A game-theoretic model of Monkeypox to assess vaccination strategies

Sri Vibhaav Bankuru[1], Samuel Kossol[1], William Hou[2], Parsa Mahmoudi[3], Jan Rychtář[4] and Dewey Taylor[4]

[1] Department of Biomedical Engineering, Virginia Commonwealth University, Richmond, VA, United States of America
[2] Department of Kinesiology and Health Sciences, Virginia Commonwealth University, Richmond, VA, United States of America
[3] Department of Biology, Virginia Commonwealth University, Richmond, VA, United States of America
[4] Department of Mathematics and Applied Mathematics, Virginia Commonwealth University, Richmond, VA, United States of America

ABSTRACT

Monkeypox (MPX) is a zoonotic disease similar to smallpox. Its fatality rate is about 11% and it is endemic to the Central and West African countries. In this paper, we analyze a compartmental model of MPX dynamics. Our goal is to see whether MPX can be controlled and eradicated by voluntary vaccinations. We show that there are three equilibria—disease free, fully endemic and previously neglected semi-endemic (with disease existing only among humans). The existence of semi-endemic equilibrium has severe implications should the MPX virus mutate to increased viral fitness in humans. We find that MPX is controllable and can be eradicated in a semi-endemic equilibrium by vaccination. However, in a fully endemic equilibrium, MPX cannot be eradicated by vaccination alone.

INTRODUCTION

Monkeypox (MPX) is a zoonotic disease that has the potential to develop into one of the most threatening human *Orthopoxvirus* infections since the eradication of smallpox (*Durski et al., 2018*). The causative agent of MPX is monkeypox virus (MPXV), found in the same genus as the variola virus (smallpox), vaccinia virus, and cowpox virus (*Shchelkunov, Marennikova & Moyer, 2006*; *Sklenovská & Van Ranst, 2018*). Common symptoms of MPX, though relatively milder than smallpox, include fever, severe headaches, skin lesions, and myalgia (*CDC, 2003*). Prevention of the disease has remained a challenge for poverty-stricken rural areas with poor infrastructure that lack necessary sanitary supervision (*Sklenovská & Van Ranst, 2018*).

MPX is endemic to Central Africa and West Africa (*Weinstein et al., 2005*; *Yinka-Ogunleye et al., 2018*). West African and Central African strains of MPXV exist, the latter of which is more virulent and symptomatically severe (*Likos et al., 2005*; *Mwamba et al., 2014*). In the Democratic Republic of the Congo (DRC) the mortality rate of the Central African strain is 11% (*Ježek et al., 1987*). Since the first case of human infection in 1970, there have been numerous outbreaks in the DRC (*Eteng et al., 2018*). Annually, the DRC reports over

Corresponding author
Jan Rychtář, rychtarj@vcu.edu

2,000 cases of suspected infections and is the only country in an endemic state (*Mwamba et al., 2014*). This estimate may be modest, as MPX is often misdiagnosed as chickenpox or other diseases that cause rashes (*Ježek et al., 1988*). Additionally, modern and robust surveillance of MPX is neglected as a consequence of limited funds and resources (*Rimoin et al., 2010*), and countries other than the DRC are not required to report all cases of MPX (*Durski et al., 2018*). Thus, the disease may be more severe than previously estimated.

In 2003, 47 cases of MPX were reported across five states in the U.S., originating from a shipment of animals from Ghana imported to Texas (*CDC, 2003*). In 2018, 3 cases of MPX were reported in the United Kingdom, making it the first time since the 2003 United States outbreak that the disease had reached a country outside of Africa (*Eteng et al., 2018*).

The clinical presentation of MPX can be found in *Di Giulio & Eckburg (2004)*. The incubation period for the virus ranges from five to 21 days. MPX infection is split into two distinct phases: the invasion period and the skin eruption period. The invasion period starts between 0–5 days and is characterized by fever, lymphadenopathy, intense asthenia, severe headaches, and myalgia. The skin eruption period occurs 1–3 days after the appearance of a fever or lymphadenopathy, and it is characterized by rash formation, which often begins on the face and spreads to the rest of the body. The rash first appears as maculopapules (lesions with flat bases) and progresses to fluid filled blisters called vesicles. The vesicles then burst, forming pustules, and a crust forms over the affected area within 10 days. The number of lesions formed can vary from a few to thousands across the body, with children reportedly experiencing more severe symptoms than adults.

The predominant mode of MPX transmission is through human-animal interaction. Direct contact with an infected animal's blood, bodily fluids, or lesions can lead to infection. Documented cases of MPX in Central and West Africa show that transmission can occur via the handling of wild animals, predominantly monkeys (*Reynolds et al., 2017*). Cultural influences, such as consumption of "bush meat," can be a potential source of transmission. Additionally, direct contact with an infected person's bodily fluids and skin lesions can lead to the transmission of the disease (*McCollum & Damon, 2013*).

Despite MPX's high case fatality rate (*Ježek et al., 1987*) there are no known cures (*Eteng et al., 2018*). Until recently, there were no disease-specific preventative measures such as vaccines, though existing smallpox vaccines have historically been around 85% successful (*Eteng et al., 2018*). However, administration of the smallpox vaccine has ceased since the disease's eradication in 1980, resulting in lowered immunity against *Orthopoxviruses* in general. This has led to a supposed increase in population susceptibility to MPXV (*Sklenovská & Van Ranst, 2018*). In 2019, the vaccine termed Jynneos® was approved by the US FDA for protection against VARV and MPXV (*Meyer, Ehmann & Smith, 2020*).

It is possible that this lack of preventative measures is partially explained by a matching lack of literature on the potential dangers of inter-human transmission of MPX (*Rimoin et al., 2010*; *Mwamba et al., 2014*; *Doshi et al., 2018*; *Sklenovská & Van Ranst, 2018*). The urgency for better research on MPX is exacerbated considering vaccination cessation and immunocompromised populations in Central Africa, so a need for comprehensive preventative strategies is apparent (*WHO, 2017*). The factors such as (1) a lack of an

effective vaccination strategy from fixed bases, (2) a shortfall in the vaccine supply and (3) logistical and security problems associated with the distance from the health centers, all contribute to the challenges of vaccinating the whole population in Central and Western Africa (*Herp et al., 2003*).

The identity of MPXV reservoir host(s) remains unknown (*Di Giulio & Eckburg, 2004*; *Falendysz et al., 2017*). The seroprevalence of MPXV was found highest in a population of moribund rope squirrels (*Funisciurus anerythrus*) in Zaire (now DRC), and the virus was also found in sun squirrels (*Heliosciurus rufobrachium*) and non-human primates in DRC (*Khodakevich et al., 1987*; *Khodakevich, Ježek & Messinger, 1988*) as well as in Gambian pouched rats (*Cricetomys gambianus*) (*Doty et al., 2017*; Doshi et al., 2019). In West Africa, African dormice (*Graphiurus sp.*) and ground squirrels (*Xerus sp.*) were identified as additional hosts (*Reynolds et al., 2010*). The majority of reported human cases originate from an interaction with an infected animal (*Arita et al., 1985*). The transmission of MPX among animals can be affected by environmental conditions (*Brown & Leggat, 2016*). Deforestation and flooding could potentially increase or decrease the MPX reservoirs, depending on how the animal population is affected by these conditions (*Brown & Leggat, 2016*). Long-distance transportation of potential MPX carriers may result in the expansion of the geographical range of the MPX reservoir, as exemplified by the 2003 US outbreak.

Currently, MPXV likely needs the animal reservoir as the human-to-human transmission chains of MPX are relatively short; the maximum number of generations reported in literature is seven (*Learned et al., 2005*). Nevertheless, as demonstrated by the case of H1N1 influenza (swine flu), some virus mutations can increase viral fitness in humans (*Elderfield et al., 2014*). We note that poxviruses have linear, double-stranded DNA genomes that vary from 130 to 230 kbp (*Moss, 2013*) and as such are evolving much slower than H1N1. Nevertheless, they can still adapt rapidly (*Elde et al., 2012*) and genetic engineering and modern molecular biology already turned a mousepox virus into an unusually lethal strain (*Jackson et al., 2001*; *Di Giulio & Eckburg, 2004*).

Epidemiologic compartmental models have been used to better understand the potential implications of disease transmission and infection (*Blackwood & Childs, 2018*; *Bidari & Goldwyn, 2019*). For MPX, the framework for a mathematical model has been tentatively set, but existing iterations have had shortcomings, failing to address some of the aforementioned aspects of the disease in their entirety. *Bhunu & Mushayabasa (2011)* introduced a basic SIR vector-borne compartmental model between humans and primates, yet deem an endemic state solely in humans as trivial. *Usman & Adamu (2017)* build upon this framework by introducing an SVEIR compartmental model to account for the disease's incubation period and potential vaccine.

