Soft Computing (2021) 25:1065–1083
https://doi.org/10.1007/s00500-020-05204-z

METHODOLOGIES AND APPLICATION

A hybrid fuzzy-stochastic multi-criteria ABC inventory classification using possibilistic chance-constrained programming

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Published online: 27 July 2020

Abstract
Inventory classification is a fundamental issue in the development of inventory policy that assigns each inventory item to several classes with different levels of importance. This classification is the main determinant of a suitable inventory control policy of inventory classes. Therefore, a great deal of research is done on solving this problem. Usually, the problem of inventory classification is considered in a multi-criteria and uncertain environment. The proposed method in this paper inspired by the notion of heterogeneous decision-making problems in which decision-makers deal with different types of data. To this aim, a mathematical modeling-based approach is proposed considering different types of uncertainty in classification information. Demand information is considered to be stochastic due to its time-varying nature and cost information is considered to be fuzzy due to its cognitive ambiguity. A hybrid algorithm based on chance-constrained and possibilistic programming is proposed to solve the problems. Considering the stochastic nature of demand information, solving the proposed model using the hybrid algorithm, the classification of items to three classes of extremely important, class A, moderately important, class B, and relatively unimportant, class C, items are determined along with a minimum inventory level required to deal with the stochasticity of demands information. The proposed approach is applied to a case study of classifying 51 inventory items. The obtained results assigned 22%, 39%, and 39% of the items to A, B, and C classes, respectively.

Keywords Multi-criteria inventory classification · Chance-constrained programming · Possibilistic programming

1 Introduction

Inventory management is a crucial organizational challenge with a noticeable impact on profitability. Usually, inventory is accounted for about 30% of a company's asset and inventory carrying costs are approximated between 20 and 25% of total inventory value (Lambert and Stock 1993; Stevenson 1999). This magnitude of financial impacts made inventory control as an essential module of managerial decision making, both in practice and theory. The total expenditures of inventories holding are reduced through managing inventories, and subsequently, the profit of the company is raised. Inventory management systems can be considered in a spectrum from the simplest form of periodic manual control to advanced real-time computerized control. However, besides its importance, inventory management is a costly and time-consuming activity. Classic inventory management models are usually developed considering a single item, while practically; real-world systems contain several inventory items (Maiti et al. 2006). Therefore, adapting suitable systems for inventory control of different items is a crucial problem. Organizations are able to apply a similar exact and advanced inventory management system for all of the inventory items. This scenario can assure a similar level of control, but its costs may not be justifiable for a set of items with lower impact and importance. Therefore, it seems reasonable to localize inventory management systems according to the role and importance of the items.

Communicated by V. Loia.

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The next logical question that arises is which inventory management system to be used for which items. This issue can be formulated as a classification problem. For efficient management of inventories, they must be classified at first (Rezaei and Salimi 2015). Classification aims to assign each inventory item into classes with different degrees of importance. In the field of inventory management, this problem is called ABC analysis. Inventory managers use ABC analysis to classify the inventory items into three categories, i.e., A means extremely important, B means moderately important, and C means relatively unimportant, considering several criteria (Liu et al. 2015). The results of ABC inventory classification are used to develop inventory control policies, determining cycle counting frequencies, slot inventory for order picking, and other managerial activities (Sople 2012).

The importance of ABC inventory classification as a determinant of inventory management policies is therefore perceptible. This importance is even intensified during the Corona virus pandemic. Many companies and supply chains dealing with stock out and interruption in their supply chain feel a required necessity to revise their inventory control policies and a new and more sensitive classification method beyond their classic classification roles.

The classic ABC categorization method classifies items based on a single criterion, i.e., the annual dollar usage. This method is performed well for a set of justly homogeneous items that are differed only in their annual usage (Ramanathan 2006). However, there may be other criteria to come into management's attention. Many pieces of research have mentioned that in addition to this criterion, such other criteria are also needed for classification (Chen et al. 2008). Therefore, ABC inventory classification is a multi-criteria problem rather than considering a single criterion.

Classification criteria have different levels of importance based on the items considered and the industry that they are applied to. Prioritizing the importance of criteria is somewhat subjective in some real-world applications. Inventory management experts assign diverse importance to the criteria based on the conditions governing the industry and the market. For example, when items suppliers assure that they will prepare the necessary items in due time, it will reduce the importance of the lead time criterion in their opinion.

