = 3$ and the AVGN decreased from 404.8 to 9.0.
III. for $\mathcal{U}_{0.15} = 0.539|(\xi^* = 75\%, \mathcal{G} = 5; \mathcal{L}_\rfloor = 0)$, we have $\mathcal{H} = 7$. However, for $\mathcal{U}_{0.15} = 5.420|(\xi^* = 75\%, \mathcal{G} = 5; \mathcal{L}_\rfloor = 0)$, we have $\mathcal{H} = 0$ and the AVGN decreased from 36.5 to 5.6.
IV. for $\mathcal{U}_{0.15} = 0.539|(\xi^* = 75\%, \mathcal{G} = 5; \mathcal{L}_\rfloor = 1)$, we have $\mathcal{H} = 43$. However, for $\mathcal{U}_{0.15} = 5.420|(\xi^* = 75\%, \mathcal{G} = 5; \mathcal{L}_\rfloor = 1)$, we have $\mathcal{H} = 2$ and the AVGN decreased from 217.0 to 10.8.

ξ^*	$\mathcal{U}_{0.15}$	$\mathcal{G}=3$				$\mathcal{G}=5$			
		Single $\mid \mathcal{L}_\rfloor = 0$		**Single** $\mid \mathcal{L}_\rfloor = 1$		**Single** $\mid \mathcal{L}_\rfloor = 0$		**Single** $\mid \mathcal{L}_\rfloor = 1$	
		$\mathcal{H}$	**AVGN**	$\mathcal{H}$	**AVGN**	$\mathcal{H}$	**AVGN**	$\mathcal{H}$	**AVGN**
75%	0.539	12	34.6	135	404.8	7	36.5	43	217.0
	0.955	5	15.0	16	47.2	3	16.3	6	28.0
	1.255	3	8.5	6	18.0	2	9.7	3	15.7
	3.835	2	6.4	4	12.0	2	7.6	2	11.2
	5.420	1	3.4	3	9.0	1	5.6	2	10.8
90%	0.539	15	46.5	224	672.4	10	49.3	72	360.4
	0.955	6	18.5	26	78.5	5	26.3	9	46.5
	1.255	3	9.4	10	30.0	3	12.6	5	25.1
	3.835	2	6.0	6	18.0	2	11.0	3	16.3
	5.420	2	5.4	4	12.0	2	9.1	2	11.5
95%	0.539	24	72.5	292	874.8	16	79.3	94	468.9
	0.955	10	31.0	34	102.1	6	32.3	12	60.4
	1.255	6	16.6	18	54.0	3	15.6	8	40.6
	3.835	3	9.3	11	33.0	3	14.0	5	26.6
	5.420	2	7.1	7	21.0	2	10.6	3	15.3
99%	0.539	32	94.7	448	1344.8	21	105.8	144	720.8
	0.955	12	36.7	52	156.9	9	45.8	19	92.9
	1.255	10	28.6	23	69.0	6	30.1	10	52.4
	3.835	6	18.3	15	45.0	4	18.5	6	31.3
	5.420	3	10.1	9	27.0	2	11.1	4	20.9

Table 1. Results under S-SGRSP.

Additionally, the AVGN increases as the ξ^* increases, for example for $\xi^* = 75\% | \mathcal{L}_\rfloor = 0$, the AVGN stated with 34.6. However, for $\xi^* = 99\% | \mathcal{L}_\rfloor = 0$, the AVGN stated with 94.7. Although the S-SGRSP with $\mathcal{L}_\rfloor = 0$ results in a smaller AVGN, as we shall see later, it also shows an undesirable operation characteristic function (OCF). As a result, we also consider the S-SGRSP with $\mathcal{L}_\rfloor = 1$.

Figure 1 presents the results under the S-SGRSP method across eight different plots. Each plot corresponds to different combinations of the parameter values: $\mathcal{G}$, $\mathcal{L}_\rfloor$, and different percentiles (75%, 90%, 95%, and 99%) of the threshold parameter ξ^*. The plots illustrate the performance metrics $\mathcal{H}$ and AVGN across these different settings. The blue lines represent the $\mathcal{H}$ values, while the red lines correspond to AVGN values. As expected, the results demonstrate that as the threshold parameter ξ^* increases, both HH and AVGN generally decrease, reflecting fewer required inspections and a lower average number of inspections as the sampling becomes more stringent. The impact of the number of groups GG and the lower control limit $\mathcal{L}_\rfloor$ is also evident; higher values of GG and $\mathcal{L}_\rfloor$ tend to result in higher $\mathcal{H}$ and AVGN, indicating that more inspections are necessary when more stringent controls are in place. These results provide a comprehensive understanding of how the S-SGRSP method performs under various parameter settings, offering valuable insights into the method's efficiency and effectiveness in different scenarios.

Specifically, the OCF associated with the Single Sampling with S-SGRSP denoted as $\left(\mathcal{H}, \mathcal{G}, \mathcal{L}_\rfloor, \mathcal{U}_\nabla\right)$ provides insight into the likelihood of accepting the batch under consideration. The derivation of the OCF provides insight into the performance of the acceptance strategy of the batches. This function acts as an indicator for assessing the efficacy of the test used to decide on a batch acceptance or rejection. The OC function of the S- SGRSP is denoted as SGRSP denoted as $\left(\mathcal{H}, \mathcal{G}, \mathcal{L}_\rfloor, \mathcal{U}_\nabla\right)$. Mathematically, it is represented by the following expression:

$$\mathrm{P}_1 = \mathrm{P}_{\mathcal{H},\Pi} = \left[\sum_{\mid=0}^{\mathcal{L}_\rfloor} \binom{\mathcal{G}}{\mid} \Pi^{\mid}(1-\Pi)^{\mathcal{G}-\mid}\right]^{\mathcal{H}}, \tag{8}$$

In this context, let's delve into the significance of $\mid$ as the number of nonconforming items within each group. Here, $\Pi = F\left(\ddagger; \mathcal{B}, \gamma, \Omega, \psi, \mathcal{V}_\nabla\right)$ represents the probability of failure, which is contingent upon various factors including $\mathcal{V}_\nabla$, the reliability parameter of interest. Importantly, Π is a decreasing function of $\mathcal{V}_\nabla$ when $\mathcal{V}_\nabla \geq \mathcal{V}_\nabla^0$ .As $\mid$ increases, indicating a greater number of nonconforming items in the sample group, the overall probability of failure Π tends to rise. This intuitive relationship reflects the idea that an increase in nonconformities within the sample group is associated with a higher likelihood of product failure or unreliability. Consequently, maintaining $\mid$ at lower levels is desirable to minimize the probability of failure and ensure product reliability.Moreover, the OCF denoted as P_1 serves as a critical metric for evaluating product quality. Specifically, P_1 represents the proportion of items conforming to the specified quality standards within the sample group. Notably, P_1 is a decreasing function of Π for a fixed termination time $\ddagger_0$. This relationship underscores the interconnectedness of various quality metrics within the testing framework. As the probability of failure Π increases, indicating a

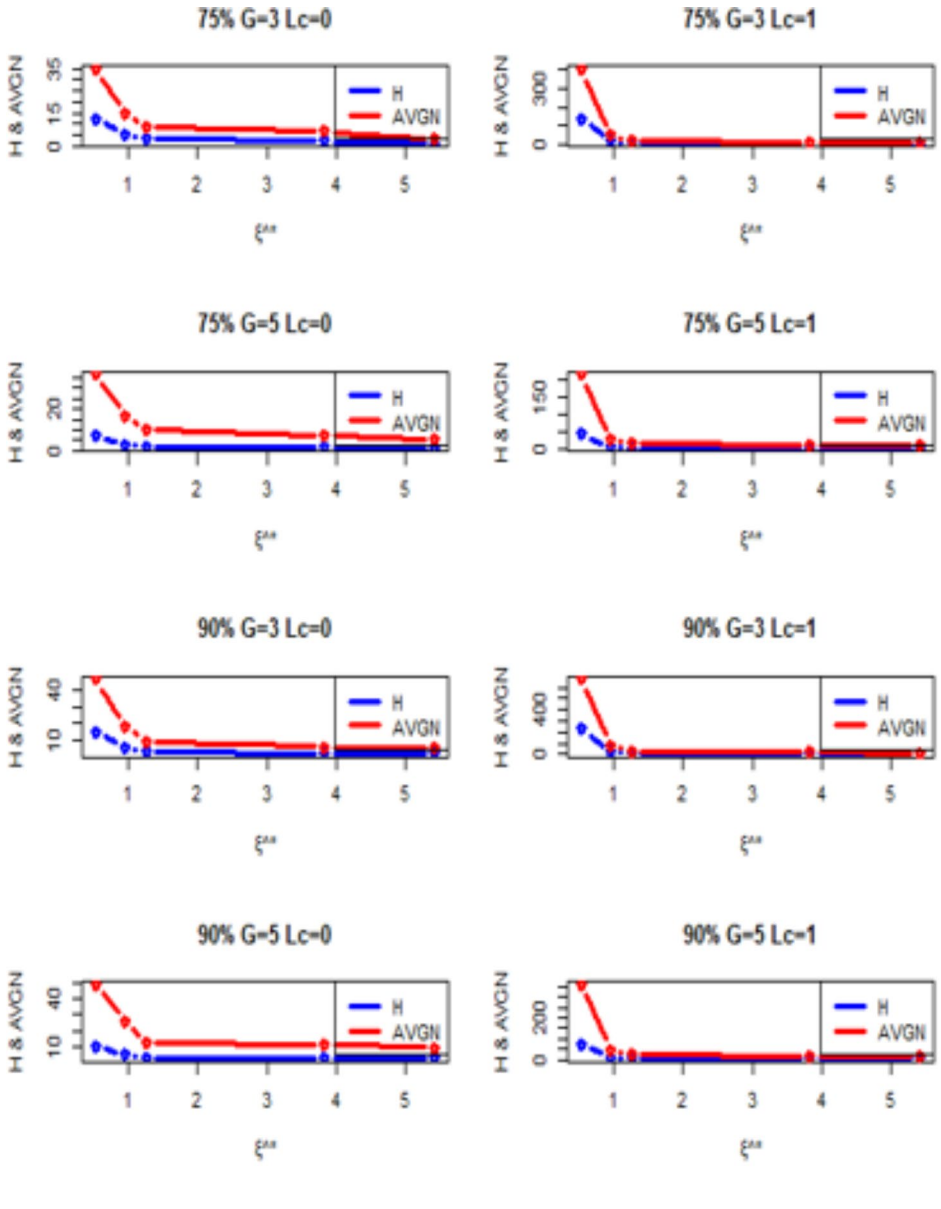

Fig. 1. Results under S-SGRSP.

higher likelihood of product unreliability, the P_1 decreases, reflecting a reduced proportion of items meeting quality standards within the sample group. Conversely, reducing the probability of failure Π by controlling factors such as $\mathcal{V}_\nabla$ or | leads to an improvement in P_1, signifying enhanced product quality and reliability.For given $\mathcal{G}$, ξ^* and $\mathcal{U}_\nabla$ the selection of $\mathcal{L}_\rfloor$ and $\mathcal{H}$ will be made on the fundament of the OCF. The following OCF results can be highlighted:

I. For $\mathcal{U}_{0.15} = 0.539$, $\mathcal{L}_\rfloor = 0$, $\mathcal{G} = 3|\xi^* = 0.75$, the OCF started with $0.479|\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 2$ and ended with $1|\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 6$.
II. For $\mathcal{U}_{0.15} = 0.539, \mathcal{L}_\rfloor = 0$, $\mathcal{G} = 5|\xi^* = 0.75$, the OCF started with $0.469|\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 2$ and ended with $1|\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 6$.
III. For $\mathcal{U}_{0.15} = 0.539$, $\mathcal{L}_\rfloor = 1$, $\mathcal{G} = 3|\xi^* = 0.75$, the OCF started with $0.838|\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 2$ and ended with $1|\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 6$.
IV. For $\mathcal{U}_{0.15} = 0.539, \mathcal{L}_\rfloor = 1$, $\mathcal{G} = 5|\xi^* = 0.75$, the OCF started with $0.831|\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 2$ and ended with $1|\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 6$.
V. Generally, the OCF values decreases as $\mathcal{U}_{0.15}$ increases for all $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 2,3,4,5,6$.
VI. The, the OCF values decreases as ξ^* increases for all $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 2,3,4,5,6$.
VII. The, the OCF values increases as $\mathcal{L}_\rfloor$ increases from 0 to 1 at $\mathcal{G} = 3$ and 5 for all $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 2,3,4,5,6$.

From the observations provided in Table 2, several key trends regarding the OCF become apparent:

1. **Effect of $\mathcal{U}_\nabla$ on OCF**:
- As the ratio $\mathcal{U}_\nabla$ increases, there is a general trend of decreasing OCF values. This implies that extending the test duration relative to the specified quality threshold $\mathcal{V}_\nabla^0$ tends to decrease the proportion of items conforming to quality standards within the sample group. This reduction in OCF suggests a higher likelihood of encountering nonconforming items as the testing duration increases.
- However, it's noteworthy that at higher values of the quality ratio (perhaps indicating a more stringent quality standard), the OCF values start to increase towards one. This suggests that although extending the test duration initially reduces the OCF, there comes a point where the quality standard becomes more consistently met as the testing duration is further extended, leading to an increase in the OCF towards optimal levels.
2. **Effect of $\mathcal{G}$ on OCF**:

ξ^*	$\mathcal{U}_{0.15}$	$\mathcal{V}_{0.15}/\mathcal{V}_{0.15}^0$									
		2	**3**	**4**	**5**	**6**	**2**	**3**	**4**	**5**	**6**
		$\mathcal{L}_\rfloor = 0, \mathcal{G} = 3$					$\mathcal{L}_\rfloor = 0, = \mathcal{G}5$				
75%	0.539	0.479	0.667	1	1	1	0.469	0.66	1	1	1
	0.955	0.375	0.591	0.712	0.785	0.83	0.343	0.564	0.691	0.768	0.816
	1.255	0.185	0.414	0.57	0.671	0.74	0.146	0.366	0.526	0.635	0.709
	3.835	0.055	0.232	0.4	0.526	0.617	0.027	0.16	0.318	0.448	0.547
	5.420	0.01	0.096	0.236	0.368	0.475	0.009	0.088	0.224	0.355	0.462
90%	0.539	0.372	0.581	1	1	1	0.35	0.562	1	1	1
	0.955	0.298	0.522	0.657	0.741	0.794	0.178	0.397	0.55	0.653	0.721
	1.255	0.155	0.377	0.537	0.643	0.716	0.084	0.274	0.437	0.556	0.642
	3.835	0.055	0.232	0.4	0.526	0.617	0.005	0.07	0.189	0.311	0.416
	5.420	0.01	0.096	0.236	0.368	0.475	0.0005	0.019	0.088	0.185	0.285
95%	0.539	0.214	0.429	1	1	1	0.185	0.396	1	1	1
	0.955	0.131	0.336	0.495	0.606	0.68	0.12	0.321	0.48	0.593	0.669
	1.255	0.038	0.181	0.336	0.462	0.557	0.046	0.2	0.358	0.483	0.577
	3.835	0.011	0.104	0.242	0.37	0.474	0.0012	0.033	0.118	0.224	0.325
	5.420	0.0024	0.045	0.148	0.247	0.333	0.0001	0.01	0.059	0.14	0.231
99%	0.539	0.133	0.331	1	1	1	0.105	0.29	1	1	1
	0.955	0.09	0.275	0.435	0.552	0.633	0.05	0.2	0.353	0.476	0.565
	1.255	0.004	0.053	0.153	0.264	0.365	0.003	0.045	0.138	0.246	0.346
	3.835	0.0002	0.012	0.062	0.142	0.231	0.0001	0.011	0.06	0.139	0.227
	5.420	0.0002	0.012	0.067	0.153	0.247	0.00008	0.008	0.051	0.128	0.216
		$\mathcal{L}_\rfloor = 1, \mathcal{G} = 3$					$\mathcal{L}_\rfloor = 1, \mathcal{G} = 5$				
75%	0.539	0.838	0.947	1	1	1	0.831	0.944	1	1	1
	0.955	0.832	0.946	0.977	0.988	0.993	0.817	0.939	0.973	0.986	0.992
	1.255	0.588	0.849	0.932	0.964	0.979	0.473	0.782	0.897	0.944	0.967
	3.835	0.208	0.602	0.801	0.891	0.934	0.137	0.498	0.725	0.841	0.901
	5.420	0.061	0.367	0.638	0.789	0.869	0.012	0.181	0.443	0.639	0.761
90%	0.539	0.745	0.914	1	1	1	0.735	0.909	1	1	1
	0.955	0.737	0.912	0.962	0.98	0.988	0.715	0.901	95%6	0.977	0.986
	1.255	0.412	0.761	0.889	0.941	0.965	0.287	0.664	0.834	0.909	0.945
	3.835	0.095	0.467	0.717	0.84	0.903	0.051	0.351	0.618	0.771	0.856
	5.420	0.024	0.263	0.549	0.729	0.83	0.0064	0.14	0.392	0.597	0.731
95%	0.539	0.682	0.889	1	1	1	0.671	0.884	1	1	1
	0.955	0.673	0.887	95%1	0.974	0.985	0.647	0.873	0.943	0.97	0.982
	1.255	0.203	0.612	0.81	0.897	0.938	0.136	0.52	0.748	0.858	0.914
	3.835	0.013	0.247	0.543	0.727	0.83	0.007	0.175	0.448	0.649	0.772
	5.420	0.0015	0.096	0.35	0.575	0.721	0.0014	0.077	0.295	0.51	0.664
99%	0.539	0.555	0.835	1	1	1	0.541	0.827	1	1	1
	0.955	0.544	0.832	0.925	0.961	0.976	0.512	0.812	0.914	0.955	0.973
	1.255	0.13	0.534	0.764	0.87	0.922	0.083	0.441	0.696	0.826	0.893
	3.835	0.0028	0.149	0.435	0.648	0.775	0.0026	0.123	0.382	0.595	0.733
	5.420	0.0002	0.049	0.26	0.491	0.657	0.0002	0.033	0.196	0.408	0.58

Table 2. OCF results.