Game theoretical models attempt to study complex scenarios in which self-interested individuals will take an action based on the decisions of the rest of the population (*Bauch & Earn, 2004*). The model is a predictive tool in populations for extracting an optimum decision-making strategy (*Chang et al., 2020*). Game theory has been applied to protection strategies to control diseases such as smallpox (*Bauch, Galvani & Earn, 2003*), toxoplasmosis (*Sykes & Rychtář, 2015*), cholera (*Kobe et al., 2018*), measles (*Shim et al., 2012*), rubella (*Shim, Kochin & Galvani, 2009*), influenza (*Galvani, Reluga & Chapman,*

2007), African sleeping sickness (*Crawford et al., 2015*), malaria (*Orwa, Mbogo & Luboobi, 2018*; *Broom, Rychtář & Spears-Gill, 2016*), (Zika *Padmanabhan, Seshaiyer & Castillo-Chavez, 2017*; *Banuelos et al., 2019*) (Polio *Cheng et al., 2020*), Ebola (*Brettin et al., 2018*), chikungunya (SRM Klein, AO Foster, DA Feagins, JT Rowell, IV Erovenko, 2019, unpublished data), meningitis (A Martinez, J Machado, E Sanchez, I Erovenko, 2019, unpublished data), typhoid (C Acosta-Alonzo, IV Erovenko, A Lancaster, H Oh, J Rychtář, D Taylor, 2020, unpublished data), Hepatitis C (*Scheckelhoff, Ejaz & Erovenko, 2019*) and Hepatitis B (*Chouhan et al., in press*) among others. In this paper, we apply a similar approach to MPX to investigate a scenario in which individuals have the option of vaccinating to reduce the chance of contracting the virus. We further evaluate vaccination strategies on an individual and population-wide level by discussing the vaccination rates required to achieve herd immunity and Nash equilibrium.

In the present study, we build on the work of *Usman & Adamu (2017)*, see also *Lauko, Pinter & TeWinkel (2018)* for a simplified SIR version of the model. The mathematical model of the MPX dynamics is shown in the next section. Then, we provide closed-form formulas for equilibrium states of MPX dynamics; the formulas provided in *Usman & Adamu (2017)* do not allow for direct calculations of the equilibria. We also show the existence of a "semi-endemic" equilibrium. This was not previously discussed in *Usman & Adamu (2017)*, although it appears in *Lauko, Pinter & TeWinkel (2018)*. We apply a game-theoretic approach to evaluate individual and population-wide vaccination strategies on the basis of cost and probabilistic disease acquisition and then we perform sensitivity analysis. We conclude the study by a discussion.

MATHEMATICAL MODEL

We adopt the compartmental epidemiological model introduced in *Usman & Adamu (2017)* and shown in Fig. 1. We consider squirrels to be the primary reservoir hosts. The population is divided into squirrels and humans, denoted by s and h subscripts, respectively. Individuals are born as Susceptible (S) at rate Λ. Susceptible humans vaccinate (move to V_h) at rate α_h. Vaccinated humans are assumed to never contract the disease in the remainder of their lifetime. Susceptible squirrels become Exposed (E_s) by coming into contact with infected squirrels with effective transmission rate β_{ss}. Susceptible humans become exposed by coming into contact with either infected squirrels (with effective transmission rate β_{sh}) or infected humans (with effective transmission rate β_{hh}). After an incubation period ν^{-1}, the exposed individuals become Infected (I). Infected individuals are infectious. They Recover (R) at rate ρ. Any individual may die due to natural causes at rate μ. Infected individuals can also die from the disease at rate d. The notation is summarized in Table 1. The model yields the following differential equations, see for example *Blackwood & Childs (2018)*.
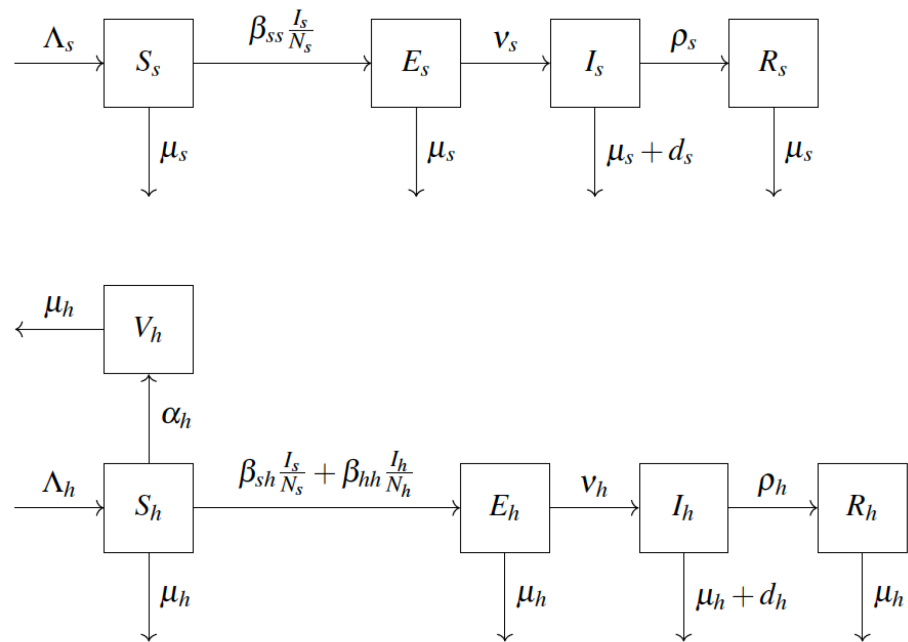
Figure 1 Scheme of mathematical model for humans and squirrels, adapted from *Usman & Adamu (2017)*.

$$\frac{dS_s}{dt} = \Lambda_s - \left(\mu_s + \beta_{ss}\frac{I_s}{N_s}\right)S_s \tag{1}$$

$$\frac{dE_s}{dt} = \beta_{ss}\frac{I_s}{N_s}S_s - (\mu_s + v_s)E_s \tag{2}$$

$$\frac{dI_s}{dt} = v_sE_s - (\mu_s + d_s + \rho_s)I_s \tag{3}$$

$$\frac{dR_s}{dt} = \rho_sI_s - \mu_sR_s \tag{4}$$

$$\frac{dS_h}{dt} = \Lambda_h - \left(\mu_h + \left(\beta_{sh}\frac{I_s}{N_s} + \beta_{hh}\frac{I_h}{N_h}\right) + \alpha_h\right)S_h \tag{5}$$

$$\frac{dV_h}{dt} = \alpha_hS_h - \mu_hV_h \tag{6}$$

$$\frac{dE_h}{dt} = \left(\beta_{sh}\frac{I_s}{N_s} + \beta_{hh}\frac{I_h}{N_h}\right)S_h - (\mu_h + v_h)E_h \tag{7}$$

$$\frac{dI_h}{dt} = v_hE_h - (\mu_h + d_h + \rho_h)I_h \tag{8}$$

$$\frac{dR_h}{dt} = \rho_hI_h - \mu_hR_h \tag{9}$$

Table 1 Model parameters. The human MPX related death rate was taken as a solution to $d_h/(d_h + \rho_h) = 0.1$ where 10% is the MPX fatality (*Ježek et al., 1987*). Similarly, squirrel MPX related death rate was taken as a solution to $d_s/(d_s + \rho_s) = 0.6$ where 60% is an estimate for the MPX fatality found between 50–75% (*Falendysz et al., 2017*). We estimated the effective squirrel-to-squirrel transmission rate as $\beta_{ss} = 40$; this yields about 24% of seropositive squirrels in the population, a number that agrees with estimates from *Khodakevich, Ježek & Messinger (1988)*. The effective transmission rates between humans was estimated as $\beta_{hh} = 32.85$ as follows. *Arita et al. (1985)* provide transmission risk $p = 0.15$ amongst household contacts and $p = 0.03$ amongst other contacts. We assumed human-to-human contact rate $\gamma = 365$ (i.e., once a day) and obtained $\beta_{hh} = 365\frac{0.15+0.03}{2} = 32.85$. The effective squirrel-to-human transmission rate was estimated to be $\beta_{sh} = 0.05$ as this yields about 1% of seropositive humans (*Khodakevich, Ježek & Messinger, 1988*). The actual cost of vaccine is \$4.85 (*Lambert de Rouvroit & Heegaard, 2016*). While the vaccine is provided for free, there are many other direct and indirect costs associated with vaccination (need to travel to the health center, associated security risk, loss of income etc., see for example *Herp et al. (2003)*) and we estimated the cost of vaccination to be 4. We note that the previously approved smallpox vaccines such as ACAM2000 could cause severe side effects (*Wollenberg & Engler, 2004*; *Nalca & Zumbrun, 2010*). It is not clear if the new vaccine, JYNNEOS, is more effective to protect against MPXV infections in humans than ACAM2000 and what the potential side effects are.