In the past 20 years, more researches have been conducted on multi-criteria inventory classification (MCIC). Various methods for categorizing inventory considering several criteria have been proposed. MCIC is one of the implicational areas of multi-criteria decision making (MCDM) (Wu and Tiao 2018; Maliene et al. 2018).

Flores et al. (1992) applied the Analytic Hierarchy Process (AHP) (Saaty 1980) to the MCIC problem. The supremacy of the AHP is that it is able to integrate a set of criteria with ease of use, but its weakness is that it majorly relies on subjective judgments in pairwise comparisons of criteria. They have used the AHP to aggregate multiple criteria in the form of a unique and consistent measure. Average unit cost and annual dollar usage have been taken as classification criteria in this study.

One of the usual methods to find a solution to the inventory problems is using mathematical optimization methods; like many other production management problems. The main steps in using optimization approach and Mathematical optimization are formulating the problem and solving the model. Defining the parameters in mathematical optimization approach is an important step. The defined parameters that are used in the model must be consistent with the real world and using the crisp numbers for parameters results in the unrealizable solution for models. Uncertainty is an intrinsic feature of real-world implementations.

Usually, the uncertainty can occur due to (1) partial or (2) approximate information (Pedrycz and Gomide 1998). Several frameworks are proposed to deal with uncertainty. Each framework has its characteristics and will be appropriate for special cases. While probability corresponds to the happening of well-defined events, fuzzy sets deal with gradual ambiguity or vagueness and describe their boundaries (Tang and Grubbström 2002; Razavi Hajiagha et al. 2019). Developing inventory control models under uncertainty is an accepted way of dealing with real-world incomplete and approximated information (Shekarian et al. 2017).

The main idea of the current study is taken from the notion of heterogeneous MCDM. In many decision-making (DM) problems, both qualitative and quantitative attributes are important. In these DM problems which precise calculation is almost unlikely, people's involvement is necessary to evaluate the attributes and assess the alternatives. In these situations, there may be different types of information such as fuzzy numbers, real numbers, and stochastic information. Heterogeneous DM methods can include multiple formats of information (Wan and Li 2013; Yu et al. 2018). In the inventory control framework, the behavior of demand along with time can be assessed with a probability distribution; while cost parameters ambiguity is often due to the lack of knowledge and does not behave stochastically. Therefore, as it is convenient, demands are taken into account as stochastic variables, while cost parameters are considered to be fuzzy numbers. This paper aims to combine them both in a singular model. Modeling inventory management in hybrid fuzzy-stochastic environments is also previously considered in some studies (Dutta et al. 2007).

The main contribution of the proposed paper is to propose a hybrid fuzzy-stochastic method to deal with the uncertain and heterogeneous nature of MCIC problems. In this context, as described above, demand-related information is considered to be stochastic due to the presence of historical data and the possibility of fitting a statistical distribution to this time-varying information. On the other hand, cost-related parameters are considered to be fuzzy information due to their cognitive uncertainty and lack of information, since usually in unstable markets, price behavior cannot be approximated by previous information. This heterogeneous formulation of the MCIC problem can be considered as the main novelty of this paper. Also, to solve the formulated multi-objective problem, a hybrid approach based on chance-constrained programming (Charnes and Cooper 1959) and possibilistic programming (Lai and Hwang 1992) is developed.

The paper is organized as follows. Section 2 includes an extensive review of previous studies. Determination of ABC classification criteria importance weights using the analytic hierarchy process is then described in Sect. 3. Mathematical problem formulation and its solving approach are explained in Sects. 4 and 5, respectively. The application of the proposed approach is then illustrated in a real-world case study. Finally, the paper is concluded in Sect. 6.

2 Literature review

Conventional ABC classification uses annual demand and item price as two classification criteria. It has been shown that categorizing items by their common order cycle could result in a similar grouping (Chakravarty 1981). Flores and Whybark (1986, 1987) are among the first researchers to consider other criteria as well. Subsequent to these authors, a significant number of MCDM methods have been used to deal with the MCIC problem.

Ramanathan (2006) proposed a weighted linear optimization method for multi-criteria ABC classification. A weighted additive function is developed to aggregate the performance of an inventory item in various criteria and a particular score, called the optimum inventory score, is approximated for items. An optimization model is proposed to determine criteria weights, and it has to be considered that this model is subject to the constraint that the sum of weights for all the items must be less than or equal to one. Four criteria of average unit cost, annual dollar usage, critical factor, and lead time are considered.