- When considering different values of the parameter $\mathcal{G}$, there is a trend of decreasing OCF values as $\mathcal{G}$ increases from 3 to 5 at a specific value of $\mathcal{L}_{\rfloor}$. This indicates that increasing the number of $\mathcal{L}_{\rfloor}$ while holding other factors constant, leads to a reduction in the proportion of items conforming to quality standards. Essentially, as the acceptance threshold for nonconforming items becomes stricter, the OCF decreases.
- Conversely, when $\mathcal{L}_{\rfloor}$ increases from zero to one while maintaining the same value of $\mathcal{G}$, there is an observed increase in the OCF values. This suggests that relaxing the acceptance threshold for nonconforming items results in a higher proportion of items conforming to quality standards within the sample group. Essentially, allowing a greater number of nonconforming items within the sample group leads to a higher OCF.

The manufacturer's primary focus may lie in enhancing the quality of the product to ensure that its acceptance probability surpasses a predetermined threshold. In instances where the producer's risk, denoted as 0.05 for illustrative purposes, is specified, determining the value of $\mathcal{V}_{\nabla}/\mathcal{V}_{\nabla}^{0}$ becomes crucial. This value guarantees that the producer's risk remains at 0.05 or lower. This objective can be achieved by meeting the following inequality:

$$\left[\sum_{\rfloor=0}^{\mathcal{L}_{\rfloor}} \binom{\mathcal{G}}{\rfloor} \Pi^{\rfloor}(1-\Pi)^{\mathcal{G}-\rfloor}\right]^{\mathcal{H}} \leq \frac{5}{100}, \tag{9}$$

For a given parameter of S-SGRSP $\left(\mathcal{H}, \mathcal{G}, \mathcal{L}_{\rfloor}, \mathcal{U}_{\nabla}\right)$, at a specified probability ξ^{*}, the values of $\mathcal{V}_{\nabla}/\mathcal{V}_{\nabla}^{0}$ satisfying the condition in (9) are illustrated in Table 3.

The insights gleaned from Table 3 shed light on the relationship between various parameters and the termination time ratio $\mathcal{V}_{\nabla}/\mathcal{V}_{\nabla}^{0}$, as well as its implications on confidence levels and optimal testing strategies:

1. **Effect of confidence level on termination time ratio**:
- It is observed that as the confidence level increases, the termination time ratio $\mathcal{V}_{\nabla}/\mathcal{V}_{\nabla}^{0}$ also increases. This suggests that higher confidence levels necessitate longer testing durations relative to the specified quality threshold $\mathcal{V}_{\nabla}^{0}$. In other words, to achieve greater confidence in meeting or exceeding quality standards, it becomes imperative to extend the testing duration accordingly.
2. **Effect of $\mathcal{L}_{\rfloor}$ on optimal ratio**:
- When considering different values of $\mathcal{L}_{\rfloor}$, the optimal termination time ratio are decreases as $\mathcal{L}_{\rfloor}$ increases from 0 to 1 at the same group size. This indicates that as the tolerance for nonconforming items becomes stricter ($\mathcal{L}_{\rfloor}$ increases), the optimal testing duration relative to the specified quality threshold decreases. In essence, stricter acceptance criteria necessitate shorter testing durations to maintain or improve overall product quality and compliance with quality standards.
3. **Effect of $\mathcal{G}$ on optimal ratio**:
- For the same $\mathcal{L}_{\rfloor}$, the optimal termination time ratio increases as $\mathcal{G}$ increases from 3 to 5. This implies that increasing the number of $\mathcal{G}$ while maintaining the same acceptance threshold for $\mathcal{L}_{\rfloor}$ requires longer testing durations relative to the specified quality threshold. In other words, when the tolerance for nonconforming items is higher, more stringent testing is needed to ensure product reliability and compliance with quality standards.

Finally, Fig. 2 gives a simple flow chart for the S-SGRSP. This figure represents the flowchart for the single sampling process. The process begins with taking one sample, and if the results meet the specified criteria, the batch is accepted; otherwise, it is rejected. This figure clearly outlines each step in the sampling process, from start to finish.

	$\mathcal{U}_{0.15}$									
	0.539	**0.955**	**1.255**	**3.835**	**5.420**	**0.539**	**0.955**	**1.255**	**3.835**	**5.420**
ξ^{*}	**Single | $\mathcal{L}_{\rfloor}=0$**					**Single | $\mathcal{L}_{\rfloor}=1$**				
$\mathcal{G}=3$										
75%	4.66	9.40	17.15	27.89	33.63	3.397	4.142	8.495	10.609	18.45
90%	6.99	12.35	19.51	31.13	39.82	4.635	6.54	10.034	13.803	22.41
95%	8.46	17.58	23.47	33.85	45.30	5.098	8.25	13.67	19.486	25.49
99%	10.30	19.45	25.40	36.22	47.90	8.551	10.65	15.76	25.527	30.06
$\mathcal{G}=5$										
75%	5.01	13.38	21.21	28.70	38.85	3.72	4.75	10.18	11.65	21.85
90%	9.45	15.69	22.79	32.53	40.57	5.39	8.22	12.57	14.07	26.38
95%	10.05	19.13	24.18	36.55	46.14	6.11	10.12	15.15	22.21	28.38
99%	11.18	20.55	27.75	37.09	48.23	10.54	14.06	18.62	27.76	33.31

Table 3. Results of the proper ratio of $\mathcal{V}_{\nabla}/\mathcal{V}_{\nabla}^{0}$ for S-GRSP.

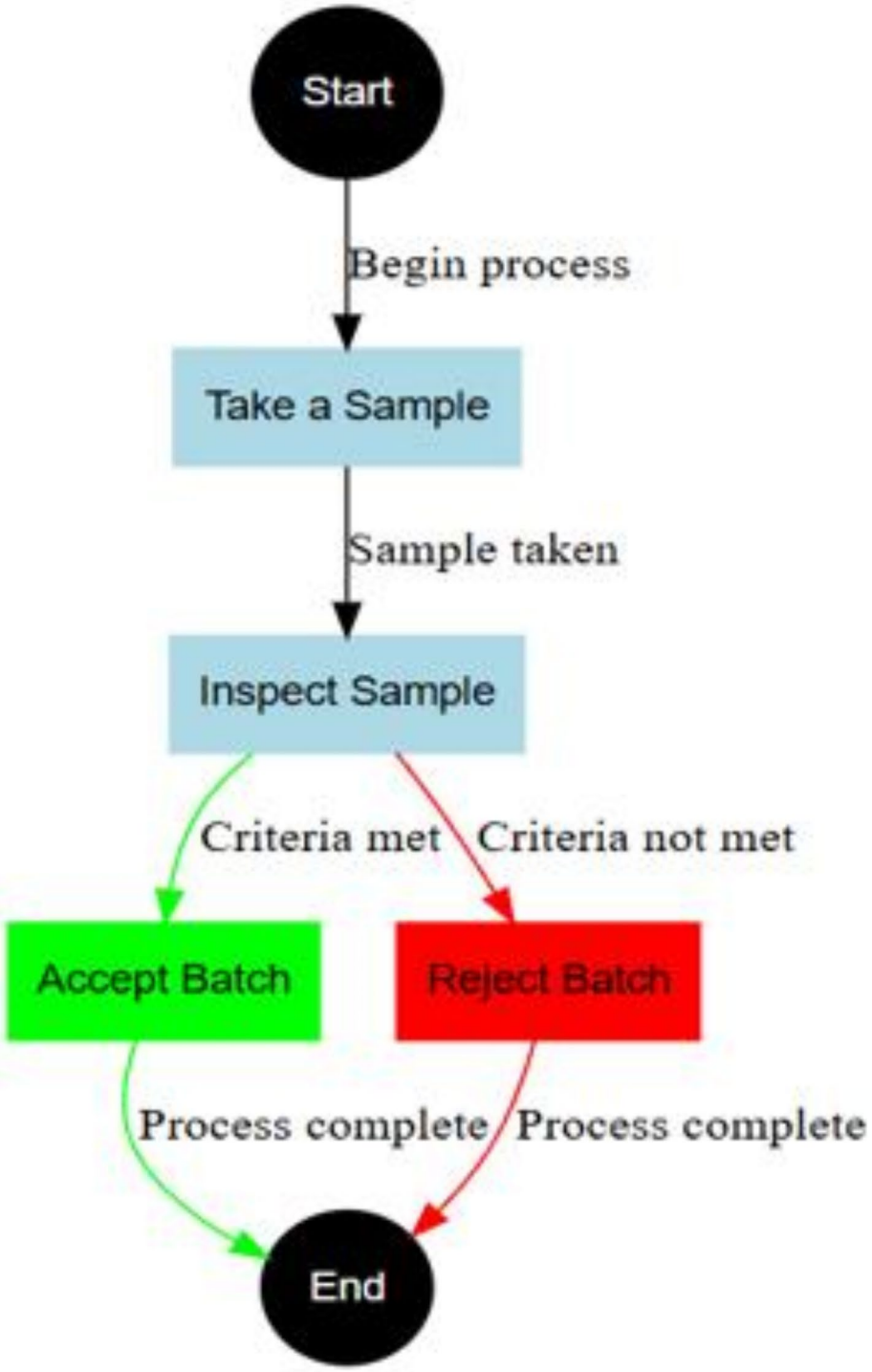

Fig. 2. flow chart for the S-SGRSP

Design of the two-stage GRSP

It's noteworthy that the determination of $\mathcal{G}$ is contingent upon the type of testers employed in the process. The proposed plan involves the parameters of $\mathcal{H}_1, \mathcal{H}_2, \mathcal{L}_{\rfloor_1}$ and $\mathcal{L}_{\rfloor_2}$ where $\mathcal{L}_{\rfloor_1} < \mathcal{L}_{\rfloor_2}$, then, the OCF for T-SGRSP is determined by

$$P_2 = \left[\sum_{|_1=0}^{\mathcal{L}_{|_1}} \binom{\mathcal{G}}{|_1} \Pi^{|_1}(1-\Pi)^{\mathcal{G}-|_1}\right]^{\mathcal{H}_1} + \left[\sum_{|_1=\mathcal{L}_{|_1}+1}^{\mathcal{L}_{|_2}} \binom{\mathcal{G}}{|_1} \Pi^{|_1}(1-\Pi)^{\mathcal{G}-|_1}\right]^{\mathcal{H}_1} \left[\sum_{|_2=0}^{\mathcal{L}_{|_2}-|_1} \binom{\mathcal{G}}{|_2} \Pi^{|_2}(1-\Pi)^{\mathcal{G}-|_2}\right]^{\mathcal{H}_2} \quad (10)$$

where Π is the probability that any item in any group will not work out before the termination time $\mathcal{Z}_0$. For a more comprehensive understanding of the methodologies and theoretical frameworks discussed, readers are encouraged to consult the following references, Aslam et al.[84] and Aslam et al.[11,12] offer foundational insights into the development and application of statistical models relevant to the topic. These studies provide detailed explanations of the underlying principles and methodologies that inform the current work. Mughal et al.[85] further expand on these concepts, introducing advanced techniques and applications within the context of quality control and reliability analysis. Rao[86] and Rao et al.[15,16] contribute additional perspectives on statistical methods and their implementation, offering a broader view of the theoretical developments in the field. Aslam et al.[87] and Azam et al.[88] provide recent advancements and case studies that reflect contemporary approaches to group inspection plans and statistical modeling. Rao and Rao[89] offer a detailed examination of specific methodologies and their practical applications, which are critical for understanding the nuances of the proposed techniques. Prasad et al.[90] and Rao et al.[91] present empirical research and comparative analyses that validate and extend the theoretical work discussed in earlier studies. Kanaparthi[92] introduces new perspectives and applications, reflecting the latest developments in the field and providing a forward-looking view of the research landscape. These references collectively provide a rich source of information and context for the methodologies discussed, offering both theoretical and practical insights that are essential for a thorough understanding of the subject matter. Some other models can be used for quality control theory and group acceptance sampling plans including Alizadeh et al.[93], Alizadeh et al.[94], Aslam et al.[95], Bourguignon et al.[96], Stephens et al.[97] and Yousof et al.[98]. The initial segment of Eq. (10) delineates the batch acceptance probability stemming from the initial stage of the two-stage group sampling Plan (T-SGRSP), whereas the subsequent segment denotes the acceptance probability of the second sample. Given consumers' preference for sampling methodologies with fewer examination limits, we direct our attention towards the following mechanism where $\mathcal{L}_{|_1}= 0$ and $\mathcal{L}_{|_2}= 1$. Consequently, the batch acceptance probability described in Eq. (10) takes on the following form:

$$P_2 = (1-\Pi)^{\mathcal{G}.\mathcal{H}_1} + (\mathcal{G}\Pi)^{.\mathcal{H}_1}(1-\Pi)^{(\mathcal{G}-1).\mathcal{H}_1+\mathcal{G}\mathcal{H}_2}. \quad (11)$$

Then, the optimal number of groups (OTNGs) $\mathcal{H}_1$ and $\mathcal{H}_2$ ensuring that $\mathcal{V}_\nabla^0 \leq \mathcal{V}_\nabla$ at the consumer's confidence level ξ^* can be calculated by

$$1-\xi^* \geq P_2, \quad (12)$$

where Π is the failure probability at $\mathcal{V}_\nabla = \mathcal{V}_\nabla^0$ and is given by Eq. (7).

When determining the number of groups needed to fulfill Eq. (12), which likely represents a decision criterion for accepting or refusing a lot, it's essential to optimize the AVGN. The AVGN represents the average number of units inspected per group to make the decision regarding lot acceptance or refusal. To establish an effective decision-making process, it's crucial to minimize the AVGN while ensuring that the condition $\mathcal{H}_1 \geq \mathcal{H}_2$ holds true. This condition likely reflects a comparison between two hypotheses or decision thresholds related to the lot's quality characteristics. Reducing the AVGN involves finding a balance between the number of groups inspected and the accuracy of the decision-making process. While increasing the number of groups inspected may enhance decision accuracy, it also escalates inspection costs and time. Conversely, reducing the number of groups inspected may decrease inspection costs but could compromise decision accuracy.