Symbol	Meaning	Value	Source
Λ_h	Human birth rate	0.0328	*CIA (2019)*
Λ_s	Squirrel birth rate	2	*Hayssen (2008)*
μ_h	Human natural death rate	1/60	*World Bank (2019)*
μ_s	Squirrel natural death rate	0.5	*Khodakevich, Ježek & Messinger (1988)*
d_h	Human MPX related death rate	3.12	*Ježek et al. (1987)*
d_s	Squirrel MPX related death rate	17.5	*Falendysz et al. (2017)*
ρ_h	Human recovery rate	28.08	*Di Giulio & Eckburg (2004)*
ρ_s	Squirrel recovery rate	12	*Falendysz et al. (2017)*
ν_h	Human infection rate	30.42	*Di Giulio & Eckburg (2004)*
ν_s	Squirrel infection rate	120	*Falendysz et al. (2017)*
α_h	Vaccination rate	variable	
β_{ss}	Squirrel-to-squirrel transmission rate	40	Assumed based on *Khodakevich, Ježek & Messinger (1988)*
β_{sh}	Squirrel-to-human transmission rate	0.05	Assumed based on *Khodakevich, Ježek & Messinger (1988)*
β_{hh}	Human-to-human transmission rate	32.85	*Arita et al. (1985)*
C_V	Cost of vaccination	4	*Herp et al. (2003)*
C_{MPX}	Cost of MPX infection	\$100	*Adam, Evans & Murray (2003)*

RESULTS

Equilibrium states of the MPX dynamics

The basic reproduction numbers were derived by *Usman & Adamu (2017)* and are given by

$$R_{0ss} = \beta_{ss} \cdot \frac{1}{\mu_s + d_s + \rho_s} \cdot \frac{\nu_s}{\mu_s + \nu_s} \tag{10}$$

$$R_{0hh} = \beta_{hh} \cdot \frac{\mu_h}{\alpha_h + \mu_h} \cdot \frac{1}{\mu_h + d_h + \rho_h} \cdot \frac{\nu_h}{\mu_h + \nu_h}. \tag{11}$$

As shown in Appendix 1, R_{0ss} corresponds to a number of secondary squirrel infections caused by a single infected squirrel in an otherwise healthy population. The meaning of R_{0hh} is similar.

There are three qualitatively distinct equilibria of the dynamics (1)–(9). First, ϵ^0 is the disease free equilibrium. It occurs when $R_{0ss} < 1$ and $R_{0hh} < 1$. Second, ϵ^* is the fully endemic equilibrium with disease occurring amongst humans as well as squirrels. The

Table 2 Different equilibria of the MPX dynamics. The formulas for N_h^* and $N_h^\dagger$ are too long for the table and are given in Appendix 1.

	Disease-free (ϵ^0)	Fully endemic (ϵ^*)	Semi-endemic ($\epsilon^\dagger$)
N_s	$\frac{\Lambda_s}{\mu_s}$	$\frac{\Lambda_s \cdot \left(\frac{\mu_s+d_s+\rho_s}{v_s}+1+\frac{\rho_s}{\mu_s}\right)}{\mu_s \cdot \left(\frac{\mu_s+d_s+\rho_s}{v_s}\right)+d_s \cdot \left(1-\frac{1}{R_{0ss}}\right)+\mu_s+\rho_s}$	$\frac{\Lambda_s}{\mu_s}$
S_s	N_s^0	$N_s^* \cdot \frac{1}{R_{0ss}}$	$N_s^\dagger$
E_s	0	$\left(\frac{\mu_s+d_s+\rho_s}{v_s}\right) \cdot I_s^*$	0
I_s	0	$\frac{\Lambda_s-\mu_s N_s^*}{d_s}$	0
R_s	0	$\frac{\rho_s}{\mu_s} \cdot I_s^*$	0
N_h	$\frac{\Lambda_h}{\mu_h}$	(45)	(59)
S_h	$\frac{\mu_h}{\mu_h+\alpha_h} \cdot N_h^0$	$\frac{(\mu_h+v_h)\left(\frac{\mu_h+d_h+\rho_h}{v_h}\right) \cdot I_h^*}{\beta_{sh}\left(\frac{I_s^*}{N_s^*}\right)+\beta_{hh}\left(\frac{I_h^*}{N_h^*}\right)}$	$\frac{(\mu_h+d_h+\rho_h)(\mu_h+v_h)}{v_h\beta_{hh}} \cdot N_h^\dagger$
V_h	$\frac{\alpha_h}{\mu_h+\alpha_h} \cdot N_h^0$	$\frac{\alpha_h}{\mu_h} \cdot S_h^*$	$\left(\frac{\alpha_h}{\mu_h}\right) \cdot S_h^\dagger$
E_h	0	$\left(\frac{\mu_h+d_h+\rho_h}{v_h}\right) \cdot I_h^*$	$\left(\frac{\mu_h+d_h+\rho_h}{v_h}\right) \cdot I_h^\dagger$
I_h	0	$\frac{\Lambda_h-\mu_h N_h^*}{d_h}$	$\frac{\Lambda_h-\mu_h N_h^\dagger}{d_h}$
R_h	0	$\frac{\rho_h}{\mu_h} \cdot I_h^*$	$\frac{\rho_h}{\mu_h} \cdot I_h^\dagger$

equilibrium is stable when $R_{0ss} > 1$. Finally, $\epsilon^\dagger$ is a semi-endemic equilibrium with disease prevalent only amongst human population. It is stable when $R_{0ss} < 1$ and $R_{0hh} > 1$.

The closed form formulas are given in Table 2. Step-by-step derivation can be found in Appendix 1.

Herd immunity and Nash equilibrium vaccination rates

The average cost of not vaccinating when the population vaccination rate is α_h is denoted $C_{notV}(\alpha_h)$ and it is given as a product of the cost of the MPX infection (C_{MPX}) and the
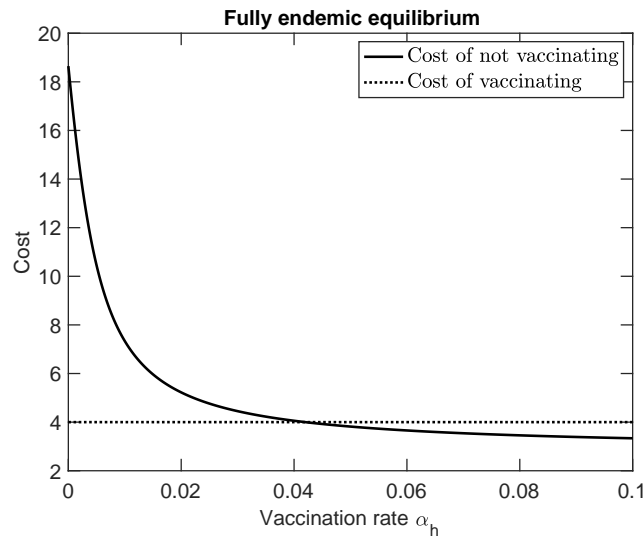
Figure 2 Cost versus vaccination rate. The vaccination rate (α_h) is varied while all other parameter values are as specified in Table 1.

probability of moving from the S_h compartment to the I_h compartment, i.e.,

$$C_{notV}(\alpha_h) = C_{MPX} \cdot \left(\frac{\left(\beta_{sh} \frac{I_s}{N_s} + \beta_{hh} \frac{I_h}{N_h} \right)}{\left(\beta_{sh} \frac{I_s}{N_s} + \beta_{hh} \frac{I_h}{N_h} \right) + \mu_h} \right) \cdot \left(\frac{v_h}{v_h + \mu_h} \right). \tag{12}$$

As the vaccination rate α_h increases, R_{0hh} decreases by Eq. (11). Furthermore, the fraction $\frac{I_h}{N_h}$ at the appropriate equilibrium also decreases. Consequently, the cost of not vaccinating decreases. In the semi-endemic equilibrium, the cost eventually becomes 0 when the vaccination rate reaches

$$\alpha_{HI} = \max \left\{ 0, \frac{v_h \beta_{hh} \mu_h}{(\mu_h + d_h + \rho_h)(\mu_h + v_h)} - \mu_h \right\}. \tag{13}$$

At that point, herd immunity is achieved and the disease is eradicated from the population. In the fully endemic equilibrium, there is always a reservoir of MPX in the squirrel population. This reservoir causes an influx of MPX infections amongst humans. Therefore the disease can never be fully eradicated and the cost of not vaccinating will never reach 0, see Fig. 2.

When the vaccination rate is such that $C_{notV}(\alpha_h) = C_V$, the vaccination rate is at Nash equilibrium, α_{NE}. When $\alpha_h < \alpha_{NE}$, it is beneficial for the individual to vaccinate; when $\alpha_h > \alpha_{NE}$, it is beneficial for the individual not to vaccinate.