Ng (2007) presented a weighted linear optimization model for MCIC that evaluated a numeric score based on classification criteria. Optimum scores of inventory items could be handily acquired without a linear optimization.

Tsai and Yeh (2008) developed an inventory classification algorithm applying particle swarm optimization (PSO). In this model, inventory items can be categorized based on a specific objective or multiple objectives. Also, this method specified the best number of inventory category, and the way items should be categorized. To specify the best composition of the parameters of the algorithm quantity, some experiments are employed. Four item properties are used for item classification: item setup cost, unit holding cost, demand per unit time, and supplier ordering cost.

Chu et al. (2008) proposed an inventory control approach called ABC–fuzzy classification (ABC–FC). This method handles variables either with nominal or non-nominal characteristics. Also, the manager's expertise is gathered and implied in inventory classification. Annual demand, unit price, usage frequency, procurement lead time, current item status, the criticality of an inventory item, and severity of the impact of the inventory are considered to be classification criteria in their case study.

Chen (2011) proposed a peer-estimation approach for MCIC. This method specified two common sets of importance degrees of criteria and the two resulting performance scores are aggregated in the most favorable and least favorable senses for each item. In this approach, the DEA cross-efficiency method is improved for solving MCDM problems. A separate model is being solved to specify the weight coefficients for this aggregation. Annual dollar usage, average unit cost, and lead time are considered to be classification criteria in this study.

Hadi-Vencheh (2010) developed the Ng-model for the MCIC problem. Along with combining multiple criteria, this method also retains the effects of weights in the final score. In this study, annual dollar usage, average unit cost, and lead time are used as classification criteria.

Hadi-Venche and Mohamadghasemi (2011) developed an integrated fuzzy analytic hierarchy process–data envelopment analysis (FAHP-DEA) method for MCIC. In this method, FAHP is used to determine criteria importance using linguistic terms. The values of the linguistic terms are specified with DEA, and then, item scores under various measures are aggregated into a final score for each item with SAW (simple additive weighting). Annual dollar usage, limitation of warehouse space, average lot cost, and lead time are considered to be classification criteria in this study.

Torabi et al. (2012) aggregated the common weight MCDM–DEA model of Hatefi and Torabi (2010) and the imprecise data envelopment analysis (IDEA) model (Zhu 2003) and proposed a new DEA-based methodology for MCIC problem. The developed method is a linear programming model with enhanced discerning power that applied a common weight approach and can deal with both quantitative and qualitative criteria. Annual dollar usage, average unit cost, critical factor, and lead time are considered to be categorization criteria in this study.

Millstein et al. (2014) developed an optimization model to optimize the number of inventory groups, their commensurate service levels, and allotment of SKUs to groups when the inventory budget is constrained. The criteria used in their example are annual demand, gross profit per unit of SKU, inventory holding cost, fixed overhead management cost, and service level.

Rezaei and Salimi (2015) proposed an interval programming model for the MCIC problem. In this method, the values of demand, overage, and shortage costs for each item are estimated as interval numbers.

Liu et al. (2015) proposed an approach based on the non-compensatory ELECTRE method for the multi-criteria ABC classification. A combination of cluster analysis and the simulated annealing algorithm is used to search for the optimal categorization. In their example, inventory items evaluated based on four criteria: average unit cost, annual RMB usage, lead time, and turnover (rate).

Fu et al. (2016) developed the Ng-model (2007) based on a distance-based decision-making method. In this method, a set of common weights corresponding to all rankings of the criteria importance are specified, and finally, an inclusive scoring scheme is provided by aggregating all rankings. Three criteria, namely, annual dollar usage (ADU), average unit cost (AUC), and lead time (LT) are considered in this study.

Baykasoglu et al. (2016) proposed a fuzzy linear assignment method for multi-attribute group decision-making problems. Due to the uncertain nature of many decision-related problems, various concepts—from fuzzy set theory—like fuzzy arithmetic and aggregation, fuzzy ranking, and fuzzy mathematical programming are combined into a fuzzy concordance-based group decision-making process. In their case study, inventory items are appraised based on three criteria: Annual demand, Unit cost/part, Annual cost.