To determine the optimal number of groups while minimizing the AVGN and satisfying the condition $\mathcal{H}_1 \geq \mathcal{H}_2$, a systematic approach such as optimization algorithms or statistical methods may be employed. These methods aim to strike a balance between decision accuracy, inspection efficiency, and resource constraints. It is noted that the AVGN in the S-SGRSP with parameters $\mathcal{G}, \mathcal{H}, \mathcal{L}_|$ is $ANGN = \mathcal{H}\mathcal{G}$. For a T-SGRSP, the AVGN is given by:

$$\mathrm{AVGN} = \mathcal{G}\mathcal{H}_2\,(1-\psi) + \mathcal{G}\mathcal{H}_1, \quad (13)$$

where ψ is the probability of planning on the first sample. The probability ψ can be expressed asψ = Pr (batch can be accepted due to the first sample) + Pr (batch will be rejected due to the first sample),then,

$$\psi = \Pr\left(|\leq \mathcal{L}_{|_1}\right) + P\left(|\geq \mathcal{R}_{e_1}\right) = 1 - \Pr\left(\mathcal{L}_{|_1} < |_1 \leq \mathcal{L}_{|_2}\right),$$

which can be expressed as

$$\psi = P\left(|\leq \mathcal{L}_{|_1}\right) + P\left(|\geq \mathcal{R}_{e_1}\right) = 1 - \left[\sum_{|_1=\mathcal{L}_{|_1}+1}^{\mathcal{L}_{|_2}} \binom{\mathcal{G}}{|_1} \Pi^{|_1}(1-\Pi)^{\mathcal{G}-|_1}\right]^{\mathcal{H}_1},$$

for $\mathcal{L}_{|_1} = 0$ and $\mathcal{L}_{|_2} = 1$, Eq. (13) becomes

$$\mathrm{AVGN} = \mathcal{G}\mathcal{H}_1 + \mathcal{G}\mathcal{H}_2\left[\mathcal{G}\Pi(1-\Pi)^{\mathcal{G}-1}\right]^{\mathcal{H}_1}.$$

Consequently, the OTNGs $\mathcal{H}_1$ and $\mathcal{H}_2$ for $\mathcal{L}_{\rfloor_1} = 0$ and $\mathcal{L}_{\rfloor_2} = 1$ in T-SGRSP will be determined by solving the following optimization problem:

$$\text{Minimize } ANGN = \mathcal{G}\mathcal{H}_1 + \mathcal{G}\mathcal{H}_2\left[\mathcal{G}\Pi(1-\Pi)^{\mathcal{G}-1}\right]^{\mathcal{H}_1}, \tag{14}$$

subject to

$$\mathrm{P}_2 \leq 1 - \xi^*, \tag{15}$$

$$\mathcal{H}_1 \geq \mathcal{H}_2 \geq 1, \tag{16}$$

$\mathcal{H}_1$ and $\mathcal{H}_2$ positive integers.

Equation (16) likely presents a constraint that ensures the number of groups in the second stage of inspection does not exceed the number in the first stage. This constraint is imposed because it may not be desirable to have a larger number of groups in the second stage compared to the first stage. By imposing the constraint that the number of groups in the second stage should not exceed that in the first stage, organizations aim to maintain a balanced and efficient inspection process. This constraint ensures that sufficient attention is given to early-stage inspections while still allowing for subsequent stages to address any remaining quality concerns. Ultimately, this approach contributes to effective quality control and assurance practices, leading to improved product quality and customer satisfaction. Table 4 shows the OTNGs required for the two-stage group sampling plan under the EXD model with $\mathcal{B} = 4$,$\gamma = 3$, $\psi = 1/8$ and $\Omega = 1/4$; ξ^*= 75%, 90%, 95% and 99%; $\mathcal{U}_\nabla = 0.539$, 0.955, 1.255, 3.835 and 5.420 at $\nabla = 0.15$ when $\mathcal{G}$ =3 and 5, as mentioned earlier for $\mathcal{L}_{\rfloor_1}$= 0 and $\mathcal{L}_{\rfloor_2} = 1$ in all cases.

Table 5 displays the OCF values according to Eq. (11) for a given proposed T-SGRSP $(\mathcal{G}, \mathcal{H}_1, \mathcal{H}_2, \mathcal{U}_\nabla)$ and ξ^* . In Table 5, we presented the OCF values of the T-SGRSP for the EXD model with $\mathcal{B} = 4$,$\gamma = 3$, $\psi = 1/8$ and $\Omega = 1/4$.Now, the following equation can be used to find the correct ratio of $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ for T-SGRSP, allowing the lot to be accepted with a producer risk of $\tau = \frac{1}{2}$

$$\mathrm{P}_2 \leq \tau, \tag{17}$$

where $\mathcal{H}_1$ and $\mathcal{H}_2$ are selected in accordance with $1 - \xi^*$ when $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0 = 1$. Table 6 lists and examines the ideal values of the ratio $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ that fulfil Eq. (17). Table 6 below presents the proper ratio of $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ in order that the lot accepted with the producer's risk of 5% for the EXD model with $\mathcal{B} = 4$,$\gamma = 3$, $\psi = 1/8$ and $\Omega = 1/4$.

		$\mathcal{G}=3$			$\mathcal{G}=5$		
ξ^*	$\mathcal{U}_{0.15}$	$\mathcal{H}_1$	$\mathcal{H}_2$	**AVGN**	$\mathcal{H}_1$	$\mathcal{H}_2$	**AVGN**
75%	0.539	11	6	32.0	7	3	32.8
	0.955	5	2	14.2	3	1	15.6
	1.255	3	2	8.6	2	1	9.6
	3.835	2	1	5.5	1	1	6.9
	5.420	2	1	4.9	1	1	5.0
90%	0.539	15	7	44.3	9	8	46.6
	0.955	6	2	17.0	5	3	23.0
	1.255	3	2	9.4	2	2	9.6
	3.835	2	1	5.9	2	2	9.1
	5.420	2	1	6.0	2	1	9.0
95%	0.539	21	14	63.9	13	7	65.6
	0.955	8	5	24.6	5	3	24.5
	1.255	4	1	12.1	3	2	12.8
	3.835	3	1	8.7	3	2	12.6
	5.420	3	1	8.1	2	2	10.4
99%	0.539	30	14	88.5	18	16	89.5
	0.955	11	5	31.9	7	6	36.6
	1.255	5	1	14.5	3	2	12.8
	3.835	3	1	9.2	3	2	13.4
	5.420	3	1	8.1	2	2	10.4

Table 4. The proper number of groups in T-SGRSP with $\mathcal{L}_{\rfloor_1} = 0$ and $\mathcal{L}_{\rfloor_2} = 1$ for the EXD model with $\mathcal{B} = 4$,$\gamma = 3$, $\psi = 1/8$ and $\Omega = 1/4$.

ξ^*	$\mathcal{U}_{0.15}$	$\mathcal{G}=3$					$\mathcal{G}=5$				
		$\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$									
		2	**3**	**4**	**5**	**6**	**2**	**3**	**4**	**5**	**6**
75%	0.539	0.506	0.688	1	1	1	0.497	0.681	1	1	1
	0.955	0.408	0.618	0.733	0.801	0.843	0.378	0.589	0.709	0.782	0.828
	1.255	0.223	0.442	0.589	0.685	75%	0.206	0.442	0.587	0.68	0.743
	3.835	0.108	0.343	0.509	0.616	0.689	0.043	0.242	0.444	0.583	0.675
	5.420	0.023	0.177	0.37	0.561	0.596	0.016	0.15	0.369	0.556	0.588
90%	0.539	0.39	0.596	1	1	1	0.371	0.58	1	1	1
	0.955	0.328	0.55	0.68	0.76	0.809	0.225	0.449	0.596	0.692	0.753
	1.255	0.184	0.399	0.552	0.655	0.726	0.203	0.412	0.559	0.652	0.714
	3.835	0.092	0.309	0.472	0.582	0.654	0.014	0.134	0.298	0.432	0.531
	5.420	0.011	0.115	0.267	0.393	0.481	0.0005	0.025	0.118	0.24	0.35
95%	0.539	0.256	0.474	1	1	1	0.247	0.464	1	1	1
	0.955	0.199	0.421	0.572	0.672	0.736	0.204	0.425	0.576	0.675	0.739
	1.255	0.102	0.292	0.454	0.571	0.655	0.102	0.294	0.45	0.563	0.645
	3.835	0.037	0.154	0.289	0.408	0.506	0.003	0.062	0.174	0.287	0.385
	5.420	0.004	0.059	0.154	0.255	0.349	0.0002	0.014	0.082	0.183	0.283
99%	0.539	0.152	0.355	1	1	1	0.149	0.351	1	1	1
	0.955	0.123	0.325	0.484	0.596	0.672	0.091	0.277	0.436	0.553	0.634
	1.255	0.061	0.225	0.385	0.509	0.6	0.101	0.294	0.45	0.563	0.645
	3.835	0.031	0.135	0.264	0.384	0.483	0.002	0.053	0.154	0.262	0.359
	5.420	0.004	0.059	0.154	0.255	0.349	0.0002	0.014	0.082	0.183	0.283

Table 5. The OCF values of the T-SGRSP for the EXD model with $\mathcal{B}=4$,$\gamma=3$, $\psi=1/8$ and $\Omega=1/4$.

ξ^*	$\mathcal{U}_{0.15}, \mathcal{G}=3$					$\mathcal{U}_{0.15}, \mathcal{G}=5$				
	0.539	**0.955**	**1.255**	**3.835**	**5.420**	**0.539**	**0.955**	**2.255**	**3.835**	**5.420**
75%	3.9	8.2	15.8	25.23	30.69	5.02	11.9	19.5	26.8	32.6
90%	4.56	11.45	18.1	29.23	38.01	7.16	12.15	20.8	30.6	40.7
95%	6.88	14.6	21.2	30.01	41.3	8.26	17.06	22.8	31.71	42.9
99%	7.91	15.9	22.4	32.05	43.95	9.11	19.13	25.01	34.32	45.12

Table 6. The proper ratio of $\mathcal{V}_\nabla/\mathcal{V}^0_\nabla$ in order that the lot accepted with the producer's risk of 5% for the EXD model with $\mathcal{B}=4$,$\gamma=3$, $\psi=1/8$ and $\Omega=1/4$.

Figure 3 below gives the flow chart for the T-SGRSP. This figure illustrates the process of secondary sampling. If the first sample does not meet the required criteria, a second sample is taken for re-evaluation. The final decision is based on the results of the second sample, where the batch can either be accepted or rejected.

Design of the multiple GRSP (M-SGRSP)

Extending the double inspection plan to include additional sample drawings to further refine the determination of the batch's disposition is a common practice. When an ultimate sample is formed to make a conclusive decision at that stage, akin to the double sampling method, these approaches are known as multiple sampling plans. Essentially, for a designated final specimen, denoted as the ‡th sample, the setup ensures that the rejection limit $\mathcal{R}_{e_\ddagger}$ equals the examination limit $\mathcal{L}_{\rfloor_\ddagger}$ plus one. Thus, double sampling plan is a special case of the multiple sampling plan where $\ddagger=2$. However, because managing and documenting all the necessary samples and groups can be challenging, M-SGRSP is often found to be challenging to implement.

The impacts of S-SGRSP and T-SGRSP have been the subject of numerous studies; however, M-SGRSP has not received as much attention. A multiple GRSP with three steps is thus offered, presuming that an item's lifetime follows the EXD model with known form parameters and is based on an amputated life-test. The OCF for multiple group inspection sampling is determined by

$$\mathrm{P}_{\rangle,\rangle=1,2,\ldots,\ddagger}=$$

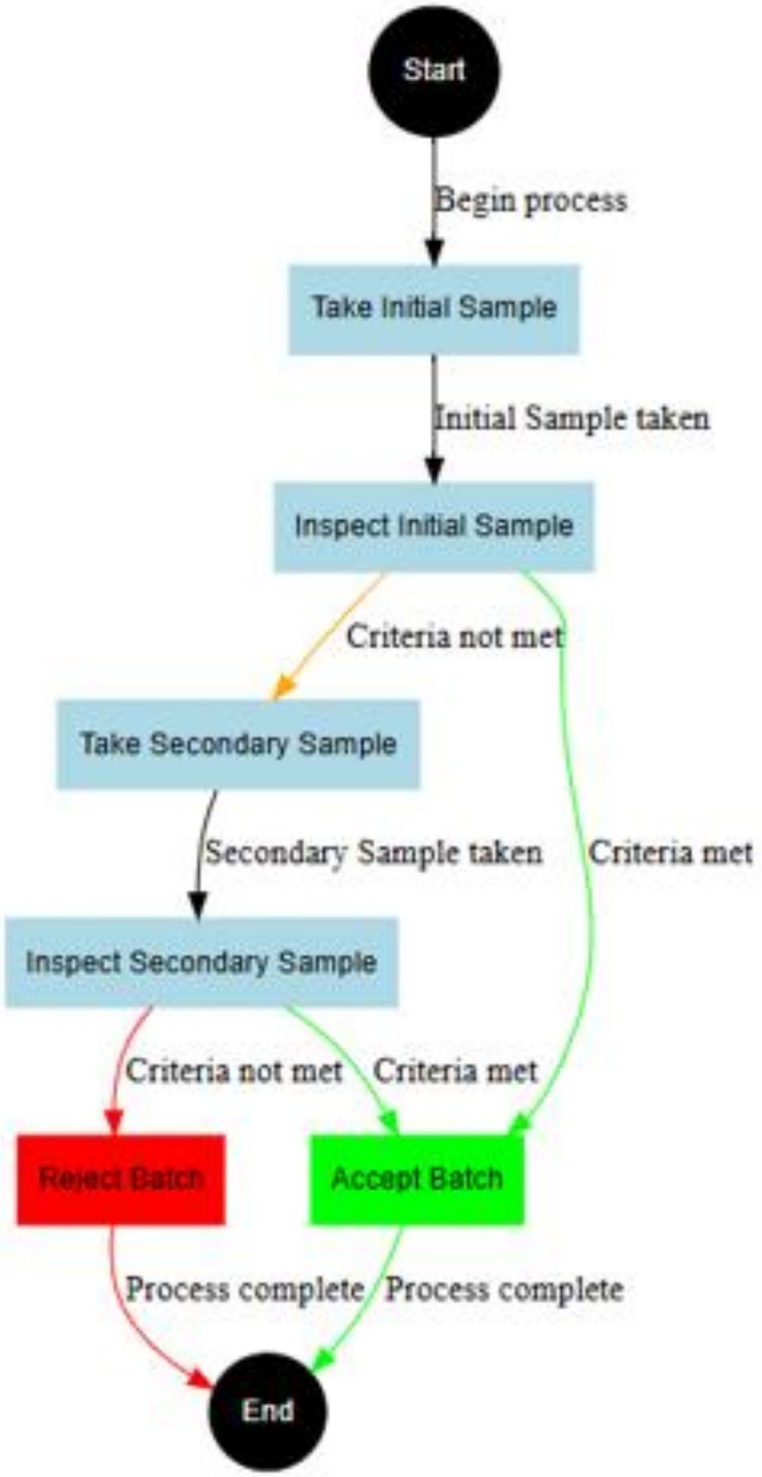

Fig. 3. The flow chart for the T-SGRSP.

$$
\begin{aligned}
&B\left(\mathcal{L}_{\mathcal{C}_1}|\mathcal{GH}_1\right)+\sum_{|_1=\mathcal{L}_{\mathcal{C}_1}+1}^{\mathcal{R}_{e_1}-1} b\left(|_1|\mathcal{GH}_1\right)\sum_{|_2=\mathcal{L}_{\mathcal{C}_2}-|_1+1}^{\mathcal{R}_{e_2}-1} b\left(|_2|\mathcal{GH}_2\right)B\left(\mathcal{L}_{\mathcal{C}_3-V_1}|\mathcal{GH}_3\right)\\
&\quad+\sum_{|_1=\mathcal{L}_{\mathcal{C}_1}+1}^{\mathcal{R}_{e_1}-1} b\left(|_1|\mathcal{GH}_1\right)\sum_{|_2=\mathcal{L}_{\mathcal{C}_2}-|_1+1}^{\mathcal{R}_{e_2}-1} b\left(|_2|\mathcal{GH}_2\right)\sum_{|_3=\mathcal{L}_{\mathcal{C}_3-V_1}+1}^{\mathcal{R}_{e_3}-1} b\left(|_3|\mathcal{GH}_3\right)B\left(\mathcal{L}_{\mathcal{C}_4-V_2}|\mathcal{GH}_3\right)\\
&\quad+\cdots+\sum_{|_1=\mathcal{L}_{\mathcal{C}_1}+1}^{\mathcal{R}_{e_1}-1} b\left(|_1|\mathcal{GH}_1\right)\sum_{|_2=\mathcal{L}_{\mathcal{C}_2}-|_1+1}^{\mathcal{R}_{e_2}-1} b\left(|_2|\mathcal{GH}_2\right)\sum_{|_3=\mathcal{L}_{\mathcal{C}_3-V_1}+1}^{\mathcal{R}_{e_3}-1} b\left(|_3|\mathcal{GH}_3\right)\\
&\sum_{|_\Delta=\mathcal{L}_{\mathcal{C}_\Delta}-V_{\Delta-1}+1}^{\mathcal{R}_{e_\Delta}-1} b\left(|_\Delta|\mathcal{GH}_\Delta\right)B\left(\mathcal{L}_{\mathcal{C}_{\ddagger-1}}-V_{\ddagger-2}|\mathcal{GH}_{\ddagger-1}\right)+\sum_{|_1=\mathcal{L}_{\mathcal{C}_1}+1}^{\mathcal{R}_{e_1}-1} b\left(|_1|\mathcal{GH}_1\right)\sum_{|_2=\mathcal{L}_{\mathcal{C}_2}-|_1+1}^{\mathcal{R}_{e_2}-1} b\left(|_2|\mathcal{GH}_2\right)\sum_{|_3=\mathcal{L}_{\mathcal{C}_3-V_1}+1}^{\mathcal{R}_{e_3}-1} b\left(|_3|\mathcal{GH}_3\right)\\
&\quad\cdots\sum_{|_\Delta=\mathcal{L}_{\mathcal{C}_\Delta}-V_{\Delta-1}+1}^{\mathcal{R}_{e_\Delta}-1} b\left(\mathcal{R}_\mathcal{G}|\mathcal{B}_\mathcal{G}\right)\sum_{\mathcal{R}_{\ddagger-1}=\mathcal{L}_{\mathcal{C}_{\ddagger-1}}-R_{\ddagger-2}+1}^{\mathcal{R}_{e_{\ddagger-1}}-1} b\left(|_{\ddagger-1}|\mathcal{GH}_{\ddagger-1}\right)\left(B\left(\mathcal{L}_{\mathcal{C}_\ddagger}-V_{\ddagger-1}|\mathcal{GH}_\ddagger\right)\right),
\end{aligned}
\tag{18}
$$

where $V_1 = |_1 + |_2$ and $V_2 = |_1 + |_2 + |_3$.