Figure 3 shows a scenario where $\beta_{hh} = 60$. While this value is unrealistically high, we investigated this hypothetical scenario to see what would happen if MPXV mutates as was the case of H1N1 influenza (swine flu) or is genetically engineered as was the case of mousepox (*Jackson et al., 2001*; *Di Giulio & Eckburg, 2004*). When β_{hh} is large enough, specifically when

$$\beta_{hh} > \frac{\alpha_h + \mu_h}{\alpha_h} \cdot (\mu_h + d_h + \rho_h) \cdot \frac{\mu_h + v_h}{v_h} \tag{14}$$

A   **B**
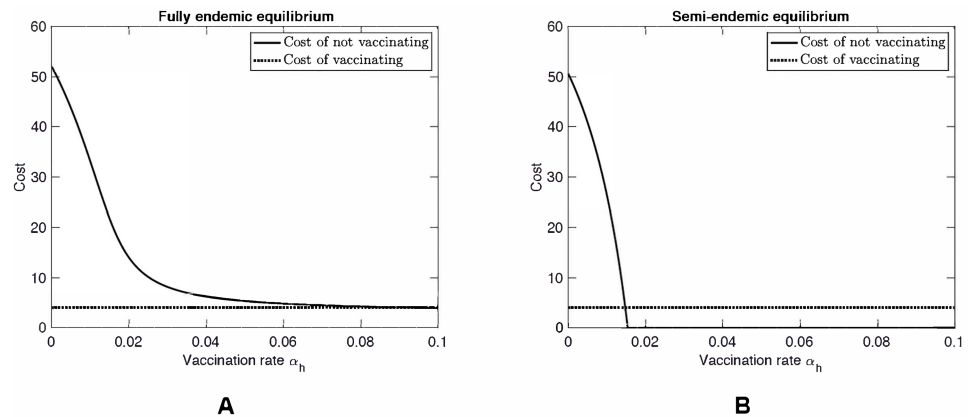
Figure 3 Costs vs. Vaccination rate when the effective human-to-human transmission rate is high, $\beta_{hh} = 60$. (A) The fully endemic state. (B) The semi-endemic state ($\beta_{ss} = 30 < (\mu_s + d_s + \rho_s) \cdot \frac{\mu_s + v_s}{v_s}$). The same scenario occurs when $\beta_{sh} = 0$ and β_{ss} is arbitrary. In both figures, the vaccination rate (α_h) is varied while all other parameter values are as specified in Table 1.

MPX no longer needs squirrels to persist in the human population. In particular, it can become endemic even in countries without natural squirrel population (i.e., even when $\beta_{sh} = 0$). At the same time, in the semi-endemic equilibrium, the disease can be controlled through vaccination. Note that there is almost no difference between the Nash equilibrium rate α_{NE} (a solution to $C_{notV}(\alpha_h) = C_V$) and the rate α_{HI} needed for herd immunity (a solution to $C_{notV}(\alpha_h) = 0$).

Sensitivity analysis

As shown in Fig. 2, as α_h increases, $C_{notV}(\alpha_h)$ approaches an asymptote. Consequently, the value of α_{NE}, a solution to $C_{notV}(\alpha_h) = C_V$ can be very sensitive to C_V when $C_V \approx 3$. Any small decrease of C_V can cause a significant increase of α_{NE}. The same sensitivity is demonstrated in Fig. 3. Figure 4 shows the sensitivity analysis and how α_{NE} depends on variation of different parameters. We can see the high sensitivity of α_{NE} on the squirrel-to-human transmission rate, β_{sh}, and on the cost of vaccination, C_V, for low values of C_V. Moreover, the figure demonstrates that α_{NE} can be quite sensitive on the effective transmission rate amongst squirrels, β_{ss} and the squirrels recovery rate, ρ_s. For $\beta_{ss} < (\mu_s + d_s + \rho_s) \cdot \frac{\mu_s + v_s}{v_s}$, there is a semi-endemic equilibrium and $\alpha_{NE} = 0$. However, as β_{ss} increases above that threshold value, α_{NE} rapidly increases and, when $\beta_{ss} > 45$, there is no Nash equilibrium vaccination rate. Similarly, when ρ_s is large enough to have a semi-endemic equilibrium, the optimal vaccination rate is 0. However, for small ρ_s, there is no Nash equilibrium and the change is relatively abrupt as in the case of β_{ss}.

Model validation

For the parameters in Table 1, the proportion of seropositive squirrels, given as $\frac{I_s + R_s}{N_s}$, is 24.44% which generally agrees with *Khodakevich, Ježek & Messinger (1988)*. Also, the proportion of seropositive people, $\frac{I_h + R_h}{N_h}$, is 1.06%, again agreeing with *Khodakevich, Ježek & Messinger (1988)*.
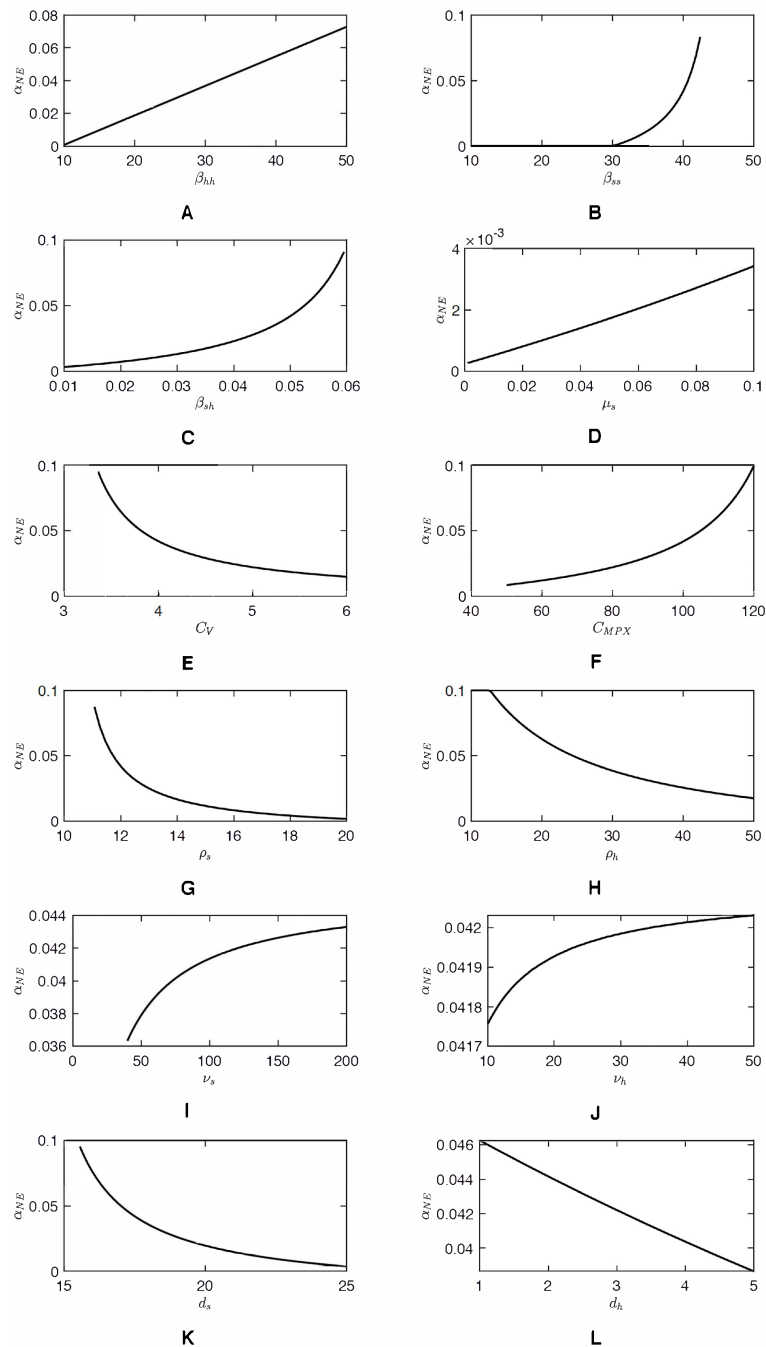
Figure 4 Dependence of α_{NE} on different parameter values (A–L). Unless varied, the parameter values are as specified in Table 1. For those parameters, $\alpha_{NE} = 0.0419$ and the sensitivity index, $SI_x = \left(\frac{x}{\alpha_{NE}}\right)\left(\frac{\partial \alpha_{NE}}{\partial x}\right)$ derivatives of α_{NE} with respect to parameters are as follows: $SI_{\beta_{hh}} = 1.415$, $SI_{\beta_{sh}} = 3.472$, $SI_{\beta_{ss}} = 10.911$, $SI_{\mu_s} = 0.932$, $SI_{\rho_s} = -7.120$, $SI_{\rho_h} = -1.258$, $SI_{\nu_s} = 0.077$, $SI_{\nu_h} = 0.004$, $SI_{d_s} = -5.917$, $SI_{d_h} = -0.141$, $SI_{C_V} = -3.518$, $SI_{C_{MPX}} = 3.603$.