Shanshan et al. (2017) extended Ng-model (2007). The importance of criteria is computed through Shannon entropy. They solved an example using three criteria: annual dollar usage, average unit cost, and lead time. They also compared their result with Ng-model.

Yang et al. (2017) developed a mixed-integer linear programming model by considering the non-stationary demand for inventory items. Demand is discretized in time horizon into several time periods, and it is supposed that the demand for an item is distributed normally. The net present value is considered to be an objective function. Sales volume, coefficient of variation in demand, number of orders, shelf life, and gross profit are considered to be categorization criteria in their case study.

Using a randomized greedy strategy, López-Soto et al. (2017) designed a multi-start constructive algorithm to train a discrete artificial neural network for solving the MCIC problem. They investigated three data sets of ABC classifications. In the first data set, the criticality factor of the part, annual usage, the annual average cost per unit, and lead time are used as classification criteria. In the next two cases; unit price, ordering cost per lot; demand, and lead time are used.

Li et al. (2019) applied a version of stochastic multi-criteria acceptability analysis (SMAA-2) by considering classification criteria following two kinds of distribution functions, namely, uniform and normal distributions. They solved a problem consisting of average unit cost, annual dollar usage, and lead time.

Hadi-Vencheh et al. (2018) considered the information uncertainty with Gaussian interval type 2 fuzzy sets. They proposed two linear programming problems to arrive criteria involved in inventory items classification and then developed a TOPSIS approach to assign items into ABC classes. In their case study, they used annual dollar usage, lead time, average lot cost, limitation of warehouse space, and availability of the substitute raw material as ABC classification criteria.

Ishizaka et al. (2018) proposed a variant of data envelopment analysis introduced as DEA Sort to solve the MCIC problem. In their method, they used information obtained from the analytic hierarchy process (AHP) method to bound criteria weights based on managers' opinions. Annual usage value, frequency of issue per year, and current stock value are used as classification criteria in their study.

İsen and Boran (2018) proposed a hybrid method of inventory item classification. In their method, they optimized fuzzy c-means clustering using a genetic algorithm. Next, items are clustered using the fuzzy c-means method. Then, the output of fuzzy c-means is entered as an input to an Adaptive Neuro-Fuzzy Inference System (ANFIS) to create fuzzy rules. Cost, size, lead time, and critical factor are used as classification criteria.

Wu et al. (2018) developed a weighted least-square dissimilarity approach to solving the MCIC problem. They proposed a mathematical programming approach to derive criteria weights. They investigated an MCIC problem where inventory items are characterized by three criteria of annual dollar usage, average unit cost, and lead time.

Lolli et al. (2019) trained supervised classifiers to classify inventory items. Their proposed method is developed to deal with intermittent demands. They tested their method on two large datasets. Demand, lead time, purchasing, and holding costs are used in their case studies as classification criteria. Agarwal and Mittal (2019) also used multi-level association rule mining for the MCIC problem. They used the concept of loss profit to rank inventory items at different levels.

Sheikh-Zadeh and Rossetti (2019) defined the concept of artificial stocking policy as a classification criterion. Their model mainly focused on repairable items. Then, a non-subjective weighted linear scoring method is developed and a heuristic partitioning method is proposed for ranking items. In this method, the cost and demand of items, depot repair cycle time, repair cycle time at the base, depot-to-base resupply time, and the probability of items being repaired at the base are considered to be classification criteria.

Kheybari et al. (2019) used a combination of methods including entropy, the technique for ordering preferences based on similarity to ideal solution (TOPSIS), and goal programming to determine classification criteria weights. Then, the value of each item is calculated. Shannon's entropy is used to determine the criteria weights and the value of each item is determined using TOPSIS. Finally, items are clustered applying goal programming. In their numerical example, average unit cost, critical factor, annual dollar usage, and lead time are used as classification criteria.

Ersalan and Tansel iÇ (2019) developed an improved decision support system (IDSS) for MCIC problem. This IDSS includes two modules one of which is assigned to specific product characteristics and the other compares and ranks inventory items. The ABC analysis module includes annual dollar usage (ADU) and analytic hierarchy process (AHP) methods. A case study based on price, demand, lead time, criticality, and volume as classification criteria indicated that the AHP method produces more realistic results.

Douissa and Jabur (2019) proposed a classification approach based on ELECTRE III and computed a score for inventory items. The non-compensatory nature of ELECTREE III prohibited items with poor performance in one or some parameters to being classified as important (A class) items.