To illustrate the efficacy of multiple inspection plans, let's consider an example where we focus on the design of the third and fourth samples. As we delve deeper into sampling multiple inspections, the complexity of these plans tends to escalate, particularly beyond the third sample. By elucidating this intricacy, we aim to shed light on how this system operates and evolves over subsequent sampling steps.

In essence, as we progress beyond the initial steps of inspection, the interplay of various factors such as sampling strategies, acceptance criteria, and the cumulative information gleaned from prior samples necessitates a more sophisticated approach to maintain the desired level of quality control. This complexity underscores the importance of understanding and effectively managing the intricacies involved in multiple inspection plans to ensure optimal performance and product quality. When M-SGRSP is characterized by $\left(\mathcal{H}_\rangle, \mathcal{G}, \mathcal{L}_{\rfloor_\rangle}, \mathcal{R}_{e_\rangle}\right)$, $\rangle = 1, 2, \ldots, 4$, consequently, the OCF for three-SGRSP is given by

$$\mathrm{P}_3 = \mathrm{P}_{\mathcal{H}_1,\Pi} + \mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\Pi} + \mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3,\Pi}$$

and

$$\mathrm{P}_4 = \mathrm{P}_{\mathcal{H}_1,\Pi} + \mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\Pi} + \mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3,\Pi} + \mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3,\mathcal{H}_4,\Pi}$$

The probabilities $\mathrm{P}_{\mathcal{H}_1,\Pi}$ and $\mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\Pi}$ are provided above, and the probabilities $\mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3,\Pi}$ and $\mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3,\mathcal{H}_4,\Pi}$ can be stated as follows:

$$
\mathrm{P}_{\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3,\Pi} = \left[\sum_{|_1=\mathcal{L}_{\rfloor_1}+1}^{\mathcal{R}_{e_1}-1}\binom{\mathcal{G}}{|_1}\Pi^{|_1}(1-\Pi)^{\mathcal{G}-|_1}\right]^{\mathcal{H}_1}\times\left[\sum_{|_2=\mathcal{L}_{\rfloor_2}-|_1+1}^{\mathcal{R}_{e_2}-1}\binom{\mathcal{G}}{|_2}\Pi^{|_2}(1-\Pi)^{\mathcal{G}-|_2}\right]^{\mathcal{H}_2}\left[\sum_{|_3=0}^{\mathcal{L}_{\rfloor_3}-V_1}\binom{\mathcal{G}}{|_3}\Pi^{|_3}(1-\Pi)^{\mathcal{G}-|_3}\right]^{\mathcal{H}_3}
\tag{19}
$$

$$
\begin{aligned}
P_{\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3,\mathcal{H}_4,\Pi} &= \left[\sum_{|_1=\mathcal{L}_{\rfloor_1}+1}^{\mathcal{R}_{e_1}-1}\binom{\mathcal{G}}{|_1}\Pi^{|_1}(1-\Pi)^{\mathcal{G}-|_1}\right]^{\mathcal{H}_1}\\
&\times\left[\sum_{|_2=\mathcal{L}_{\rfloor_2}-|_1+1}^{\mathcal{R}_{e_2}-1}\binom{\mathcal{G}}{|_2}\Pi^{|_2}(1-\Pi)^{\mathcal{G}-|_2}\right]^{\mathcal{H}_2}\\
&\times\left\{\left[\sum_{|_3=\mathcal{L}_{\rfloor_3}-V_1+1}^{\mathcal{R}_{e_3}-1}\binom{\mathcal{G}}{|_3}\Pi^{|_3}(1-\Pi)^{\mathcal{G}-|_3}\right]^{\mathcal{H}_3}\left[\sum_{|_4=0}^{\mathcal{L}_{\rfloor_3}-V_2}\binom{\mathcal{G}}{|_4}\Pi^{|_4}(1-\Pi)^{\mathcal{G}-|_4}\right]^{\mathcal{H}_4}\right\}
\end{aligned}
\tag{20}
$$

Equation (19) delineates two distinct expressions: the initial one elucidates the acceptance probability originating from the first stage, while the subsequent set encapsulates the acceptance probability stemming from the combined second and third steps. Similarly, Eq. (20) presents two sets of expressions: the first set illustrates the acceptance probability resultant from the four-stage process. To illustrate the application of these formulas, consider the following techniques:

I. Utilize the formulas provided in Eqs. (19) and (20) to compute the acceptance probabilities for specific scenarios. This involves plugging in relevant values such as sample sizes, acceptance criteria, and quality levels into the equations to obtain precise probabilities.

II. Present numerical examples to demonstrate the practical application of the formulas. Choose sample scenarios with varying parameters and calculate the acceptance probabilities using the provided equations. This helps in understanding how changes in input values impact the acceptance probabilities.
III. Visualize the acceptance probabilities obtained from the formulas using graphs or charts. Plot the acceptance probabilities against relevant variables such as sample sizes or quality levels to observe trends and patterns. This graphical representation aids in comprehending the behavior of the acceptance probabilities under different conditions.
IV. Compare the acceptance probabilities derived from multiple-stage inspection plans with those obtained from S-SGRSP. Highlight the differences in acceptance probabilities, efficiency, and effectiveness of the inspection strategies under consideration. This comparative analysis provides insights into the advantages of employing multiple-stage inspection plans.

By employing these techniques, one can effectively demonstrate the utilization and implications of the provided formulas for assessing acceptance probabilities in multiple-stage inspection plans.

Plans	Plan with 3 steps			Plan with 4 steps			
Steps	N_1	N_2	N_3	N_1	N_2	N_3	N_4
Acceptance limits	0	1	2	0	1	2	3
Rejection limits	2	3	3	2	3	4	4

The Statistical Research Group (1948) devised the control table method to compute the Operating Characteristic Function (OCF) and other relevant indicators for Multi-Stage Group Sampling and Reinspection Plans (M-SGRSP). To employ this method effectively, it is essential to begin by determining the probabilities associated with each event at every stage of the sample selection process. These probabilities can be derived from the provided table and tree diagrams. Subsequently, the contents of the control table for plans involving three or four steps can be established accordingly. It is important to note that in this context, "$\mathrm{A}_{\Delta\nabla}$" and "$\mathrm{R}_{\Delta\nabla}$" represent the probabilities of acceptance and refusal, respectively. Here, "Δ" signifies faulty units, "∇" denotes the sample number, and "$p_{\Delta\nabla}$" indicates uncertainty. Table 7 below illustrates a stepwise decision process based on the number of defective items detected. As defects increase, the likelihood of accepting the batch decreases, and the process moves towards rejection. Conversely, if fewer defects are found, the process is more likely to be accepted. This structured approach helps in determining when to accept, reject, or continue inspecting based on the evolving probability and expected outcomes at each step.

Table 7 describes the decision-making process for a quality control or acceptance sampling scenario, where the outcome depends on the number of defective items identified at each step. The table is structured to show how the process evolves through different steps (1, 2, 3) based on the level of defectiveness detected and the resulting actions.

Figure 4 presents the control chart, which outlines the sequential decision process through various steps. The chart is designed to monitor the quality or process control by assessing whether the process stays within the predefined limits or if corrective actions are required. The evaluation of four-SGRSP is possibly best illustrated by a probability tree, as shown in Fig. 4.where

$$E\left(\emptyset|\mathsf{N}_{(.)}\right)=\left[1-B\left(\emptyset-1|\mathcal{GH}_{(.)}\right)\right],$$

$$B\left(\varnothing|\mathsf{N}_{(.)}\right)=\sum_{|=0}^{\varnothing}\left(\begin{array}{c}\mathcal{GH}_{(.)}\\|\end{array}\right)\Pi^{|}(1-\Pi)^{\mathcal{GH}_{(.)}-\varnothing},$$

The control chart in Fig. 2 provides a structured framework for decision-making at each step of the process. It highlights how the probabilities and expected values evolve, helping in determining whether the process remains under control or requires intervention. This chart serves as a visual tool to understand the dynamics of the process and ensures that timely actions are taken to maintain quality standards.

Steps / Defective	1	2	3
3		$p_{11}E\left(1\|\mathcal{GH}_2\right)=R_{32}$ Reject	$p_{22}.E\left(0\|\mathcal{GH}_3\right)=R_{33}$ Reject
2	$E\left(1\|\mathcal{GH}_1\right)=R_{21}$ Reject	$p_{11}.b\left(1\|\mathcal{GH}_2\right)=p_{22}$ uncertainty	$p_{22}.b\left(0\|\mathcal{GH}_3\right)=\mathrm{A}_{23}$ Accept
1	$b\left(1\|\mathcal{GH}_1\right)=p_{11}$ uncertainty	$p_{11}.b\left(0\|\mathcal{GH}_2\right)=\mathrm{A}_{12}$ Accept	
0	$b\left(0\|\mathcal{GH}_1\right)=\mathrm{A}_{01}$ Accept		

Table 7. Stepwise decision process based on the number of defective items detected.

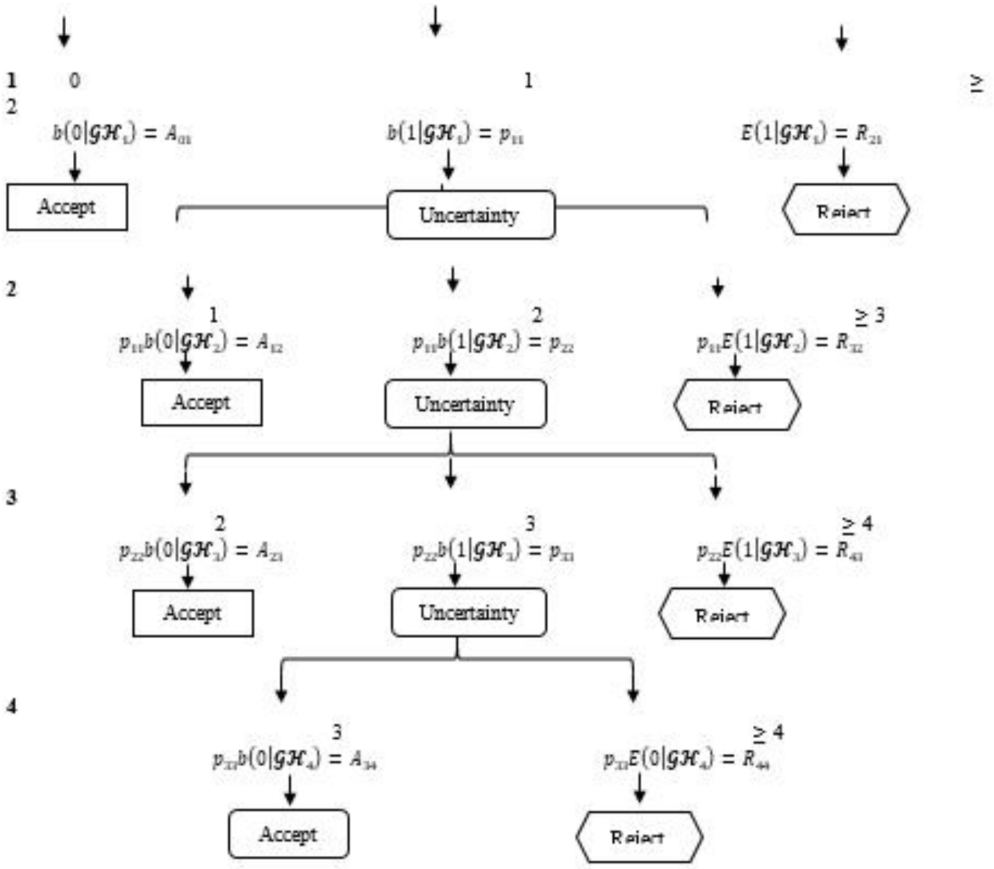

Fig. 4. Control Chart.

Plan	Plan with 3- stage			Plan with 4- stage			
Steps	**1**	**2**	**3**	**1**	**2**	**3**	**4**
Accept, $A_{\rfloor\rangle}$	A_{01}	A_{12}	A_{23}	A_{01}	A_{12}	A_{23}	A_{34}
Uncertainty, $\mathcal{D}_{\rangle}$	p_{11}	p_{22}	0	p_{11}	p_{22}	p_{33}	0
Reject, $R_{e_{\rangle}}$	R_{21}	R_{32}	R_{33}	R_{21}	R_{32}	R_{33}	R_{44}

Table 8. Quality Control table for M-GRSP with three and four steps.

Table 8 effectively illustrates the structure and criteria of the M-GRSP across different stages. The comparison between the three-stage and four-stage plans highlights how the complexity and granularity of the quality control process can be adjusted to better fit different inspection scenarios. The inclusion of uncertainty and rejection thresholds provides a complete picture of the decision-making process in each plan, aiding in the assessment of their effectiveness and suitability for various applications. The probabilities associated with Figs. 1 and 2 are introduced in Table 8, which displays the probability of accepting ($\boldsymbol{A}_{\rfloor\rangle}$), uncertainty ($\mathcal{D}_{\rangle}$) and rejecting ($\boldsymbol{R}_{e_{\rangle}}$) of each occurrence at each stage. The following formulas can be acquired from Table 8 regarding the probabilities of acceptance for plans with three and four steps respectively.

$$\mathrm{P}_3 = \left[(1-\Pi)^{\mathcal{G}}\right]^{\mathcal{H}_1} + (\mathcal{G}.\Pi)^{\mathcal{H}_1}(1-\Pi)^{\mathcal{G}.(\mathcal{H}_1+\mathcal{H}_2)-\mathcal{H}_1} + \left[\mathcal{G}.\Pi(1-\Pi)^{\mathcal{G}-1}\right]^{\mathcal{H}_1+\mathcal{H}_2}\left[(1-\Pi)^{\mathcal{G}}\right]^{\mathcal{H}_3}. \tag{21}$$

and

$$\mathrm{P}_4 = (1-\Pi)^{\mathcal{G}\mathcal{H}_1} + (\mathcal{G}.\Pi)^{\mathcal{H}_1}(1-\Pi)^{\mathcal{G}.(\mathcal{H}_1+\mathcal{H}_2)-\mathcal{H}_1}\begin{bmatrix} 1+(\mathcal{G}.\Pi)^{\mathcal{H}_2}(1-\Pi)^{\mathcal{G}\mathcal{H}_3-\mathcal{H}_2} \\ +(\mathcal{G}.\Pi)^{\mathcal{H}_2+\mathcal{H}_3}(1-\Pi)^{\mathcal{G}(\mathcal{H}_3+\mathcal{H}_4)-(\mathcal{H}_2+\mathcal{H}_3)} \end{bmatrix} \tag{22}$$

where Π is the proportion of product quality at $\mathcal{V}_{\nabla} = \mathcal{V}_{\nabla}^{0}$ and is given by Eq. (7). The proper number of groups for M-GRSP with three and four steps, respectively, to guarantee that the $\mathcal{V}_{\nabla} \geq \mathcal{V}_{\nabla}^{0}$ at the customer's error $(1 - \xi^{*})$, may be found using the following inequality:

$$\mathrm{P}_3 \leq 1 - \xi^{*}, \tag{23}$$

and

$$\mathrm{P}_4 \leq 1 - \xi^{*}. \tag{24}$$

Because there may be numerous solutions of plan parameters that achieve Eqs. (23) and (24), we intend to use the condition $\mathcal{H}_1 \geq \cdots \geq \mathcal{H}_4$ to reduce the AVGN in order to obtain the appropriate solutions. The following procedure can be used to obtain the AVGN:

$$\mathrm{AVGN} = \mathcal{GH}_1 + \mathcal{D}_1 \left(\mathcal{GH}_2 + \mathcal{D}_2 \left\{\mathsf{N}_3 + \mathcal{D}_3 \left[\ldots \left(\mathcal{GH}_m + 0\right) \ldots\right]\right\}\right) \tag{25}$$