DISCUSSION

The phylogenetic relatedness between MPXV and variola virus grants the smallpox vaccine an 85% effectiveness in preventing MPX (*Reynolds & Damon, 2012*). Poxviruses from the *Orthopoxvirus* genus have cross-reactive antibodies, meaning that vaccinated individuals would have a much lower risk of infection and mortality compared to unvaccinated individuals (*Louten, 2016*). The imperfect prevention rate of the vaccine was omitted in the design of the mathematical model for the sake of simplicity. As noted in *Wu, Fu & Wang (2011)*, imperfect protection aggravates the dilemma of voluntary protective actions as lower vaccine effectiveness can lead to better vaccine coverage and smaller free-riding effects; however, the impact of the epidemic can be harder to mitigate.

It is of interest to identify and evaluate possible preventative measures in addition to vaccination that would have a measurable effect on the transmission of MPX. For instance, decreasing the animal-to-human contact and launching an education campaign about dangers of eating raw meat which seems to be the main culprit behind squirrel-to-human transmission *Khodakevich, Ježek & Messinger (1988)* would significantly decrease the animal-to-human transmission rate. It could still come at a considerable individual cost (such as decrease of meat supply) but it would not require a complex or well-developed healthcare infrastructure needed for the vaccination, thus providing the general population with an easily accessible preventative measure. The mathematical model for such a measure would become more complex. The main idea would follow the spirit of *Kobe et al. (2018)* that investigated a situation for cholera prevention where individuals could either vaccinate or avoid drinking potentially contaminated water.

The reservoir host for monkeypox remains unclear (*Di Giulio & Eckburg, 2004*; *Falendysz et al., 2017*). We focused on a moribund rope squirrel, *Funisciurus anerythrus*, but we note that the disease has also been confirmed in other animals (*Arita et al., 1985*; *Khodakevich, Ježek & Messinger, 1988*; *Reynolds et al., 2010*). As noted in *Falendysz et al. (2017)*, in a recent outbreak of MPX in DRC, no association was found between contact with rope squirrels and human infection (*Nolen et al., 2015*). Additionally, a recent survey of 34 villages in the Tshuapa region of DRC did not detect contact with a rope squirrel carcass in the previous 30 days, although they reported contacts with red-legged sun squirrel, *Heliosciurus rufobrachium*, (*Monroe et al., 2015*) which was identified in *Khodakevich, Ježek & Messinger (1988)* as another frequent host of monkeypox.

CONCLUSIONS

We modeled MPX dynamics using the compartmental model of *Usman & Adamu (2017)*. As one of our major contributions, we provided closed form formulas of the equilibrium states of the dynamics. Moreover, we also showed a potential existence of the semi-endemic equilibrium, in which there is no infection in the squirrel population and the disease still persists in the human population. Currently, MPX does not seem to have the viral fitness to become endemic solely through human transmission. Yet, simple mutations in viral proteins could still occur and increase successive inter-human cases as seen in the H1N1

virus outbreak (*Le et al., 2009*). Should this mutation occur, a careful understanding of the semi-endemic equilibrium will be needed.

In addition, we applied a game-theoretical approach to assess vaccination decision-making developed by *Bauch & Earn (2004)*. Individuals in any population susceptible to MPX have the choice to vaccinate against the disease or risk the possibility of contracting the disease. Naturally, it is in the individual's best interest to choose the option with the smaller expected cost. The model quantifies the costs and benefits of getting smallpox vaccine. We found that the optimal vaccination rate is about 0.04, i.e., individuals should vaccinate about once every 25 years.

We must note that the parameter values we used are only estimates based on available literature. In reality, the parameters may be quite different, in large part because the reservoir hosts are different as discussed above. The performed sensitivity analysis allows us to gain insight into how our results depend on the specific parameter values. We observed that the optimal vaccination rate, α_{NE}, is about 10 times more sensitive to parameters related to animal hosts than to a corresponding parameter related to humans. It is therefore important to establish more accurate parameters. Consequently, greater efforts are needed to track the true prevalence and recurrent cases of MPX in all populations rather than relying on suspected cases.

Though not perfect in practice, mathematical modeling of diseases remains a powerful tool that grants a more profound understanding how MPX operates under certain conditions. The scope of epidemiological modeling and game-theoretic cost analysis is wide. As cases of MPX become increasingly reported among humans (*Antwerpen et al., 2019*), we hope that the models may serve as a predictive tool to better study the spread of MPX.

ACKNOWLEDGEMENTS

S.V.B., S.K., W.H., and P.M. worked on this manuscript as part of the course MATH/BIOL 380—Introduction to mathematical biology. They wish to acknowledge the help and support of their classmates.

APPENDIX 1. STEP-BY-STEP SOLUTIONS TO EQUILIBRIUM STATES

In this section we find the equilibrium states of the system (1)–(9). We will look for solutions of the following system.

$$0 = \Lambda_s - \left(\mu_s + \beta_{ss}\frac{I_s}{N_s}\right)S_s \tag{15}$$

$$0 = \beta_{ss}\frac{I_s}{N_s}S_s - (\mu_s + \nu_s)E_s \tag{16}$$

$$0 = \nu_s E_s - (\mu_s + d_s + \rho_s)I_s \tag{17}$$

$$0 = \rho_s I_s - \mu_s R_s \tag{18}$$

$$0 = \Lambda_h - \left(\mu_h + \left(\beta_{sh}\frac{I_s}{N_s} + \beta_{hh}\frac{I_h}{N_h}\right) + \alpha_h\right)S_h \tag{19}$$

$$0 = \alpha_h S_h - \mu_h V_h \tag{20}$$

$$0 = \left(\beta_{sh}\frac{I_s}{N_s} + \beta_{hh}\frac{I_h}{N_h}\right)S_h - (\mu_h + \nu_h)E_h \tag{21}$$

$$0 = \nu_h E_h - (\mu_h + d_h + \rho_h)I_h \tag{22}$$

$$0 = \rho_h I_h - \mu_h R_h \tag{23}$$

where (15)–(18) are equations for the squirrels and (19)–(23) are for the humans.

We will distinguish three equilibrium states ϵ^0, ϵ^* and $\epsilon^\dagger$ depending on the existence of infection among squirrels and humans. However, by (16), (18), (20), (21) and (23), no matter which equilibrium state, the following formulas will always be valid.

$$E_s = \frac{\mu_s + d_s + \rho_s}{\nu_s}I_s \tag{24}$$

$$R_s = \frac{\rho_s}{\mu_s}I_s \tag{25}$$

$$V_h = \frac{\alpha_h}{\mu_h}S_h \tag{26}$$

$$E_h = \frac{\mu_h + d_h + \rho_h}{\nu_h}I_h \tag{27}$$

$$R_h = \frac{\rho_h}{\mu_h}I_h \tag{28}$$

Disease-free equilibrium, ϵ^0

Assume $I_h^0 = I_s^0 = 0$. It follows from (24) that $E_s^0 = 0$, from (25) that $R_s^0 = 0$ and from (15) that $S_s^0 = \frac{\Lambda_s}{\mu_s}$. Also, by (27), $E_h^0 = 0$ and, by (28), $R_h^0 = 0$. It follows from (19) that

$$S_h^0 = \frac{\Lambda_h}{\alpha_h + \mu_h}. \tag{29}$$

and thus, by (26),

$$V_h^0 = \frac{\Lambda_h}{\mu_h} \cdot \frac{\alpha_h}{\alpha_h + \mu_h}. \tag{30}$$

The stability of ϵ^0 was discussed and the basic reproduction numbers were derived in *Usman & Adamu (2017)* using the next-generation matrix method of *Van den Driessche*

& *Watmough (2002)*. Here we present an alternative derivation of the basic reproduction numbers.

Assume there is an infected squirrel in an otherwise disease-free population. The squirrel stays infected for a period of $(\mu_s + d_s + \rho_s)^{-1}$ during which it exposes susceptible individuals at the rate $\beta_{ss} \frac{S_s^0}{N_s^0} = \beta_{ss}$. The newly exposed individuals end up in the I_s compartment with probability $\frac{v_s}{\mu_s + v_s}$. Consequently, the number of secondary infections from a single infected squirrel in an otherwise disease free equilibrium is given by

$$R_{0ss} = \beta_{ss} \cdot \left(\frac{1}{\mu_s + d_s + \rho_s} \right) \cdot \left(\frac{v_s}{\mu_s + v_s} \right). \tag{31}$$

Similarly, we can derive that the number of secondary infections caused by a single infected human in otherwise disease-free population is

$$R_{0hh} = \beta_{hh} \frac{S_h^0}{N_h^0} \cdot \left(\frac{1}{\mu_h + d_h + \rho_h} \right) \cdot \left(\frac{v_h}{\mu_h + v_h} \right) = \frac{v_h \beta_{hh} \mu_h}{(\mu_h + d_h + \rho_h)(\mu_h + v_h)(\alpha_h + \mu_h)}. \tag{32}$$