The above studies are summarized in Table 1. According to this table, it can be concluded that:

1. Research into multi-criteria ABC classification has drawn considerable attention until recent years taking into account the researches published in 2018 and 2019;
2. Two main streams are being ruled in multi-criteria ABC classification problems. A class of studies applied multi-attribute decision-making techniques, e.g., AHP, TOPSIS, and outranking methods. In another class, MCIC problem analysis includes formulating the problem as a mathematical programming one and then, developing methods and algorithms to solve this problem. The third stream including artificial intelligence and machine learning does also seem to be rising;
3. Since the MCIC problem required investigation of each inventory item according to different criteria—some are subjective and some partially known—uncertainty seems inevitable in these problems. However, considering 26 papers investigated, the frequency of different types of uncertainty is being considered and shown in Fig. 1.

Based on Fig. 1, it is evident that while uncertainty seems axiomatic, the main flow of papers considered MCIC problems as deterministic.

4. Considering criteria used in MCIC case studies, the criteria can be classified into four categories, including cost-related criteria, demand- or usage-related criteria, lead time, critical factor, and other factors. The frequency of criteria in these categories is shown in Fig. 2. Each category includes several criteria as illustrated in Table 2. The first category involves cost-related criteria (numbers in parenthesis indicate the frequency of the criterion being used).

3 Modeling of the hybrid multi-objective fuzzy-stochastic problem

3.1 Problem definition

Suppose that there are N stocks keeping units (SKUs). Each SKU_i's, $i = 1, 2, \ldots, N$, demand followed a normal distribution $\hat{d}_i$ with mean $\bar{d}_i$ and standard deviation of σ_i. Each SKU_i has a known lead time L_i. To develop a suitable inventory management system appropriate for SKUs status, the inventory manager wants to classify the N inventory items. It is assumed that a generic inventory policy, i.e., service level, is required for each group. Setting a high service level for all the SKUs will guarantee the highest desirability, but it is not practical since doing so incurs a significant amount of expenditure. The inventory holding cost for SKU_i is $\tilde{c}_i$ per unitestimated ambiguously as a fuzzy parameter. There is an overall budget of B assigned to inventory management in the considered time horizon and the inventory manager should design their inventory policy accordingly. Therefore, the inventory manager decided to classify SKUs into three subgroups of A, B, and C. To achieve this aim, a hybrid fuzzy-stochastic multi-criteria ABC inventory classification (HFSMCIC) model is proposed.

Table 1 A synthesis on previous studies of MCIC problem

Author	Year	Criteria uncertainty type				Analytical method(s)	Criteria used
		Fuzzy	Stochastic	Interval	Robust		
Ramanathan	2006	–	–	–	–	Weighted linear optimization	Average unit cost; Annual dollar usage; Critical factor; Lead time
Tsi and Yeh	2008	–	–	–	–	Particle swarm optimization	Setup cost; Unit holding cost; Demand per unit time; Supplier ordering cost
Chu et al.	2008	✔	–	–	–	Fuzzy classification analysis	Demand Unit price Usage frequency Procurement lead time Current item status Criticality Severity of the inventory running out
Hadi-Venche	2010	–	–	–	–	Nonlinear programming model	Annual dollar usage; Average unit cost; Lead time
Chen	2011	–	–	–	–	Peer-estimation approach	Annual dollar usage; Average unit cost; Lead time
Hadi-Venche and Mohamadghasemi	2011	✔	–	–	–	Fuzzy AHP-DEA approach	Annual dollar usage; Limitation of warehouse space; Average lot cost; Lead time
Torabi et al.	2012	–	–	–	–	Linear programming(DEA)	Annual dollar usage; Average unit cost; Lead time; Critical factor
Millstein et al.	2014	–	–	–	–	Mixed-integer linear program	Annual demand; Gross profit per unit of SKU; Inventory holding cost; Fixed overhead cost; Service level
Rezaei and Salimi	2015	–	–	✔	–	Parametric linear programming with interval number	Demand; Overage cost; Shortage cost.
Liu et al.	2015	–	–	–	–	Outranking model clustering analysis and the simulated annealing algorithm	Average unit cost; Annual RMB usage; Lead time; Turnover (rate)
Fu et al.	2016	–	–	✔	–	Distance-based decision-making method	Annual dollar usage; Average unit cost; Lead time
Baykasoglu et al.	2016	✔	–	–	–	Fuzzy linear assignment method	Durability Availability Criticality Replenishment time Total annual cost