Here $\mathcal{D}_{)}$ is described as the probability of failing to make the ruling at $\rangle^{\mathrm{th}}$ stage. According to Table 8, the AVGN for M-GISP with three and four steps respectively in Eq. (25) reduces to

$$\mathrm{AVGN} = \mathcal{GH}_1 + \mathcal{D}_1 \left[\mathcal{GH}_2 + \mathcal{D}_2 \left(\mathcal{GH}_3\right)\right], \tag{26}$$

and

$$\mathrm{AVGN} = \mathcal{GH}_1 + \mathcal{D}_1 \left\{\mathcal{GH}_2 + \mathcal{D}_2 \left[\mathcal{GH}_3 + \mathcal{D}_3 \left(\mathcal{GH}_4\right)\right]\right\}, \tag{27}$$

where

$$B\left(\varnothing | \mathsf{N}_{(.)}\right) = \sum_{|=0}^{\varnothing} \left(\begin{matrix} \mathcal{GH}_{(.)} \\ | \end{matrix} \right) \Pi^{|} (1 - \Pi)^{\mathcal{GH}_{(.)} - \varnothing},$$

,

$$\mathcal{D}_0 = 1$$

and

$$\mathcal{D}_{\ddagger} = 0.$$

Table 8 can be used to identify the probabilities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$, respectively, as shown below.

$$\mathcal{D}_1 | \mathcal{H}_1 = \left[\mathcal{G}\Pi (1 - \Pi)^{\mathcal{G}-1}\right]^{\mathcal{H}_1} \tag{28}$$

$$\mathcal{D}_2 | \mathcal{H}_1, \mathcal{H}_2 = \left[\mathcal{G}\Pi (1 - \Pi)^{\mathcal{G}-1}\right]^{\mathcal{H}_1 + \mathcal{H}_2}, \tag{29}$$

and

$$\mathcal{D}_3 | \mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3 = \left[\mathcal{G}\Pi (1 - \Pi)^{\mathcal{G}-1}\right]^{\mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3}. \tag{30}$$

Numerous sampling strategies outlined in the academic literature aim to minimize AVGN. These strategies aim to streamline the sampling process, ensuring that the AVGN is kept to a minimum. This pursuit of minimizing AVGN is motivated by several key advantages it offers. Firstly, a sampling strategy characterized by a minimal AVGN inherently implies reduced examination costs and examination time. This reduction is particularly advantageous from both economic and operational standpoints, as it optimizes resource allocation and enhances efficiency. Furthermore, minimizing the AVGN holds particular significance in the context of proposed multiple-stage sampling inspection plans for distributions governed by EXD under amputated life-tests. In such scenarios, where reliability and durability are paramount, minimizing the AVGN becomes imperative. By doing so, the sampling plan can effectively identify potential defects or failures early in the product lifecycle, facilitating timely interventions and corrective measures. Consequently, products can be brought to market with enhanced quality and reliability, thereby bolstering customer satisfaction and brand reputation.

The following non-linear problem will be performed for the purpose of determining the proper number of groups in M-SGIP with three and four steps respectively.

$$\text{Minimize } \mathrm{AVGN} = \mathcal{GH}_1 + \mathcal{D}_1 \left[\mathcal{GH}_2 + \mathcal{D}_2 \left(\mathcal{GH}_3\right)\right]. \tag{31}$$

Subject to

$$\mathrm{P}_3 \leq 1 - \xi^{*} \tag{32}$$

$$\mathcal{H}_1 \geq \mathcal{H}_2 \geq \mathcal{H}_3 \geq 1 \tag{33}$$

$\mathcal{H}_1$, $\mathcal{H}_2$ and $\mathcal{H}_3$ positive integerAlso,

$$\text{Minimize } ANGN = \mathcal{GH}_1 + \mathcal{D}_1\left\{\mathcal{GH}_2 + \mathcal{D}_2\left[\mathcal{GH}_3 + \mathcal{D}_3\left(\mathcal{GH}_4\right)\right]\right\}. \tag{34}$$

Subject to

$$\mathrm{P}_4 \leq 1 - \xi^* \tag{35}$$

$$\mathcal{H}_1 \geq \mathcal{H}_2 \geq \mathcal{H}_3 \geq \mathcal{H}_4 \geq 1 \tag{36}$$

$\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$ and $\mathcal{H}_4$ positive integer

The proper number of group for $\mathcal{H}_1$,$\mathcal{H}_2$, $\mathcal{H}_3$ and $\mathcal{H}_4$ and AVGN for the M-SGRSP with three-stage under the EXD model with $\mathcal{B} = 4$,$\gamma = 3$, $\psi = {}^1/_8$ and $\Omega = {}^1/_4$; $\mathcal{L}_{\rfloor_1} = 0$, $\mathcal{L}_{\rfloor_2}$=1, $\mathcal{L}_{\rfloor_3}$= 2 and $\mathcal{R}_{e_1}$= 2, $\mathcal{R}_{e_2}$= $\mathcal{R}_{e_3}$= 3; ξ^*= 75%, 90%, 95% 99%; $\mathcal{U}_\nabla = 0.539$, 1.125, 1.255, 3.835 and 5.420 at $\nabla = 0.15$ when $\mathcal{G}$ =3 and 5 are show in Table 9.

Table 9 effectively illustrates the relationship between confidence levels, group sizes, and the parameters of the EXD model, providing valuable guidance for optimizing sampling plans in practical applications. Table 10 provides a detailed overview of the optimal number of Single-Stage Group Sampling Plans (M-SGRSP) with four stages for the Extended Dagum (EXD) model, given the parameters $\mathcal{B} = 4$,$\gamma = 3$, $\psi = {}^1/_8$ and $\Omega = {}^1/_4$. The table is organized by different ξ^* and illustrates the required number of groups across four stages for different parameter values $\mathcal{U}_{0.15}$.

In conclusion, Table 10 shows that, in comparison to the three-stage plan, the four-stage M-SGRSP strategy provides a more thorough and sophisticated quality control mechanism, particularly when it comes to handling bigger sample numbers and reaching greater confidence levels. The outcomes demonstrate the advantages of adding a step to improve the sample process' accuracy and dependability. Analysis of Tables 1, 4, 9, and 10 reveals several noteworthy trends. Firstly, it becomes evident that as the $\mathcal{G}$ increases, there is a corresponding decrease in the number of groups required for the sampling process. This relationship underscores the efficiency gained through larger group sizes, presumably due to reduced variability and more representative sampling. Conversely, it is observed that the AVGN increases with larger $\mathcal{G}$. This observation suggests that while larger group sizes may lead to fewer groups overall, each group contains a greater number of items, potentially enhancing the precision of the sampling process. Furthermore, the number of groups diminishes as the termination time ratio ($\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$) increases. This finding indicates that extending the termination time allows for fewer groups to be required to achieve the desired level of confidence, likely due to the increased opportunity for observing failures over an extended period.

ξ^*	$\mathcal{U}_{0.15}$	$\mathcal{G}=3$				$\mathcal{G}=5$			
		$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	**AVGN**	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	**AVGN**
75%	0.539	9	9	9	27.5	6	6	6	31.6
	0.955	4	2	2	13.2	3	2	1	15.2
	1.255	3	2	1	8.5	2	1	1	10.0
	3.835	2	1	1	5.3	1	1	1	7.6
	5.420	2	1	1	4.8	1	1	1	5.3
90%	0.539	14	12	12	41.1	9	6	6	44.6
	0.955	5	4	4	16.1	4	3	1	20.5
	1.255	3	2	1	8.9	2	1	1	10.3
	3.835	2	2	1	5.5	2	1	1	9.5
	5.420	2	1	1	6.2	2	1	1	9.1
95%	0.539	17	17	17	50.8	11	11	11	54.7
	0.955	6	4	4	17.6	4	3	1	20.0
	1.255	3	2	1	10.3	2	2	1	11.9
	3.835	3	2	1	7.6	2	1	1	10.2
	5.420	3	1	1	7.5	2	1	1	9.6
99%	0.539	27	24	24	80.6	17	12	12	84.4
	0.955	10	8	8	30.4	6	2	2	31.9
	1.255	3	2	1	10.0	3	2	1	12.7
	3.835	3	2	1	8.7	3	1	1	13.3
	5.420	2	1	1	6.6	2	1	1	10.0

Table 9. The proper number of M-SGRSP with three-stage for the EXD model with $\mathcal{B} = 4$,$\gamma = 3$, $\psi = {}^1/_8$ and $\Omega = {}^1/_4$.

ξ^*	$\mathcal{U}_{0.15}$	$\mathcal{G}=3$					$\mathcal{G}=5$				
		$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	AVGN	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	AVGN
75%	0.539	8	2	2	2	22.6	5	2	2	2	25.4
	0.955	4	2	2	2	11.9	3	2	2	2	15.1
	1.255	3	1	1	1	8.1	2	2	2	2	9.4
	3.835	2	1	1	1	5.3	2	1	1	1	5.6
	5.420	1	1	1	1	4.4	1	1	1	1	5.5
90%	0.539	13	13	5	5	38.3	8	2	2	2	40.8
	0.955	5	2	2	2	14.3	4	2	2	2	20.0
	1.255	3	2	1	1	8.3	2	2	2	2	9.3
	3.835	2	1	1	1	5.2	2	2	2	2	7.9
	5.420	2	1	1	1	5.7	2	1	1	1	7.5
95%	0.539	16	15	5	5	48.8	10	2	2	2	50.3
	0.955	6	2	2	2	16.6	3	2	2	2	17.3
	1.255	3	2	2	2	9.0	2	1	1	1	11.0
	3.835	3	1	1	1	7.5	2	1	1	1	9.9
	5.420	2	1	1	1	6.0	2	1	1	1	7.5
99%	0.539	25	5	5	5	75.0	16	7	7	7	78.1
	0.955	9	3	3	3	26.4	6	4	4	4	27.8
	1.255	3	2	1	1	9.3	2	1	1	1	11.3
	3.835	3	1	1	1	8.5	2	1	1	1	10.4
	5.420	2	1	1	1	5.9	2	1	1	1	7.5

Table 10. The proper number of M-SGRSP with four-stage for the EXD model with $\mathcal{B}=4$,$\gamma=3$, $\psi={}^1/_8$ and $\Omega={}^1/_4$.

Similarly, the number of groups is seen to rise with the consumer's confidence level (ξ^*) increasing. This outcome suggests that higher confidence levels necessitate a larger number of groups to meet the stringent reliability requirements, emphasizing the trade-off between confidence and sample size.

To provide deeper insights into the OCF, Tables 10 and 11 display the OCF values derived from Eqs. (21) and (22) for various sampling plans characterized by parameters such as $\mathcal{G}$, $\mathcal{H}_\rangle$, $\mathcal{L}_{|_\ddagger}$, $\mathcal{R}_{e_\ddagger}$, and $\mathcal{U}_\nabla$. These tables offer a comprehensive overview of the performance of the sampling plans under different conditions, facilitating informed decision-making in reliability analysis and quality control processes.

Notice that Π is a function of $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ as indicated in Eq. (5), then $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ is the smallest positive number for which Π satisfies the following inequalities.

$$\mathrm{P}_3 \leq \tau, \tag{37}$$

$$\mathrm{P}_4 \leq \tau \tag{38}$$

Thus, for a given the proposed plans $\left(\mathcal{G},\mathcal{H}_\rangle,\mathcal{L}_{|_\ddagger},\mathcal{R}_{e_\ddagger},\mathcal{U}_\nabla\right)$, $\rangle=1,\ldots,4$ at the specified value of ξ^*, the values of $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ satisfying inequalities (37) and (38) are presented in Tables 12 and 13. Table 12 (due to P_3) provides a clear picture of how the ratio $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ varies with confidence levels, $\mathcal{U}_{0.15}$, and sample sizes. The increasing ratios with higher confidence levels and $\mathcal{U}_{0.15}$ reflect the growing need for adjustments in variability control. The impact of sample size is also evident, with larger sample sizes requiring higher ratios to achieve similar levels of confidence. This table is essential for understanding how different parameters and confidence levels influence the variability and control measures in sampling plans. Table 13 (due to P_4) provides a detailed view of how the ratio $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ varies with different confidence levels, $\mathcal{U}_{0.15}$, and sample sizes. The increasing ratios with higher confidence levels and $\mathcal{U}_{0.15}$ indicate the need for greater adjustments in variability control. The impact of sample size is also evident, with larger sample sizes necessitating higher ratios to maintain accuracy and control. This table is crucial for understanding the required adjustments in variability relative to a reference value based on different parameter settings and confidence levels.

Figure 5 gives the flow chart for the M-GRS. This figure shows the process of multiple sampling. It begins with testing the first sample, and if the criteria are not met, additional samples are taken. The final decision is based on the results of all the samples inspected. This figure helps clarify how multiple sampling operations are managed to ensure accurate results.

Explanation and examples of the application of the tables

Example (1) Consider that an item's lifetime ($\mathcal{Z}_0$) follows the EXD model with parameters $\mathcal{B}=4$,$\gamma=3$, $\psi={}^1/_8$ and $\Omega={}^1/_4$. Let's say that light bulb makers want to determine if the 15th percentile lifetime of their product exceeds the required lifetime,$\tau_0=1000$ h with $\xi^*=75\%$ confidence level. Let's say they want to experiment for 955 h using group size with 3 items each. This results in the termination time ratio being $\mathcal{U}_{0.15}=0.955$ and Table 1

ξ^*	$\mathcal{U}_{0.15}$	$\mathcal{G}=3$					$\mathcal{G}=5$				
		$\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$									
		2	**3**	**4**	**5**	**6**	**2**	**3**	**4**	**5**	**6**
75%	0.539	0.557	0.725	1	1	1	0.51	0.691	1	1	1
	0.955	0.424	0.631	0.743	0.809	0.849	0.395	0.601	0.719	0.79	0.834
	1.255	0.232	0.45	0.595	0.69	0.755	0.214	0.439	0.576	0.667	0.731
	3.835	0.162	0.417	0.563	0.653	0.715	0.093	0.398	0.556	0.637	0.708
	5.420	0.031	0.223	0.421	0.548	0.63	0.013	0.141	0.376	0.528	0.614
90%	0.539	0.417	0.618	1	1	1	0.387	0.594	1	1	1
	0.955	0.35	0.569	0.695	0.772	0.819	0.268	0.492	0.633	0.721	0.778
	1.255	0.209	0.425	0.574	0.673	0.741	0.206	0.424	0.562	0.654	0.72
	3.835	0.119	0.352	0.495	0.591	0.655	0.018	0.166	0.332	0.452	0.539
	5.420	0.011	0.125	0.275	0.393	0.484	0.0007	0.037	0.159	0.292	0.399
95%	0.539	0.339	0.552	1	1	1	0.312	0.528	1	1	1
	0.955	0.316	0.539	0.671	0.752	0.803	0.278	0.502	0.641	0.728	0.783
	1.255	0.153	0.361	0.519	0.627	0.702	0.136	0.338	0.487	0.594	0.67
	3.835	0.046	0.199	0.347	0.466	0.558	0.015	0.144	0.297	0.414	0.502
	5.420	0.006	0.084	0.197	0.3	0.391	0.0004	0.03	0.137	0.26	0.363
99%	0.539	0.18	0.39	1	1	1	0.166	0.373	1	1	1
	0.955	0.136	0.343	0.502	0.611	0.685	0.123	0.325	0.484	0.596	0.672
	1.255	0.163	0.372	0.529	0.636	0.709	0.116	0.306	0.456	0.567	0.648
	3.835	0.032	0.154	0.29	0.41	0.508	0.005	0.075	0.181	0.282	0.373
	5.420	0.009	0.108	0.246	0.36	0.451	0.0003	0.026	0.124	0.241	0.341

Table 11. The OCF values of M- S GRSP with three-stage for the EXD model with $\mathcal{B}=4$,$\gamma=3$, $\psi={}^1/_8$ and $\Omega={}^1/_4$.

ξ^*	$\mathcal{U}_{0.15}, \mathcal{G}=3$					$\mathcal{U}_{0.15}, \mathcal{G}=5$				
	0.539	**0.955**	**1.255**	**3.835**	**5.420**	**0.539**	**0.955**	**1.255**	**3.835**	**5.420**
75%	3.01	7.8	14.89	22.9	28.1	4.29	10.4	17.2	24.3	30.14
90%	3.79	9.2	16.8	28.1	36.3	5.02	11.6	19.9	29.9	38.1
95%	5.12	13.1	19.6	27.3	38.9	6.08	13.9	20.3	28.06	39.7
99%	6.77	14.2	20.37	30.9	40.68	7.05	16.1	23.4	33.36	43.9

Table 12. The proper ratio of $\mathcal{V}_\nabla/\mathcal{V}^0_\nabla$ due to P_3.