Case when $I_s > 0$

By adding (15)–(18), we get $0 = \Lambda_s - \mu_s N_s^* - d_s I_s^*$ which yields

$$I_s^* = \frac{\Lambda_s - \mu_s N_s^*}{d_s}. \tag{33}$$

By (24), (16) becomes

$$0 = \frac{\beta_{ss} I_s^* S_s^*}{N_s^*} - (\mu_s + v_s) \cdot \frac{\mu_s + d_s + \rho_s}{v_s} \cdot I_s^* \tag{34}$$

and since we are assuming $I_s^* > 0$, we can divide by I_s and get

$$S_s^* = N_s^* \cdot \frac{(\mu_s + v_s)(\mu_s + d_s + \rho_s)}{\beta_{ss} v_s} = \frac{N_s^*}{R_{0ss}}. \tag{35}$$

Consequently,

$$\begin{aligned} N_s^* &= S_s^* + E_s^* + I_s^* + R_s^* \\ &= \frac{N_s^*}{R_{0ss}} + \left(\frac{\mu_s + d_s + \rho_s}{v_s} + 1 + \frac{\rho_s}{\mu_s} \right) \cdot \left(\frac{\Lambda_s - \mu_s N_s^*}{d_s} \right) \end{aligned} \tag{36}$$

which yields

$$N_s^* = \frac{\Lambda_s \cdot \left(\frac{\mu_s + d_s + \rho_s}{v_s} + 1 + \frac{\rho_s}{\mu_s} \right)}{\mu_s \cdot \left(\frac{\mu_s + d_s + \rho_s}{v_s} \right) + \mu_s + \rho_s + d_s \left(1 - \frac{1}{R_{0ss}} \right)}. \tag{37}$$

Fully endemic equilibrium, ϵ^*, human population

Adding (19)–(23) yields

$$0 = \Lambda_h - \mu_h N_h^* - d_h I_h^* \tag{38}$$

and consequently

$$I_h^* = \frac{\Lambda_h - \mu_h N_h^*}{d_h}. \tag{39}$$

Substituting (39) into (23), (21), (19), (20) we get

$$R_h^* = \frac{\rho_h}{\mu_h} \cdot \left(\frac{\Lambda_h - \mu_h N_h^*}{d_h} \right) \tag{40}$$

$$E_h^* = \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \cdot \left(\frac{\Lambda_h - \mu_h N_h^*}{d_h} \right) \tag{41}$$

$$S_h^* = \frac{(\mu_h + v_h) \cdot \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \cdot \left(\frac{\Lambda_h - \mu_h N_h^*}{d_h} \right)}{\beta_{sh} \left(\frac{I_s^*}{N_s^*} \right) + \beta_{hh} \frac{\frac{\Lambda_h - \mu_h N_h^*}{d_h}}{N_h^*}} \tag{42}$$

$$V_h^* = \frac{\alpha_h}{\mu_h} \cdot \left(\frac{(\mu_h + v_h) \cdot \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \cdot \left(\frac{\Lambda_h - \mu_h N_h^*}{d_h} \right)}{\beta_{sh} \left(\frac{I_s^*}{N_s^*} \right) + \beta_{hh} \frac{\frac{\Lambda_h - \mu_h N_h^*}{d_h}}{N_h^*}} \right). \tag{43}$$

Since $N_h^* = S_h^* + V_h^* + E_h^* + I_h^* + R_h^*$, we get

$$N_h^* = \frac{\rho_h}{\mu_h} \left(\frac{\Lambda_h - \mu_h N_h^*}{d_h} \right) + \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \cdot \left(\frac{\Lambda_h - \mu_h N_h^*}{d_h} \right)$$

$$+ \frac{\Lambda_h - \mu_h N_h^*}{d_h} + \frac{(\mu_h + v_h) \cdot \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \cdot \left(\frac{\Lambda_h - \mu_h N_h^*}{d_h} \right)}{\beta_{sh} \left(\frac{I_s^*}{N_s^*} \right) + \beta_{hh} \frac{\frac{\Lambda_h - \mu_h N_h^*}{d_h}}{N_h^*}}$$

$$+ \frac{\alpha_h}{\mu_h} \cdot \left(\frac{(\mu_h + v_h) \cdot \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \cdot \left(\frac{\Lambda_h - \mu_h N_h^*}{d_h} \right)}{\beta_{sh} \left(\frac{I_s^*}{N_s^*} \right) + \beta_{hh} \frac{\frac{\Lambda_h - \mu_h N_h^*}{d_h}}{N_h^*}} \right). \tag{44}$$

This yields that N_h^* is a positive root of the equation $AN_h^2 + BN_h + C = 0$, i.e.,

$$N_h^* = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \tag{45}$$

where

$$A = g \cdot d_h + k \cdot c \cdot d_h \cdot \mu_h \left(1 + \frac{\alpha_h}{\mu_h} \right) + g \cdot \mu_h (1 + z + k) \tag{46}$$

$$B = d_h \cdot \beta_{hh} \cdot \Lambda_h - \left(g \cdot \Lambda_h - \beta_{hh} \cdot \Lambda_h \cdot \mu_h \right)(1 + z + k) - k \cdot c \cdot d_h \cdot \Lambda_h \left(1 + \frac{\alpha_h}{\mu_h} \right) \tag{47}$$

$$C = -\Lambda_h^2 \cdot \beta_{hh} (1 + z + k) \tag{48}$$

and

$$k = \frac{\mu_h + d_h + \rho_h}{v_h} \tag{49}$$

$$g = d_h \beta_{sh} \left(\frac{I_s^*}{N_s^*} \right) - \beta_{hh} \mu_h \tag{50}$$

$$c = \mu_h + v_h \tag{51}$$

$$z = \frac{\rho_h}{\mu_h}. \tag{52}$$

The positive solution for N_h^* from (45) can then be recursively substituted into (39), (40), (41), (42), and (43) to get closed-form formulas for equilibrium values.

Semi-endemic equilibrium

Here, $I_s = 0$. As above, by adding all equations (19)–(23) we get $0 = \Lambda_h - \mu_h N_h^\dagger - d_h I_h^\dagger$ which yields

$$I_h^\dagger = \frac{\Lambda_h - \mu_h N_h^\dagger}{d_h}. \tag{53}$$

Substituting Eq. (53) into Eqs. (23), (21), (19) we get

$$R_h^\dagger = \frac{\rho_h}{\mu_h} \cdot \left(\frac{\Lambda_h - \mu_h N_h^\dagger}{d_h} \right) \tag{54}$$

$$E_h^\dagger = \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \cdot \left(\frac{\Lambda_h - \mu_h N_h^\dagger}{d_h} \right) \tag{55}$$

$$S_h^\dagger = \frac{(\mu_h + v_h) \cdot \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \cdot \left(\frac{\Lambda_h - \mu_h N_h^\dagger}{d_h} \right)}{\beta_{sh} \left(\frac{I_s^\dagger}{N_s^\dagger} \right) + \beta_{hh} \frac{\frac{\Lambda_h - \mu_h N_h^\dagger}{d_h}}{N_h^\dagger}}. \tag{56}$$

Since $I_s^\dagger = 0$, we get

$$S_h^\dagger = N_h^\dagger \cdot \frac{(\mu_h + d_h + \rho_h)(\mu_h + v_h)}{v_h \beta_{hh}} \tag{57}$$

and consequently, by (20),

$$V_h^\dagger = N_h^\dagger \cdot \left(\frac{\alpha_h}{\mu_h} \right) \cdot \frac{(\mu_h + d_h + \rho_h)(\mu_h + v_h)}{v_h \beta_{hh}}. \tag{58}$$

Finally, summing up (53)–(58), we obtain the closed-form formula

$$N_h^\dagger = \frac{\Lambda_h \cdot \left(\frac{\rho_h}{\mu_h} + \frac{\mu_h + d_h + \rho_h}{v_h} + 1 \right)}{d_h + \mu_h \cdot \left(\frac{\rho_h}{\mu_h} + \frac{\mu_h + d_h + \rho_h}{v_h} + 1 \right) - d_h \cdot \left(\frac{\mu_h + d_h + \rho_h}{v_h} \left(\frac{\mu_h + v_h}{\beta_{hh}} \right) + \frac{\alpha_h}{\mu_h} \left(\frac{\mu_h + d_h + \rho_h}{v_h} \right) \left(\frac{\mu_h + v_h}{\beta_{hh}} \right) \right)} \tag{59}$$

which can be used to calculate equilibrium values.

Funding
Support for publishing this article was provided by the VCU Libraries Open Access Publishing Fund. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Grant Disclosures
The following grant information was disclosed by the authors:
VCU Libraries Open Access Publishing Fund.

Competing Interests
The authors declare there are no competing interests.

Author Contributions

- Sri Vibhaav Bankuru, Samuel Kossol, William Hou, Parsa Mahmoudi, Jan Rychtář and Dewey Taylor conceived and designed the experiments, performed the experiments, analyzed the data, prepared figures and/or tables, authored or reviewed drafts of the paper, and approved the final draft.