ξ^*	$\mathcal{U}_{0.15}, \mathcal{G}=3$					$\mathcal{U}_{0.15}, \mathcal{G}=5$				
	0.539	**0.955**	**1.255**	**3.835**	**5.420**	**0.539**	**0.955**	**1.255**	**3.835**	**5.420**
75%	2.455	6.89	14.8	22.6	26.5	3.36	8.92	16.9	23.6	27.2
90%	3.095	8.04	15.1	25.6	35.6	4.12	8.99	17.18	27.6	36.9
95%	4.35	10.1	17.9	26.1	37.2	5.23	11.87	18.8	26.16	38.4
99%	5.01	12.9	19.7	29.3	38.8	6.89	14.5	21.1	31.6	40.1

Table 13. The proper ratio of $\mathcal{V}_\nabla/\mathcal{V}^0_\nabla$ due to P_4.

shows that the appropriate group needed is $\mathcal{H}=5$. The equivalent sample of size, n of 15 and distributed 3 items to 5 groups to test for 955 h for the assay limit $\mathcal{L}_\rfloor=0$. If there are no failed items during the first 955 h and the experimenter can claim that the 15th percentile is at least 1000 h, the batch is accepted. If not, halt the experiment and reject the entire lot. For the S-SGRSP ($\mathcal{G}$, $\mathcal{H}$, $\mathcal{L}_\rfloor$ and $\mathcal{U}_{0.25}$) = (3, 5, 0, 0.955) at ξ^*= 75%, the OCF values are shown in Table 2. This demonstrates that the producer's risk is around 0.409 when the genuine 15th percentile is thrice the set 15th percentile and approximately 0.17 when $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}=6$. For various combinations of ($\mathcal{G}$, $\mathcal{L}_\rfloor$, and $\mathcal{U}_{0.15}$), Table 3 can be used to determine the value of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$ such that the producer's risk is limited to 5%. To give an example, when $\mathcal{U}_{0.15}=0.955$, ξ^*= 75%, $\mathcal{L}_\rfloor=0$ and $\mathcal{G}=3$, the value of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$ is 9.403. This implies that for the batch to be approved with 75% confidence level, the product must have a 15th percentile life of 9.40 times the stated 15th percentile life.

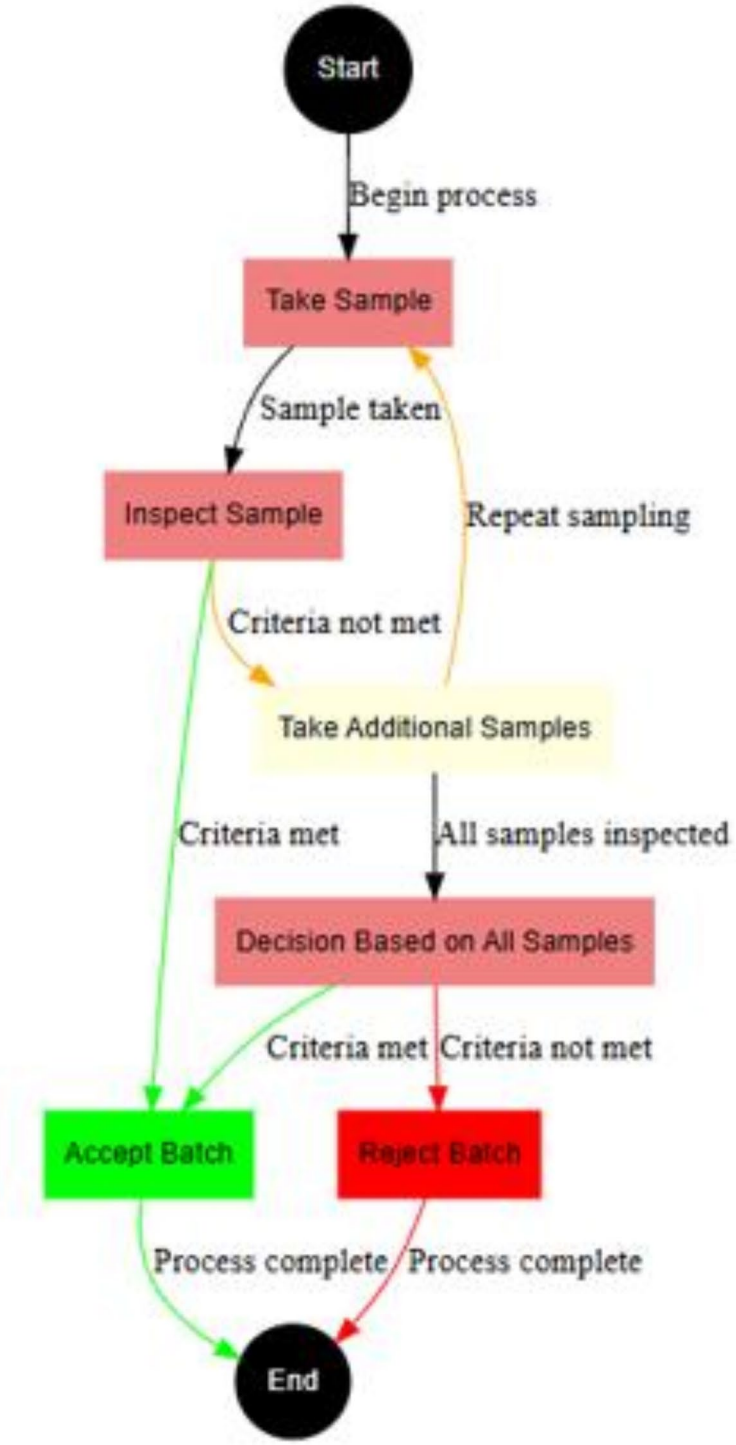

Fig. 5. The flow chart for the M-GRS.

Example (2) Consider that an item's lifetime ($\mathcal{Z}_0$) follows the EXD model with parameters $\mathcal{B} = 4$,$\gamma = 3$, $\psi = {}^1/_8$ and $\Omega = {}^1/_4$. Assume a tester wants to establish the veritable unknown 15th percentile lifetime of the product undergoing examination is at least 1000 h with $\xi^* = 75\%$ confidence level at $\mathcal{L}_{]1} = 0$ and $\mathcal{L}_{]2} = 1$. Using a group size outfitted with 3 items, a tester wishes to stop an experiment at 955 h. This information reveals that $\mathcal{U}_{0.15} = 0.955$ h. The proper number of groups needed are $\mathcal{H}_1 = 5$ and $\mathcal{H}_2 = 2$, according to Table 4. This is how the plan will be carried out: The first sample of size $\mathsf{N}_1 = 15$ is drawn, and 3 items are given to each of the 5 testers. If there are no failures during the experiment, the product is accepted. If more than one failure occurs prior to 955 h, the

product is rejected. When exactly one failure is noticed, a second sample of size $N_2 = 10$ is drawn, 3 items are distributed to each of the 2 testers, and they are all given the identical test. If there have been one or fewer failures overall, accept the product; if not, reject it. For $\mathcal{L}_{\rfloor_1} = 0$ and $\mathcal{L}_{\rfloor_2} = 1$, the OCF values for T-SGRSP under the EXD model are $(\mathcal{G}, \mathcal{H}_1, \mathcal{H}_2$ and $\mathcal{U}_{0.15}) = (3, 5, 2$ and $0.955)$ with $\xi^* = 75\%$ are shown in Table 5. The probability of accepting the batch increases up to 84.3% if the genuine 15th percentile life is 6 times the set life. Assume that the products have the EXD model with parameters $\mathcal{B} = 4, \gamma = 3, \psi = 1/8, \Omega = 1/4$ and consumers want to refuse a faulty batch with $\xi^* = 75\%$. If the T-SGRSP is predicated on the inspection limit of $\mathcal{L}_{\rfloor_1} = 0$ and $\mathcal{L}_{\rfloor_2} = 1$, what should the genuine 15th percentile life of products have been with the producer's risk of 5%. Table 6 reveals that the proper ratio of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$ is 8.20. The maker's product must therefore have a 15th percentile life of at least 820 h in order for it to be accepted with a probability of 75% under T-SGRSP.

Example (3) Suppose that the lifespan of the product follows the EXD model with $\mathcal{B} = 4$, $\gamma = 3$, $\psi = 1/8$ and $\Omega = 1/4$. It is wanted to apply M-SGRSP with three steps to inspect that the 15th percentile lifetime is at least 1000 h and the fitter wants to run a test for 955 h using testers equipped with $\mathcal{G} = 3$ items each if the consumer's risk of acceptance should be below 25%. As a result, the termination multiplier,$\mathcal{U}_{0.15} = 0.955$ and from Table 9 the proper number of groups needed are $\mathcal{H}_1 = 4$, $\mathcal{H}_2 = 2$ and $\mathcal{H}_3 = 2$. This plan will be implemented as follows: The first sample of size $N_1 = 12$ is selected and divided into 4 testers. The batch is accepted if no flawed items are discovered in phase one. The batch is inadmissible if two or more flawed items are discovered. If one flawed item is discovered, a second sample of size $N_2 = 6$ is chosen and distributed to 2 testers. The batch is accepted if the total number of flawed items in phases one and two is one or fewer. The batch is denied if the total number of flawed items is three or greater. If there are two flawed items, a third sample of size $N_3 = 6$ is towed and allocated to 2 testers. If the total number of flawed items in phases two and three is two or less. The batch is denied if the total number of flawed items is three or greater. The OCF values for M-SGRSP with three-stage under the EXD model is described by $(\mathcal{G}, \mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ and $\mathcal{U}_{0.15}) = (3, 4, 2, 2$ and $0.955)$ with $\xi^* = 75\%$ are display in Table 10. When the genuine 15th percentile life is thrice the set 15th percentile life $\left(\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 3\right)$, the corresponding producer's risk is roughly 0.369 or less, but it is approximately 0.151 when $\left(\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 6\right)$. If we must determine the ratio related to the producer's risk $\tau = 0.05$, we are able to locate it in Table 14. As an illustration, when $(\mathcal{G}, \mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ and $\mathcal{U}_{0.15}) = (3, 4, 2, 2$ and $0.955)$ with $\xi^* = 75\%$, the ratio of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$ is 7.80. This indicates that the item must have a 15th percentile life of at least 7.80 times the specified 15th percentile life of 1000 h to allow the batch to be accepted with a probability of 75%.

Example (4) Let $\mathcal{Z}_0$ be a random variable describing the product's lifetime and have an EXD model with shape parameters $\mathcal{B} = 4$, $\gamma = 3$, $\psi = 1/8$ and $\Omega = 1/4$. Additionally, assume that a fitter would like to implement the proposed M-SGRSP with four steps to establish the genuine 15th percentile lifetime for the product is at least 1000 h and the experiment will be halted at 955h using group size with three items each $(\mathcal{G} = 3)$ with consumer's error $1 - \xi^* = 0.25$ under assay limits $\mathcal{L}_{\rfloor_1} = 0$, $\mathcal{L}_{\rfloor_2} = 1$, $\mathcal{L}_{\rfloor_3} = 2$, $\mathcal{L}_{\rfloor_4} = 3$ and rejection limits $\mathcal{R}_{e_1} = 2$, $\mathcal{R}_{e_2} = 3$, $\mathcal{R}_{e_3} = \mathcal{R}_{e_4}$

ξ^*	$\mathcal{U}_{0.15}$	$\mathcal{G} = 3$					$\mathcal{G} = 5$				
		$\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$									
		2	3	4	5	6	2	3	4	5	6
75%	0.539	0.618	0.768	1	1	1	0.582	0.743	1	1	1
	0.955	0.461	0.659	0.764	0.826	0.863	0.343	0.559	0.686	0.765	0.813
	1.255	0.261	0.472	0.612	0.703	0.765	0.227	0.467	0.606	0.693	0.753
	3.835	0.169	0.432	0.574	0.66	0.721	0.034	0.231	0.433	0.564	0.649
	5.420	0.045	0.286	0.506	0.629	0.7	0.01	0.127	0.358	0.562	0.698
90%	0.539	0.442	0.639	1	1	1	0.419	0.62	1	1	1
	0.955	0.392	0.605	0.723	0.794	0.837	0.277	0.5	0.639	0.727	0.782
	1.255	0.251	0.462	0.604	0.697	0.76	0.227	0.47	0.61	0.698	0.757
	3.835	0.126	0.354	0.496	0.593	0.664	0.026	0.195	0.387	0.523	0.615
	5.420	0.013	0.142	0.311	0.435	0.526	0.002	0.059	0.22	0.385	0.505
95%	0.539	0.353	0.565	1	1	1	0.343	0.556	1	1	1
	0.955	0.337	0.558	0.686	0.764	0.813	0.335	0.553	0.681	0.76	0.81
	1.255	0.209	0.422	0.572	0.671	0.739	0.175	0.386	0.527	0.626	0.697
	3.835	0.066	0.226	0.364	0.476	0.564	0.018	0.165	0.325	0.438	0.523
	5.420	0.011	0.13	0.291	0.413	0.505	0.002	0.059	0.22	0.385	0.505
99%	0.539	0.203	0.416	1	1	1	0.190	0.402	1	1	1
	0.955	0.194	0.395	0.549	0.652	0.72	0.162	0.377	0.533	0.638	0.708
	1.255	0.174	0.403	0.555	0.657	0.728	0.16	0.373	0.516	0.617	0.689
	3.835	0.054	0.184	0.311	0.425	0.519	0.014	0.147	0.299	0.412	0.498
	5.420	0.01	0.122	0.283	0.411	0.507	0.002	0.059	0.22	0.385	0.505

Table 14. The OCF values of M- S GRSP with four-stage for the EXD model with $\mathcal{B} = 4, \gamma = 3, \psi = 1/8$ and $\Omega = 1/4$.

= 4. As a result, $\mathcal{U}_{0.15} = 0.955$. Table 9 shows the appropriate number of four-stage group sampling plans for the problem at hand, with $\mathcal{H}_1$= 4, $\mathcal{H}_2$= 2, $\mathcal{H}_3$ = 2 and $\mathcal{H}_4$ = 2. This approach is implemented as follows: From the batch, an initial sample of size $\mathsf{N}_1 = 12$ is drawn and divided into 4 testers. If no faulty products are discovered during the first phase, the batch is accepted. If two or more defective items are discovered, the batch is denied. If one flawed item is discovered, a second sample of $\mathsf{N}_2 = 6$ is taken and allocated to 2 testers. The batch is accepted if the total number of flawed items in phases one and two is one or fewer. If there are three or more flawed items in the batch, the batch is denied. If there are two flawed items, a third sample of size $\mathsf{N}_3 = 6$ is picked and distributed to 2 testers. If the total number is two or less in phases two and three, the batch is accepted. If the total number of flawed items is four or more, the batch is denied. If the total number of flawed items is three, a fourth sample of size $\mathsf{N}_4 = 6$ is towed and assigned to 2 testers. If the total number of flaws is three or fewer in phases three and four, the batch is accepted. Otherwise, the batch is denied. For $\mathcal{L}_{\rfloor_1} = 0$, $\mathcal{L}_{\rfloor_2}$= 1, $\mathcal{L}_{\rfloor_3}$= 2 and $\mathcal{L}_{\rfloor_4}$= 3, the OCF values for M-SGRSP with four steps under the EXD model is ($\mathcal{G}$,$\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$, $\mathcal{H}_4$ and $\mathcal{U}_{0.15}$) = (3, 4, 2, 2, 2 and 0.955) with $\xi^* = 75\%$ are display in Table 11.

We observe that the related producer's risk is about 0.341 if the genuine 15th percentile lifetime is thrice the set lifespan $\left(\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 3\right)$, and approximately 0.137 when $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 6$. For $\mathcal{L}_{\rfloor_1} = 0$, $\mathcal{L}_{\rfloor_2}$= 1,$\mathcal{L}_{\rfloor_3}$= 2, $\mathcal{L}_{\rfloor_4}$ = 2 and $\mathcal{R}_{e_1}$= 2, $\mathcal{R}_{e_2}$= 3, $\mathcal{R}_{e_3}$= $\mathcal{R}_{e_4}$= 4 and hence of $\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$, $\mathcal{H}_4$. Table 12 illustrates the proper ratio of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.25}$ to keep the producer's risk below 5%. Thus, for $\mathcal{G} = 3$, $\mathcal{U}_{0.15} = 0.955$ and ξ^*= 75% the value of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$ is 6.89; indicating that the item must have a 15th percentile lifespan of at least 6.89 times the set 15th percentile lifespan of 1000 h in order for the batch to be accepted with probability 75%. Generally, we can highlight the following results:

I. Based on the first application, when ($\mathcal{G}$,$\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$ and $\mathcal{U}_{0.15}$) = (3, 4, 2, 2 and 0.955)$|\xi^* = 75\%$, the ratio of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$ is 7.80, which means that the item must have a 15th percentile life of at least 7.80 times the specified 15th percentile life of 1000 h in order to allow the batch to be accepted with a probability of 75%.
II. Due to the second application and assuming that the products have the EXD model with parameters $\mathcal{B}$ = 4,$\gamma = 3$, $\psi = 1/8$, $\Omega = 1/4$. and consumers want to refuse a faulty batch with ξ^*= 75% and $\mathcal{L}_{\rfloor_1} = 0$ and $\mathcal{L}_{\rfloor_2}$= 1. Then, Table 6 reveals that the proper ratio of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$ is 8.20. The maker's product must therefore have a 15th percentile life of at least 820h in order for it to be accepted with a probability of 75% under T-SGRSP.
III. According to the third application, when ($\mathcal{G}$,$\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$ and $\mathcal{U}_{0.15}$) = (3, 4, 2, 2 and 0.955)$|\xi^* = 75\%$, the ratio of $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$ is 7.80. This result means that the item must have a 15th percentile life of at least 7.80 times the specified 15th percentile life of 1000 h in order to allow the batch to be accepted with a probability of 75%.
IV. In view of the fourth application, it is observed that the related producer's risk is about 0.341 if the genuine 15th percentile lifetime is thrice the set lifespan $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 3$, and approximately 0.137 when $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15} = 6$. For $\mathcal{L}_{\rfloor_1} = 0$, $\mathcal{L}_{\rfloor_2}$= 1,$\mathcal{L}_{\rfloor_3}$= 2, $\mathcal{L}_{\rfloor_4}$= 2 and $\mathcal{R}_{e_1}$= 2, $\mathcal{R}_{e_2}$= 3, $\mathcal{R}_{e_3}$= $\mathcal{R}_{e_4}$= 4 and hence of $\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$, $\mathcal{H}_4$.

Comparisons of one-stage and iterative GRSP

The important goal of iterative sampling is to reduce the estimated sample size, which is particularly important when working with extreme lot quality, such as very good or very poor. By iteratively improving the sampling procedure, the iterative technique aims to strike the ideal balance between sample size and dependability evaluation. Comparing the AVGN functions of iterative and single-stage plans is a crucial component of this strategy to make sure they produce about the same OCF. This makes it easier to compare quantitatively, which helps stakeholders decide which sampling approach is best. We examine the S-SGRSP, M-SGRSP, and Two-Stage Group Sampling Plan (T-SGRSP) in comparison in this section. The evaluation of the AVGN and OCF values connected to each kind of group sampling plan is the main focus of this comparison. In the event, for example, that the product lifespan is following an EXD, the next course of action is to consider selecting an S-SGRSP, T-SGRSP, or M-SGRSP. Here, the main goal is to determine the sample plan that balances resource efficiency and dependability assessment by optimizing the OCF and minimising the AVGN. Stakeholders can learn more about the relative benefits and trade-offs of each sampling technique in the context of their unique quality control requirements by carrying out these comparisons. This empirical analysis aids in selecting the most appropriate sampling plan tailored to the unique characteristics of the product and the desired level of quality assurance. Ultimately, the goal is to optimize the sampling process to effectively mitigate risks associated with extreme lot qualities while ensuring efficient resource utilization. Here we compare type of proposed plans for $\mathcal{U}_{0.15}$= 0.955, ξ^*= 75% and 15th percentile lifetime quality level $\mathcal{V}_\nabla/\mathcal{V}^0_\nabla = 3$. Table 15 provides the values of OCF and AVSN for S-SGRSP, T-SGRSP and M-SGRSP, where the presumed group size was set as $\mathcal{G} = 4$.

It has been witnessed from the table that the multiple GRSPs with four-stage (i.e. M-SGRSP) are better than the S-SGRSP, two-stage GRSPs and three- GRSPs in terms of AVGN = 11.9, OCF = 0.659 values and $\mathcal{V}_{0.15}/\mathcal{V}^0_{0.15}$. Nevertheless, there are a number of disadvantages to multiple sampling, including expensive expenses, difficult administrative tasks, a complex system requiring a lot of administrative work, inspectors' incapacity to implement the algorithm correctly, and administrative difficulties. The OCF increased from 0.591$|\mathcal{L}_\rfloor = 0$ under the single- stage to 0.659 under the four- stage. On the other hand, the AVGN decreased from 15.0$|\mathcal{L}_\rfloor = 0$ under the single- stage to 11.9 under the four- stage. However, the $\mathcal{V}_\nabla/\mathcal{V}^0_\nabla$ decreased from 9.40$|\mathcal{L}_\rfloor = 0$ under the single- stage to 6.89.

Conclusion and discussion

This research introduces and evaluates a range of group sampling inspection plans specifically designed to assess the batch acceptance of manufactured parts subjected to amputated life-tests. Amputated life-testing is

Type of plan	OCF	AVGN	$\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$
Single- stage with $\mathcal{L}_\rfloor = 0$	0.591	15.0	9.40
Two- stage	0.618	14.2	8.20
Three- stage	0.631	13.2	7.80
Four- stage	0.659	11.9	6.89

Table 15. Summarizing of OCF, AVGN and $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ values for types of GRSPs.

a method commonly used by manufacturers to accelerate testing procedures without sacrificing accuracy. The study investigates several types of sampling plans, including single-stage, two-stage, and multiple-stage group inspection plans, with a particular emphasis on three-stage plans. The focus is on determining the optimal group sizes for these sampling plans, especially for distributions that follow the Extended Dagum (EXD) distribution with known shape parameters. The study addresses various consumer risk levels and other parameters to ensure the accuracy and reliability of the testing process. It evaluates the Operating Characteristic Function (OCF) values and the associated producer risk for these plans, providing a thorough analysis of their performance. Additionally, the research establishes the minimum sample sizes required for the first, second, and third samples to meet specified mean and median lifetimes at a predetermined confidence level for customers. This includes a detailed examination of the operational characteristic values and producer risk associated with the proposed plans. The design of the two-stage and multiple group sampling inspection plans involves solving a nonlinear optimization problem aimed at minimizing the average number of groups while satisfying constraints related to acceptance probabilities. The study finds that multiple group sampling plans are generally preferred over single-stage and two-stage plans due to their ability to reduce the number of groups needed, which enhances efficiency and effectiveness. Numerical cases and tables are presented to illustrate the proposed plans, providing a clear and practical understanding of their implementation and effectiveness in real-world scenarios. This comprehensive analysis aids in selecting and applying the most suitable sampling plans for ensuring high-quality manufacturing and accurate testing outcomes.

In this work, the design of the single, double, and multiple stage group inspection plans is presented with their corresponding mathematical algorithms. For the single-stage group inspection plans, some experimental results are presented to show the importance and usage of the stage group inspection plans under ξ^*= 75%, 90%, 95% 99% and $\mathcal{U}_\nabla = 0.539, 0.955, 2.255, 3.835, 5.420$. For the two stage group inspection plans, the PNGs are determined under some critical values, also the OCF values and the optimal values of the ratio $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ are calculated under a certain value of the corresponding parameters values. The quality control table for multiple stage group inspection plans with three and four steps are presented. Also, the OCF values and the optimal values of the ratio $\mathcal{V}_\nabla/\mathcal{V}_\nabla^0$ are calculated under certain corresponding parameters for three and four stage group inspection plans. Furthermore, we presented four applications to illustrate the importance and the applicability of the single, double, and multiple stage group inspection plans under the EXD model. Finally, an applicable comparison of the one stage and the iterative group sampling plans with some numerical illustrations is presented.

In the current study, we operate under the assumption of fixed sample sizes within each group, which simplifies the analysis and maintains consistency across groups. However, if we were to consider dynamic group sizes, incorporating a Markov property could indeed provide a useful framework for modeling the success or failure of each group. The Markov property, which relies on the idea that future states depend only on the current state and not on the sequence of events that preceded it, would allow us to map the progression of each group's sampling outcomes over time. This could offer a more adaptive approach, where the sampling strategy could adjust based on the observed success or failure rates within the groups. The following issues and future points could be addressed:

I. Extending the sampling strategy to a random walk is an intriguing idea, especially when aiming to achieve comprehensive coverage of the population. A random walk approach could allow the sampling process to move more freely across the population, potentially providing better coverage and reducing the risk of sampling bias. This would be particularly useful in scenarios where the population is heterogeneous, and certain subpopulations might be underrepresented with fixed sampling strategies.
II. While both approaches offer potential benefits, they also introduce additional complexity. The implementation of dynamic group sizes with a Markovian framework would require careful consideration of the transition probabilities and the criteria for adjusting group sizes. Similarly, a random walk approach would need to ensure that the sampling process remains efficient, and that the entire population is adequately represented without over-sampling certain areas. In summary, while our current model focuses on fixed sample sizes for simplicity and consistency, your suggestion to explore dynamic group sizes with a Markov property, and potentially extend sampling to a random walk, opens up exciting avenues for future research. These approaches could provide more flexible and adaptive sampling strategies, particularly in complex or heterogeneous populations. We appreciate your suggestion and will consider these possibilities in the context of further studies.
III. The EXD model introduced may involve complex parameterization, making it challenging to interpret results and apply the model in practice. Future studies could focus on simplifying the model or developing user-friendly tools for easier interpretation and application.

IV. The methods for determining optimal sample sizes might rely on specific assumptions or conditions that may not hold in all practical scenarios. Future studies could investigate adaptive methods for sample size determination that can handle varying conditions and provide more robust results.
V. The analysis of termination time ratios may not fully account for all practical factors, such as operational constraints or variations in inspection processes. Future studies could extend the analysis to incorporate additional real-world factors and constraints to enhance the practical applicability of the results.
VI. The proposed methods and results might be limited to specific types of group inspection plans or data distributions. Future studies could validate the methods across different types of inspection plans and datasets to ensure broader applicability and robustness.
VII. The methods may not adequately address uncertainty and variability in the data, potentially affecting the reliability of the results. Future studies could incorporate techniques to better handle uncertainty and variability, such as Bayesian approaches or robust optimization methods. The paper may not provide a comprehensive comparison with existing methods or alternative approaches, making it difficult to assess the relative advantages and limitations. Conduct detailed comparative studies with other established methods to highlight the strengths and limitations of the proposed approach.
VIII. The computational requirements for implementing the proposed methods might be high, limiting their practical use in large-scale applications. Future studies could explore methods to improve computational efficiency and scalability, potentially through algorithmic enhancements or approximations.
IX. The sensitivity of the results to different parameter values and assumptions may not be thoroughly examined. Future studies could perform sensitivity analyses to assess how changes in model parameters and assumptions affect the results and conclusions.
X. The methods may lack practical guidance for implementation in real scenarios, such as specific procedural steps or guidelines. Future studies could develop practical guidelines and case studies to facilitate the implementation of the proposed methods in various industrial settings.
XI. Addressing these shortcomings can lead to significant improvements in the proposed methods and enhance their applicability and reliability. Future versions of the study should aim to refine the model, expand its applicability, and provide practical solutions to the identified limitations. Thank you for considering these aspects, and we will incorporate these insights into future research to strengthen the overall contribution of our work.

Data availability

The data is involved in the paper.

Received: 27 April 2024; Accepted: 30 September 2024
Published online: 15 October 2024

References

1. Vlcek, B. L., Hendricks, R. C. & Zaretsky, E. V. Monte Carlo simulation of sudden death bearing testing. *Tribol. Trans.* **47**(2), 188–199 (2004).
2. Balasooriya, U. Failure–censored reliability sampling plans for the exponential distribution. *J. Stat. Comput. Simul.* **52**(4), 337–349 (1995).
3. Pascual, F. G. & Meeker, W. Q. The modified sudden death test: planning life-tests with a limited number of test positions. *J. Test. Eval.* **26**(5), 434–443 (1998).
4. Jun, C. H., Balamurali, S. & Lee, S. H. Variable sampling plans for Weibull distributed lifetimes under sudden death testing. *IEEE Trans. Reliab.* **55**(1), 53–58 (2006).
5. Aslam, M., Jun, C. H. & Ahmad, M. A group sampling plan based on truncated life-test for gamma distributed items. *Pakistan J. Stat.* **25**(3), 333–340 (2009).
6. Aslam, M. & Jun, C. H. A group acceptance sampling plans for truncated life-tests based on the inverse Rayleigh and log-logistic distributions. *Pakistan J. Stat.* **25**(2), 107–119 (2009).
7. Aslam, M. & Jun, C. H. A group acceptance sampling plan for truncated life-test having Weibull distribution. *J. Appl. Stat.* **36**(9), 1021–1027 (2009).
8. Rao, G. S. A group acceptance sampling plans for lifetimes following a Marshall-Olkin extended exponential distribution. *Appl. Appl. Math. Int. J. (AAM)* **6**(2), 13 (2011).
9. Rao, G. S. A group acceptance sampling plans for lifetimes following a generalized exponential distribution. *Econ. Qual. Control* **24**(1), 75–85 (2009).
10. Rao, G. S. A group acceptance sampling plans based on truncated life-tests for Marshall-Olkin extended Lomax distribution. *Elect. J. Appl. Stat. Anal.* **3**(1), 18–27 (2010).
11. Aslam, M., Jun, C. H. & Ahmad, M. A two-stage group sampling plan based on truncated life-tests for a general distribution. *J. Stat. Comput. Simul.* **81**(12), 1927–1938 (2011).
12. Aslam, M., Kundu, D., Jun, C. H. & Ahmad, M. Time truncated group acceptance sampling plans for generalized exponential distribution. *J. Test. Eval.* **39**(4), 671–677 (2011).
13. Mughal, A. R. & Ismail, M. An economic reliability efficient group acceptance sampling plans for family Pareto distributions. *Res. J. Appl. Sci. Eng. Technol* **6**(24), 4646–4652 (2013).
14. Mughal, A. R., Zain, Z. & Aziz, N. Time truncated group chain sampling strategy for pareto distribution of the 2nd kind. *Res. J. Appl. Sci. Eng. Technol.* **10**(4), 471–474 (2015).
15. Rao, B. S., Kumar, C. & Rosaiah, K. Group acceptance sampling plans for life-tests based on Half Normal distribution. *Sri Lankan J. Appl. Stat.* **15**(3), 221–231 (2014).
16. Rao, G. S., Rosaiah, K., Babu, M. S., & Kumar, D. S. A two-stage group sampling plan based on truncated life-tests for a exponentiated Fréchet distribution. *Eur. Sci. J.* **10**(33). (2014).
17. Rosaiah, K., Rao, G. & Prasad, S. V. S. V. S. V. A group acceptance sampling plans based on truncated life-tests for Type-II generalized log-logistic distribution. *In Prob Stat Forum* **9**, 88–94 (2016).
18. Yiğiter, A., Hamurkaroğlu, C., & Danacıoğlu, N. Group acceptance sampling plans based on time truncated life-tests for compound Weibull-exponential distribution. *Int. J. Qual. Reliab. Manag.* (2021).