Data Availability
The Matlab code used to generate the figures is available as a Supplemental File.

Supplemental Information
Supplemental information for this article can be found online at http://dx.doi.org/10.7717/peerj.9272#supplemental-information.

REFERENCES

Adam T, Evans DB, Murray CJ. 2003. Econometric estimation of country-specific hospital costs. *Cost Effectiveness and Resource Allocation* **1**(1):3 DOI 10.1186/1478-7547-1-3.

Antwerpen MH, Georgi E, Nikolic A, Zoeller G, Wohlsein P, Baumgärtner W, Peyrefitte C, Charrel R, Meyer H. 2019. Use of next generation sequencing to study two cowpox virus outbreaks. *PeerJ* **7**:e6561 DOI 10.7717/peerj.6561.

Arita I, Ježek Z, Khodakevich L, Ruti K. 1985. Human monkeypox: a newly emerged orthopoxvirus zoonosis in the tropical rain forests of Africa. *The American Journal of Tropical Medicine and Hygiene* **34**(4):781–789 DOI 10.4269/ajtmh.1985.34.781.

Banuelos S, Martinez M, Mitchell C, Prieto-Langarica A. 2019. Using mathematical modelling to investigate the effect of the sexual behaviour of asymptomatic individuals and vector control measures on zika. *Letters in Biomathematics* **6**(1):1–19.

Bauch CT, Earn DJ. 2004. Vaccination and the theory of games. *Proceedings of the National Academy of Sciences of the United States of America* **101(36)**:13391–13394 DOI 10.1073/pnas.0403823101.

Bauch CT, Galvani AP, Earn DJ. 2003. Group interest versus self-interest in smallpox vaccination policy. *Proceedings of the National Academy of Sciences of the United States of America* **100(18)**:10564–10567 DOI 10.1073/pnas.1731324100.

Bhunu C, Mushayabasa S. 2011. Modelling the transmission dynamics of Pox-like infections. *IAENG International Journal of Applied Mathematics* **41(2)**:141–149.

Bidari S, Goldwyn EE. 2019. Stochastic models of influenza outbreaks on a college campus. *Letters in Biomathematics* **6(2)**:1–14.

Blackwood JC, Childs LM. 2018. An introduction to compartmental modeling for the budding infectious disease modeler. *Letters in Biomathematics* **5(1)**:195–221 DOI 10.30707/LiB5.1Blackwood.

Brettin A, Rossi-Goldthorpe R, Weishaar K, Erovenko IV. 2018. Ebola could be eradicated through voluntary vaccination. *Royal Society Open Science* **5(1)**:171591 DOI 10.1098/rsos.171591.

Broom M, Rychtář J, Spears-Gill T. 2016. The game-theoretical model of using insecticide- treated bed-nets to fight malaria. *Applied Mathematics* **7(09)**:852–860.

Brown K, Leggat PA. 2016. Human monkeypox: current state of knowledge and implications for the future. *Tropical Medicine and Infectious Disease* **1(1)**:8 DOI 10.3390/tropicalmed1010008.

Centers for Disease Control (CDC). 2003. Multistate outbreak of monkeypox–Illinois, Indiana, and Wisconsin, 2003. *Morbidity and Mortality Weekly Report*. 537.

Chang SL, Piraveenan M, Pattison P, Prokopenko M. 2020. Game theoretic modelling of infectious disease dynamics and intervention methods: a review. *Journal of Biological Dynamics* **14(1)**:57–89 DOI 10.1080/17513758.2020.1720322.

Cheng E, Gambhirrao N, Patel R, Zhowandai A, Rychtář J, Taylor D. 2020. A game-theoretical analysis of Poliomyelitis vaccination. *Journal of Theoretical Biology* **499**:110298 DOI 10.1016/j.jtbi.2020.110298.

Chouhan A, Maiwand S, Ngo M, Putalapattu V, Rychtář J, Taylor D. 2019. Game-theoretical model of retroactive Hepatitis B vaccination in China. *Bulletin of Mathematical Biology* In Press DOI 10.1007/s11538-020-00748-5.

CIA. 2019. The World Factbook - birth rate. *Available at https://www.cia.gov/library/publications/the-world-factbook/fields/345.html* (accessed on 13 December 2019).

Crawford K, Lancaster A, Oh H, Rychtář J. 2015. A voluntary use of insecticide-treated cattle can eliminate African sleeping sickness. *Letters in Biomathematics* **2(1)**:91–101 DOI 10.30707/LiB2.1Crawford.

Di Giulio DB, Eckburg PB. 2004. Human monkeypox: an emerging zoonosis. *The Lancet Infectious Diseases* **4(1)**:15–25 DOI 10.1016/S1473-3099(03)00856-9.

Doshi RH, Guagliardo SAJ, Dzabatou-Babeaux A, Likouayoulou C, Ndakala N, Moses C, Olson V, McCollum AM, Petersen BW. 2018. Strengthening of surveillance during monkeypox outbreak, Republic of the Congo, 2017. *Emerging Infectious Diseases* **24(6)**:1158–1160 DOI 10.3201/eid2406.180248.

Doty JB, Malekani JM, Kalemba LN, Stanley WT, Monroe BP, Nakazawa YU, Mauldin MR, Bakambana TL, Liyandja Dja Liyandja T, Braden ZH. 2017. Assessing monkeypox virus prevalence in small mammals at the human–animal interface in the Democratic Republic of the Congo. *Viruses* **9(10)**:283 DOI 10.3390/v9100283.

Durski KN, McCollum AM, Nakazawa Y, Petersen BW, Reynolds MG, Briand S, Djingarey MH, Olson V, Damon IK, Khalakdina A. 2018. Emergence of monkey-pox West and Central Africa, 1970–2017. *Morbidity and Mortality Weekly Report* **67(10)**:306–310 DOI 10.15585/mmwr.mm6710a5.

Elde NC, Child SJ, Eickbush MT, Kitzman JO, Rogers KS, Shendure J, Geballe AP, Malik HS. 2012. Poxviruses deploy genomic accordions to adapt rapidly against host antiviral defenses. *Cell* **150(4)**:831–841 DOI 10.1016/j.cell.2012.05.049.

Elderfield RA, Watson SJ, Godlee A, Adamson WE, Thompson CI, Dunning J, Fernandez-Alonso M, Blumenkrantz D, Hussell T, Zambon M. 2014. Accumula-tion of human-adapting mutations during circulation of A (H1N1) pdm09 influenza virus in humans in the United Kingdom. *Journal of Virology* **88(22)**:13269–13283 DOI 10.1128/JVI.01636-14.

Eteng W-E, Mandra A, Doty J, Yinka-Ogunleye A, Aruna S, Reynolds MG, Mc-Collum AM, Davidson W, Wilkins K, Saleh M. 2018. Notes from the Field: Responding to an Outbreak of Monkeypox Using the One Health Approach-Nigeria, 2017–2018. *Morbidity and Mortality Weekly Report* **67(37)**:1040–1041 DOI 10.15585/mmwr.mm6737a5.

Falendysz EA, Lopera JG, Doty JB, Nakazawa Y, Crill C, Lorenzsonn F, Lems NK, Ronderos MD, Mejia A, Malekani JM. 2017. Characterization of Monkeypox virus infection in African rope squirrels (Funisciurus sp.). *PLOS Neglected Tropical Diseases* **11(8)**:e0005809 DOI 10.1371/journal.pntd.0005809.

Galvani AP, Reluga TC, Chapman GB. 2007. Long-standing influenza vaccination policy is in accord with individual self-interest but not with the utilitarian optimum. *Proceedings of the National Academy of Sciences of the United States of America* **104(13)**:5692–5697 DOI 10.1073/pnas.0606774104.

Hayssen V. 2008. Reproductive effort in squirrels: ecological, phylogenetic, al-lometric, and latitudinal patterns. *Journal of Mammalogy* **89(3)**:582–606 DOI 10.1644/07-MAMM-A-064R2.1.

Herp MV, Parqué V, Rackley E, Ford N. 2003. Mortality, violence and lack of access to healthcare in the Democratic Republic of Congo. *Disasters* **27(2)**:141–153 DOI 10.1111/1467-7717.00225.

Jackson RJ, Ramsay AJ, Christensen CD, Beaton S, Hall DF, Ramshaw IA. 2001. Expression of mouse interleukin-4 by a recombinant ectromelia virus suppresses cytolytic lymphocyte responses and overcomes genetic resistance to mousepox. *Journal of Virology* **75(3)**:1205–1210 DOI 10.1128/JVI.75.3.1205-1210.2001.

Ježek Z, Szczeniowski M, Paluku K, Mutombo M, Grab B. 1988. Human monkeypox: confusion with chickenpox. *Acta Tropica* **45(4)**:297–307.

Ježek Z, Szczeniowski M, Paluku K, Mutombo M. 1987. Human monkeypox: clinical features of 282 patients. *Journal of Infectious Diseases* **156(2)**:293–298 DOI 10.1093/infdis/156.2.293.