19. Almarashi, A. M., Khan, K., Chesneau, C. & Jamal, F. Group acceptance sampling plan using Marshall-Olkin kumaraswamy exponential (MOKw-E) distribution. *Processes*. **9**, 1066 (2021).
20. Aziz, N., Zain, Z., Jamaludin, A. F. & Hashim, E. Time truncated modified group chain sampling plans for marshall olkin extended lomax distribution. *Int. J. Adv. Sci. Technol.* **29**(6s), 1770–1777 (2020).
21. Ali, M. M., Ali, I., Yousof, H. M. & Ibrahim, M. *G families of probability distributions: theory and practices* (CRC Press, Taylor & Francis Group, 2022).
22. Saber, M. M., Hamedani, G. G., Yousof, H. M. But, N. S., Ahmed, B. & Yousof, H. M. *A family of continuous probability distributions: theory, characterizations, properties and different copulas*. (CRC Press, Taylor & Francis Group, 2022)
23. Ahmed, B. & Yousof, H. A new group acceptance sampling plans based on percentiles for the Weibull Fréchet model. *Stat. Optim. Inform. Comput.* **11**(2), 409–421 (2023).
24. Tashkandy, Y., Emam, W., Ali, M. M., Yousof, H. M. & Ahmed, B. Quality control testing with experimental practical illustrations under the modified lindley distribution using single, double, and multiple acceptance sampling plans. *Mathematics*. **11**(9), 2184 (2023).
25. Mohamed, H. S., Ali, M. M. & Yousof, H. M. The lindley gompertz model for estimating the survival rates: properties and applications in insurance. *Ann. Data Sci*. https://doi.org/10.1007/s40745-022-00451-3 (2022).
26. Mohamed, H. S., Cordeiro, G. M., Minkah, R., Yousof, H. M. & Ibrahim, M. A size-of-loss model for the negatively skewed insurance claims data: applications, risk analysis using different methods and statistical forecasting. *J. Appl. Stat.* forthcoming. (2022b).
27. Mohamed, H. S., Cordeiro, G. M. & Yousof, H. M. The synthetic autoregressive model for the insurance claims payment data: modeling and future prediction. *Stat. Optim. Info. Comput.* forthcoming. (2022c).
28. Liaqat, M., Saeed, N., Saleem, K., Aslam, M., & Sherwani, R. A. K. A modified group chain sampling plan for lifetime following kumaraswamy generalized power weibull distribution with minimum angle approach. Scientia Iranica. (2023).
29. Teh, M. A. P., Aziz, N. & Zain, Z. New group chain acceptance sampling plans (NGCHSP-1) using minimum angle method for generalized exponential distribution. *JSM* **50**(4), 1121–1129 (2021).
30. Rha, H., Kao, M. H. & Pan, R. Design optimal sampling plans for functional regression models. *Comput. Stat. Data Anal.* **146**, 106925 (2020).
31. Wu, C. W., Darmawan, A. & Liu, S. W. Stage-independent multiple sampling plan by variables inspection for lot determination based on the process capability index Cpk. *Int. J. Prod. Res.* **61**(10), 3171–3183 (2023).
32. Fayomi, A. & Khan, K. A group acceptance sampling plan for 'another generalized transmuted exponential distribution'based on truncated lifetimes. *Qual. Reliab. Eng. Int.* **40**(1), 145–153 (2024).
33. Marques, R. A. M., Maciel, A. C., Costa, A. F. B. & Santos, K. R. D. S. The design of the mixed repetitive sampling plans based on the Cpk index. *Int. J. Qual. Reliab. Manage.* **41**(2), 674–697 (2024).
34. Hussain, N. et al. An acceptance sampling plan for the odd exponential-logarithmic Fréchet distribution: applications to quality control data. *Cogent Engineering* **11**(1), 2304497 (2024).
35. Narayan, A. et al. UncertainSCI: uncertainty quantification for computational models in biomedicine and bioengineering. *Comput Biol Med.* **152**, 106407. https://doi.org/10.1016/j.compbiomed.2022.106407 (2023).
36. Loizidou, K., Elia, R. & Pitris, C. Computer-aided breast cancer detection and classification in mammography: a comprehensive review. *Comput Biol Med.* **153**, 106554. https://doi.org/10.1016/j.compbiomed.2023.106554 (2023).
37. Tripathi, H. & Saha, M. Modified chain group sampling inspection plan under item failure scenario based on time truncated scheme. *IJSAEM* **15**(3), 1305–1314 (2024).
38. Liu, S. W., & Wu, C. W. An efficient partial sampling inspection for lot sentencing based on process yield. Ann. Oper. Res, 1–20. (2023).
39. Hou, D., Zhou, W., Zhang, Q., Zhang, K. & Fang, J. A comparative study of different variable selection methods based on numerical simulation and empirical analysis. *PeerJ Comput. Sci.* **9**, e1522 (2023).
40. Zhao, A. & Yu, Y. Multi-view computable online learner modeling by heterogeneous network: an AI-enabled perspective. *Info. Sci.* **645**, 119363 (2023).
41. Yousof, H. M., Afify, A. Z., Abd El Hadi, N. E., Hamedani, G. G., & Butt, N. S. On six-parameter Fréchet distribution: properties and applications. *PJSOR*, 281–299 (2016).
42. Yousof, H. M., Altun, E., & Hamedani, G. G. A new extension of fréchet distribution with regression models, residual analysis and characterizations. *J. Data Sci.* **16** (4)(2018).
43. Mansour, M. M. et al. A new log-logistic lifetime model with mathematical properties, copula, modified goodness-of-fit test for validation and real data modeling. *Mathematics* **8**(9), 1508 (2020).
44. Mansour, M. M. et al. A new exponentiated Weibull distribution's extension: copula, mathematical properties and applications. *Contrib. Math.* **1**(2020), 57–66. https://doi.org/10.47443/cm.2020.0018 (2020).
45. Mansour, M. et al. A generalization of the exponentiated Weibull model with properties, Copula and application. *Eurasian Bull. Math.* **3**(2), 84–102 (2020).
46. Mansour, M. et al. A new parametric life distribution with modified bagdonavičius-nikulin goodness-of-fit test for censored validation, properties, applications, and different estimation methods. *Entropy* **22**(5), 592 (2020).
47. Mansour, M., Yousof, H. M., Shehata, W. A. M. & Ibrahim, M. A new two parameter Burr XII distribution: properties, copula, different estimation methods and modeling acute bone cancer data. *J. Nonlinear Sci. Appl.* **13**(5), 223–238 (2020).
48. Mansour, M. M., Butt, N. S., Yousof, H. M., Ansari, S. I. & Ibrahim, M. A generalization of reciprocal exponential model: clayton copula, statistical properties and modeling skewed and symmetric real data sets. *Pakistan J. Stat. Oper. Res.* **16**(2), 373–386 (2020).
49. Korkmaz, M. Ç., Altun, E., Yousof, H. M., Afify, A. Z. & Nadarajah, S. The Burr X pareto distribution: properties, applications and VaR estimation. *J. Risk Financ. Manage.* **11**(1), 1 (2018).
50. Korkmaz, M. Ç. et al. The Weibull Marshall-Olkin family: regression model and application to censored data. *Commun. Stat. Theory Methods* **48**(16), 4171–4194 (2019).
51. Korkmaz, M. Ç., Yousof, H. M. & Ali, M. M. Some theoretical and computational aspects of the odd Lindley Fréchet distribution. *İstatistikçiler Dergisi: İstatistik ve Aktüerya* **10**(2), 129–140 (2017).
52. Korkmaz, M. C., Altun, E., Chesneau, C. & Yousof, H. M. On the unit-Chen distribution with associated quantile regression and applications. *Math. Slovaca* **72**(3), 765–786 (2022).
53. Elgohari, H., & Yousof, H. A generalization of lomax distribution with properties, copula and real data applications. *Pakistan J. Stat. Oper. Res.* 697–711.(2020).
54. Rasekhi, M., Altun, E., Alizadeh, M. & Yousof, H. M. The odd log-logistic weibull-g family of distributions with regression and financial risk models. *J. Oper. Res. Soc. China* **10**(1), 133–158 (2022).
55. Rasekhi, M., Saber, M. M. & Yousof, H. M. Bayesian and classical inference of reliability in multicomponent stress-strength under the generalized logistic model. *Commun. Stat. Theory Methods* **50**(21), 5114–5125 (2020).
56. Hamed, M. S., Cordeiro, G. M. & Yousof, H. M. A new compound lomax model: properties, copulas, modeling and risk analysis utilizing the negatively skewed insurance claims data. *Pakistan J. Stat. Oper. Res.* **18**(3), 601–631 (2022).
57. Shehata, W. A. M. & Yousof, H. M. A novel two-parameter Nadarajah-Haghighi extension: properties, copulas, modeling real data and different estimation methods. *Stat. Optim. Info. Comput.* **10**(3), 725–749 (2022).
58. Shehata, W. A. M. & Yousof, H. M. The four-parameter exponentiated Weibull model with Copula, properties and real data modeling. *Pakistan J. Stat. Oper. Res.* **17**(3), 649–667 (2021).

59. Shehata, W. A. M., Yousof, H. M. & Aboraya, M. A novel generator of continuous probability distributions for the asymmetric left-skewed bimodal real-life data with properties and copulas. *Pakistan J. Stat. Oper. Res.* **17**(4), 943–961. https://doi.org/10.18187/pjsor.v17i4.3903 (2021).
60. Shehata, W. A. M., Butt, N. S., Yousof, H. & Aboraya, M. A new lifetime parametric model for the survival and relief times with copulas and properties. *Pakistan J. Stat. Oper. Res.* **18**(1), 249–272 (2022).
61. Shrahili, M., Elbatal, I. & Yousof, H. M. Asymmetric density for risk claim-size data: prediction and bimodal data applications. *Symmetry* **13**, 2357 (2021).
62. Alizadeh, M. et al. The odd log-logistic Topp-Leone G family of distributions: heteroscedastic regression models and applications. *Comput. Stat.* **33**(3), 1217–1244 (2018).
63. Alizadeh, M., Rasekhi, M., Yousof, H. M. & Hamedani, G. G. The transmuted Weibull G family of distributions. *Hacettepe J. Math. Stat.* **47**(6), 1–20 (2018).
64. Alizadeh, M., Rasekhi, M., Yousof, H. M., Ramires, T. G. & Hamedani, G. G. Extended exponentiated Nadarajah-Haghighi model: Mathematical properties, characterizations and applications. *Studia Scientiarum Mathematicarum Hungarica* **55**(4), 498–522 (2018).
65. Alizadeh, M., Yousof, H. M., Rasekhi, M. & Altun, E. The odd log-logistic Poisson-G Family of distributions. *J. Math. Ext.* **12**(1), 81–104 (2018).
66. Alizadeh, M., Jamal, F., Yousof, H. M., Khanahmadi, M. & Hamedani, G. G. Flexible Weibull generated family of distributions: characterizations, mathematical properties and applications. *Univ. Politehnica Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **82**(1), 145–150 (2020).
67. Alizadeh, M., Yousof, H. M., Jahanshahi, S. M. A., Najibi, S. M. & Hamedani, G. G. The transmuted odd log-logistic-G family of distributions. *J. Stat. Manage. Syst.* **23**(4), 1–27 (2020).
68. Alizadeh, M., Rasekhi, M., Yousof, H. M., Hamedani, G. & Ataei, A. The odd log-logistic transmuted-G family of distributions: properties, characterization, applications and different methods of estimation. *Stat. Optim. Inform. Comput.* **10**(3), 904–924 (2022).
69. Hashempour, M., Alizadeh, M. and Yousof, H. M. (2023). A new lindley extension: estimation, risk assessment and analysis under bimodal right skewed precipitation data. *Ann. Data Sci.* 1–40. (2023).
70. Elbatal, I. et al. A new losses (revenues) probability model with entropy analysis, applications and case studies for value-at-risk modeling and mean of order-P analysis. *AIMS Math.* **9**(3), 7169–7211 (2024).
71. Aljadani, A., Mansour, M. M. & Yousof, H. M. A novel model for finance and reliability applications: theory, practices and financial peaks over a random threshold value-at-risk analysis. *Pak. J. Stat. Oper. Res.* **20**(3), 489–515 (2024).
72. Alizadeh, M., Afshari, M., Contreras-Reyes, J. E., Mazarei, D. & Yousof, H. M. The extended gompertz model applications, mean of order P assessment and statistical threshold risk analysis based on extreme stresses data. *IEEE Trans. Reliab.* https://doi.org/10.1109/TR.2024.3425278 (2024).
73. Schilling, E. G. & Neubauer, D. V. *Acceptance Sampling in Quality Control*. (Chapman and Hall/CRC, 2009).
74. Aslam, M., Azam, M. & Jun, C. Multiple dependent state repetitive group sampling plan for Burr XII distribution. *Qual. Eng.* **28**(2), 231–323 (2016).
75. Goode, H.P. & Kao, J.H.K. Sampling plans based on the Weibull distribution. In *Proceedings of Seventh National Symposium on Reliability and Quality Control*, 24–40 (1961).
76. Fertig, K. W. & Mann, N. R. Life-test sampling plans for two-parameter Weibull populations. *Technometrics* **22**(2), 165–177 (1980).
77. Gupta, S. S. & Groll, P. A. Gamma distribution in acceptance sampling based on life-test. *J. Am. Stat. Assoc.* **56**(296), 942–970 (1961).
78. Kantam, R. R. L. & Rosaiah, K. Half logistic distribution in acceptance sampling based on life-tests. *IAPQR Trans.* **23**(2), 117–125 (1998).
79. Balakrishnan, N., Lieiva, V. & López, J. Acceptance sampling plans from truncated life-tests based on the generalized Birnbaum-Saunders distribution. *Commun. Stat. Simul. Comput.* **36**(3), 643–656 (2007).
80. Tsai, T. & Wu, S. Acceptance sampling based on truncated life-tests for generalized Rayleigh distribution. *J. Appl. Stat.* **33**(6), 595–600 (2006).
81. Lio, Y. L., Tsai, T. R. & Wu, S. H. Acceptance sampling plans from truncated life-tests based on the Burr Type XII percentiles. *J. Chin. Inst. Indus. Eng.* **27**(4), 270–280 (2010).
82. Gui, W. & Aslam, M. Acceptance sampling plans based on truncated life-tests for weighted exponential distribution. *Commun. Stat. Simul. Comput.* **46**(3), 2138–2151 (2017).
83. Ahmed, B., Ali, M. M. & Yousof, H. M. A Novel G Family for Single Acceptance Sampling Plan with Application in Quality and Risk Decisions. Annals of Data Science, forthcoming. *Stat. Optim. Inform. Comput.* (2022).
84. Aslam, M., Jun, C. H., Rasool, M. & Ahmad, M. A time truncated two-stage group sampling plan for Weibull distribution. *Commun. Korean Stat. Soc.* **17**(1), 89–98 (2010).
85. Mughal, A. R., Hanif, M., Imran, A. A., Rafi, M. & Ahmad, M. Economic reliability two-stage group sampling plan for truncated life-test having weibull distribution. *Eur. J. Sci. Res.* **54**(4), 593–599 (2011).
86. Rao, G. S. A two-stage group sampling plan based on truncated life-tests for a MO extended exponential distribution. *Int. J. Qual. Eng. Technol.* **3**(4), 319–331 (2013).
87. Aslam, M., Srinivasa Rao, G. & Khan, N. Single-stage and two-stage total failure-based group-sampling plans for the Weibull distribution under neutrosophic statistics. *Complex Intell. Syst.* **7**(2), 891–900 (2021).
88. Azam, M., Aslam, M., Balamurali, S. & Javaid, A. Two stage group acceptance sampling plan for half normal percentiles. *J King Saud Univ. Sci.* **27**(3), 239–243 (2015).
89. Rao, B. S. & Rao, G. S. A two-stage group acceptance sampling plan based on life-tests for half logistic distribution. *Model. Assist. Stat. Appl.* **11**(3), 203–211 (2016).
90. Prasad, S. V. S. V. S. V., Rosaiah, K., & Rao, G. S. A two Stage group sampling plans based on truncated life-tests for Type-II Generalized Log-Logistic distribution. *Int. J. Sci. Res.* **5**(6) (2018).
91. Rao, G. S., Kalyani, K., Rosaiah, K. & Sivakumar, D. C. U. A time-truncated two-stage group acceptance sampling plan for odds exponential log-logistic distribution. *Life Cycle Reliab. Saf. Eng.* **8**(4), 337–345 (2019).
92. Kanaparthi, R. A two-stage group sampling plan based on truncated life-tests for Exponentiated Half Logistic distribution. *CMST* **26**(2), 55–64 (2020).
93. Alizadeh, M., Afshariy, M., Karamikabir, H. & Yousof, H. M. The odd log-logistic burr-X family of distributions: properties and applications. *J. Stat. Theory Appl.* **20**(2), 228–241 (2021).
94. Alizadeh, M., Afshari, M., Ranjbar, V., Merovci, F., & Yousof, H. M. A novel XGamma extension: applications and actuarial risk analysis under the reinsurance data. *São Paulo J. Math. Sci.* 1–31 (2023).
95. Aslam, M., Lio, Y. L. & Jun, C. H. Repetitive acceptance sampling plans for burr type XII percentiles. *Int. J. Adv. Manuf. Technol.* **68**(1), 495–507 (2013).
96. Bourguignon, M., Silva, R. B. & Cordeiro, G. M. The Weibull-G family of probability distributions. *J. Data Sci.* **12**(1), 53–68 (2014).
97. Stephens, K. S. *The handbook of applied acceptance sampling plans* (ASQ Quality Press, Milwaukee, 2001).
98. Yousof, H. M., Aljadani, A., Mansour, M. M. & Abd Elrazik, E. M. A New pareto model: risk application, reliability MOOP and PORT value-at-risk analysis. *PJSOR* **20**(3), 383–407 (2024).

Acknowledgements
This research project was supported by the Researchers Supporting Project Number (RSP2024R488), King Saud University, Riyadh, Saudi Arabia.

Author contributions
All authors participated and had equal shares in all stages of preparing the paper and made essential contributions.

to it. All authors have read and agreed to the published version of the manuscript.

Competing interests
The authors declare no competing interests.

Additional information
Correspondence and requests for materials should be addressed to G.T.M.

Reprints and permissions information is available at www.nature.com/reprints.

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.