Khodakevich L, Ježek Z, Messinger D. 1988. Monkeypox virus: ecology and public health significance. *Bulletin of the World Health Organization* **66(6)**:747–752.

Khodakevich L, Szczeniowski M, Ježek Z, Marennikova S, Nakano J, Messinger D. 1987. The role of squirrels in sustaining monkeypox virus transmission. *Tropical and Geographical Medicine* **39(2)**:115–122.

Kobe J, Pritchard N, Short Z, Erovenko IV, Rychtář J, Rowell JT. 2018. A game-theoretic model of cholera with optimal personal protection strategies. *Bulletin of Mathematical Biology* **80(10)**:2580–2599 DOI 10.1007/s11538-018-0476-5.

Lambert de Rouvroit A, Heegaard ED. 2016. Total costs associated with replicating and non-replicating smallpox vaccines. *Global Security: Health, Science and Policy* **1(1)**:3–9.

Lauko I, Pinter G, TeWinkel RE. 2018. Equilibrium analysis for an epidemic model with a reservoir for infection. *Letters in Biomathematics* **5(1)**:255–274 DOI 10.30707/LiB5.1Lauko.

Le QM, Sakai-Tagawa Y, Ozawa M, Ito M, Kawaoka Y. 2009. Selection of H5N1 influenza virus PB2 during replication in humans. *Journal of Virology* **83(10)**:5278–5281 DOI 10.1128/JVI.00063-09.

Learned LA, Reynolds MG, Wassa DW, Li Y, Olson VA, Karem K, Stempora LL, Braden ZH, Kline R, Likos A. 2005. Extended interhuman transmission of monkeypox in a hospital community in the Republic of the Congo, 2003. *The American Journal of Tropical Medicine and Hygiene* **73(2)**:428–434 DOI 10.4269/ajtmh.2005.73.428.

Likos AM, Sammons SA, Olson VA, Frace AM, Li Y, Olsen-Rasmussen M, Davidson W, Galloway R, Khristova ML, Reynolds MG. 2005. A tale of two clades: monkeypox viruses. *Journal of General Virology* **86(10)**:2661–2672 DOI 10.1099/vir.0.81215-0.

Louten J. 2016. *Essential human virology.* London: Academic Press.

McCollum AM, Damon IK. 2013. Human monkeypox. *Clinical Infectious Diseases* **58(2)**:260–267.

Meyer H, Ehmann R, Smith GL. 2020. Smallpox in the post-eradication era. *Viruses* **12(2)**:138 DOI 10.3390/v12020138.

Monroe BP, Doty JB, Moses C, Ibata S, Reynolds M, Carroll D. 2015. Collection and utilization of animal carcasses associated with zoonotic disease in Tshuapa District, the Democratic Republic of the Congo, 2012. *Journal of Wildlife Diseases* **51(3)**:734–738 DOI 10.7589/2014-05-140.

Moss B. 2013. Poxvirus DNA replication. *Cold Spring Harbor Perspectives in Biology* **5(9)**:a010199 DOI 10.1101/cshperspect.a010199.

Mwamba D, Kebela B, Shongo R, Pukuta E, Kayembe N. 2014. Profil épidemiologique du monkeypox en RDC, 2010–2014. *Annals of African Medicine* **8**:1855–1860.

Nalca A, Zumbrun EE. 2010. ACAM2000: the new smallpox vaccine for United States Strategic National Stockpile. *Drug Design, Development and Therapy* **4**:71.

Nolen LD, Osadebe L, Katomba J, Likofata J, Mukadi D, Monroe B, Doty J, Malekani J, Kabamba J, Bomponda PL. 2015. Introduction of monkeypox into a community and household: risk factors and zoonotic reservoirs in the Democratic Republic of the Congo. *The American Journal of Tropical Medicine and Hygiene* **93(2)**:410–415 DOI 10.4269/ajtmh.15-0168.

Orwa TO, Mbogo RW, Luboobi LS. 2018. Mathematical model for the in-host malaria dynamics subject to malaria vaccines. *Letters in Biomathematics* **5(1)**:222–251 DOI 10.1080/23737867.2018.1526132.

Padmanabhan P, Seshaiyer P, Castillo-Chavez C. 2017. Mathematical modeling, analysis and simulation of the spread of Zika with influence of sexual transmission and preventive measures. *Letters in Biomathematics* **4(1)**:148–166 DOI 10.30707/LiB4.1Padmanabhan.

Reynolds MG, Carroll DS, Olson VA, Hughes C, Galley J, Likos A, Montgomery JM, Suu-Ire R, Kwasi MO, Root JJ. 2010. A silent enzootic of an orthopoxvirus in Ghana, West Africa: evidence for multi-species involvement in the absence of widespread human disease. *The American Journal of Tropical Medicine and Hygiene* **82(4)**:746–754 DOI 10.4269/ajtmh.2010.09-0716.

Reynolds MG, Damon IK. 2012. Outbreaks of human monkeypox after cessation of smallpox vaccination. *Trends in Microbiology* **20(2)**:80–87 DOI 10.1016/j.tim.2011.12.001.

Reynolds MG, McCollum AM, Nguete B, Shongo Lushima R, Petersen BW. 2017. Improving the care and treatment of monkeypox patients in low-resource settings: applying evidence from contemporary biomedical and smallpox biodefense research. *Viruses* **9(12)**:380 DOI 10.3390/v9120380.

Rimoin AW, Mulembakani PM, Johnston SC, Smith JOL, Kisalu NK, Kinkela TL, Blumberg S, Thomassen HA, Pike BL, Fair JN. 2010. Major increase in human monkeypox incidence 30 years after smallpox vaccination campaigns cease in the Democratic Republic of Congo. *Proceedings of the National Academy of Sciences of the United States of America* **107(37)**:16262–16267 DOI 10.1073/pnas.1005769107.

Scheckelhoff K, Ejaz A, Erovenko IV. 2019. A game-theoretic model of optimal clean equipment usage to prevent hepatitis C among injecting drug users. Preprint.

Shchelkunov SN, Marennikova SS, Moyer RW. 2006. *Orthopoxviruses pathogenic for humans.* New York: Springer Science & Business Media.

Shim E, Grefenstette JJ, Albert SM, Cakouros BE, Burke DS. 2012. A game dynamic model for vaccine skeptics and vaccine believers: measles as an example. *Journal of Theoretical Biology* **295**:194–203 DOI 10.1016/j.jtbi.2011.11.005.

Shim E, Kochin B, Galvani A. 2009. Insights from epidemiological game theory into gender-specific vaccination against rubella. *Mathematical Biosciences and Engineering: MBE* **6(4)**:839–854 DOI 10.3934/mbe.2009.6.839.

Sklenovská N, Van Ranst M. 2018. Emergence of monkeypox as the most important orthopoxvirus infection in humans. *Frontiers in Public Health* **6**:241.

Sykes D, Rychtář J. 2015. A game-theoretic approach to valuating toxoplasmosis vaccination strategies. *Theoretical Population Biology* **105**:33–38 DOI 10.1016/j.tpb.2015.08.003.

Usman S, Adamu II. 2017. Modeling the transmission dynamics of the monkeypox virus infection with treatment and vaccination interventions. *Journal of Applied Mathematics and Physics* **5(12)**:2335–2353 DOI 10.4236/jamp.2017.512191.

Van den Driessche P, Watmough J. 2002. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences* **180(1–2)**:29–48 DOI 10.1016/S0025-5564(02)00108-6.

Weinstein RA, Nalca A, Rimoin AW, Bavari S, Whitehouse CA. 2005. Reemergence of monkeypox: prevalence, diagnostics, and countermeasures. *Clinical Infectious Diseases* **41(12)**:1765–1771 DOI 10.1086/498155.

WHO. 2017. Weekly Bulletin on Outbreaks and other Emergencies: week 18: 29 April–05 May 2017. *Weekly Bulletin on Outbreak and other Emergencies* 1–14.

Wollenberg A, Engler R. 2004. Smallpox, vaccination and adverse reactions to small-pox vaccine. *Current Opinion in Allergy and Clinical Immunology* **4(4)**:271–275 DOI 10.1097/01.all.0000136758.66442.28.

World Bank. 2019. Life expectancy at birth. *Available at* https://data.worldbank.org/indicator/SP.DYN.LE00.IN?cid=GPD_10 (accessed on 13 December 2019).

Wu B, Fu F, Wang L. 2011. Imperfect vaccine aggravates the long-standing dilemma of voluntary vaccination. *PLOS ONE* **6(6)**:e20577 DOI 10.1371/journal.pone.0020577.

Yinka-Ogunleye A, Aruna O, Ogoina D, Aworabhi N, Eteng W, Badaru S, Mohammed A, Agenyi J, Etebu E, Numbere T-W. 2018. Reemergence of human monkeypox in Nigeria, 2017. *Emerging Infectious Diseases* **24(6)**:1149.