Table 1 (continued)

Author	Year	Criteria uncertainty type				Analytical method(s)	Criteria used
		Fuzzy	Stochastic	Interval	Robust		
Yang et al.	2017	–	✔	–	–	Mixed-integer linear programming	Sales volume; Coefficient of variation in demand; Number of orders; Shelf life; Gross profit
Li et al.	2017	–	✔	–	–	Stochastic multi-criteria acceptability analysis	Average unit cost; Annual dollar usage; Lead time
Shanshan et al.	2017	–	–	–	–	Shannon entropy, Mathematical programming	Annual dollar usage; Average unit cost; Lead Time
López-Soto et al.	2017	–	–	–	–	Artificial neural network	Annual usage; Annual average cost per unit; Criticality factor of the part; Lead time; Unit price; Ordering cost per lot
Hadi-Vencheh et al.	2018	✔	–	–	–	Linear programming, TOPSIS	Annual dollar usage; Lead time; Average lot cost; Limitation of warehouse space; Availability of the substitute raw material
Ishizaka et al.	2018	–	–	–	–	DEA, AHP	Annual usage value; Frequency of issue per year; Current stock value
İsen and Boran	2018	–	–	–	–	Fuzzy c-means, genetic algorithm, ANFIS	Cost; Size; Lead time; Critical factor
Wu et al.	2018	–	–	–	–	Mathematical programming	Annual dollar usage; Average unit cost; Lead Time
Kheybari et al.	2019	–	–	–	–	Shannon' entropy, TOPSIS, goal programming	Average unit cost Critical factor Annual dollar usage Lead time
Lolli et al.	2019	–	–	–	–	Machine learning	Demand Lead time Purchasing cost Holding cost
Sheikh-Zadeh and Rossetti	2019	–	–	–	–	Mathematical modeling	Cost Demand Depot repair cycle time Repair cycle time at base Depot-to-base resupply time Probability of items being repaired at base

Table 1 (continued)

Author	Year	Criteria uncertainty type				Analytical method(s)	Criteria used
		Fuzzy	Stochastic	Interval	Robust		
Ersalan and Tansel iÇ	2019	–	–	–	–	Improved decision support system	Price Demand Lead time Criticality Volume
Douissa and Jabeur	2019					ELECTRE III	Annual Dollar Usage Annual Unit Cost Lead Time Critical Factor
Proposed method		✔	✔	–	–	Mathematical programming	Stochastic demand Unit price Current stock value Lead time Criticality factor of the part

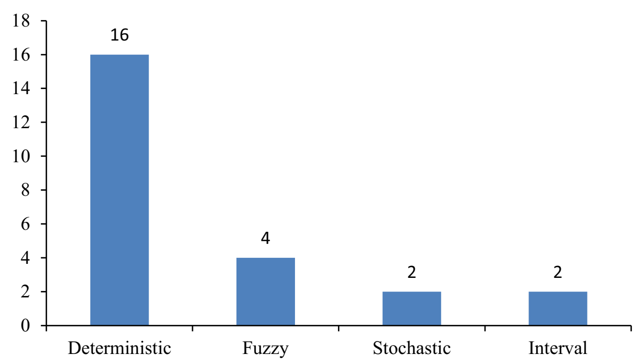

Fig. 1 Frequency distribution of information uncertainty in MCIC problems

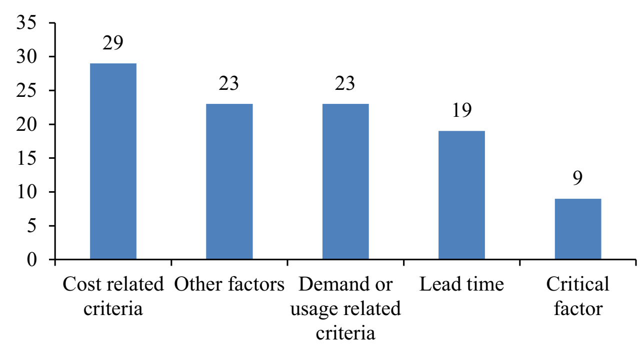

Fig. 2 Frequency of criteria being used for MCIC

3.2 Multi-criteria formulation of ABC inventory classification problem

Before proceeding to the modeling of inventory classification problem, it is notable that the multi-criteria nature of ABC inventory classification problems requires investigating each inventory item according to several criteria. Suppose that there is a set of K criteria, $\{C_1, C_2, \ldots, C_K\}$. Also, suppose that performance of inventory items (SKUs) in these criteria are illustrated in the following decision matrix:

	C_1	C_2	$\cdots$	C_K
SKU_1	x_{11}	x_{12}	$\cdots$	x_{1K}
SKU_2	x_{21}	x_{22}	$\cdots$	x_{2K}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
SKU_N	x_{N1}	x_{N2}	$\cdots$	x_{NK}

where x_{ik} denoted the performance of SKU_i with regard to criterion C_j. To obtain an aggregated performance for SKU_i, i.e., F_i, the above matrix information could be aggregated as

$$F_i = \sum_{j=1}^{K} u_j r_{ij} \tag{1}$$

In Eq. (1),

Table 2 The criteria being used in MCIC

Class	Criterion	Frequency	Class	Criterion	Frequency
Other factors	Current item status	2	Cost-related criteria	Average unit cost	13
	Durability	2		Inventory holding cost	3
	Frequency of issue per year	2		Average cost	2
	Gross profit	2		Average lot cost	2
	Limitation of warehouse space	2		Ordering cost per lot	2
	Sales volume	2		Unit price	2
	SIZE	1		Shortage cost	1
	Availability	1		Fixed overhead cost	1
	Availability of the substitute	1		Setup cost	1
	Coefficient of variation in demand	1		Annual average cost per unit	1
	Price	1		Total annual cost	1
	SERVICE level	1	Demand-related criteria	Annual dollar usage	14
	Severity of inventory running out	1		Annual demand	8
	Turnover rate	1		Usage frequency	1
	Repair cycle time at base	1	Lead time		19
	Depot-to-base resupply time	1	Critical factor		9
	Probability of items being repaired at base	1			

$$r_{ij} = \frac{r_{ij} - r_{j*}}{r_j^* - r_{j*}} \tag{2}$$

Is the normalized performance of SKU$_i$ with regard to criterion C_j, where r_j^* and r_{j*} are the ideal and anti-ideal performances over criterion j. also, $u_j, j = 1, 2, \ldots, K$ are the weights of criteria. In this paper, AHP is used to derive these weights. Constructing a set of pairwise matrices for a group of experts, $E_l, l = 1, 2, \ldots, L$, each expert provides their pairwise matrix as illustrated in Table 3.

Where, $a_{ij}^l \in \{1/9, 1/8, \ldots, 8, 9\} - \{0\}$, according to the scale proposed by Saaty (1977, 1990), is the ratio of preference of C_i to C_j from the viewpoint of lth expert. Using the geometric averaging operator, these pairwise matrices are aggregated as $P = [a_{ij}], a_{ij} = \left(a_{ij}^1 \cdot a_{ij}^2 \cdot \cdots \cdot a_{ij}^L\right)^{1/L}$. Saaty (1977) proposed using eigenvectors corresponding largest eigenvalue of pairwise matrix. Using this method in this paper, the criteria weighting vector $(u_1, u_2, \ldots, u_K)$ is determined and applying this vector to Eq. (1), the overall performance of each SKU based on all the classification criteria is then calculated. This overall performance vector $(f_1, f_2, \ldots, f_N)$ will be used in the next sections to extend the inventory classification problem.

Table 3 Pairwise comparison matrix of criteria

		C_1	C_2	$\cdots$	C_K
P^l	C_1	1	a_{12}^l	$\cdots$	a_{1K}^l
	C_2	a_{21}^l	1	$\cdots$	a_{2K}^l
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	C_K	a_{K1}^l	a_{K2}^l	$\cdots$	1

3.3 Formulation as a multi-objective programming problem

Considering N SKUs to be classified in 3 classes of A (j = 1), B (j = 2), and C (j = 3), in this section, the HFSMCIC problem is formulated as a multi-objective programming problem.

The first objective seeks to maximize the total value of SKUs classification. Each SKU$_i$ has a stochastic demand of $\hat{d}_i$ with an evaluated performance of F_i, obtained using Eq. (1). Also, decision maker considered a service level of $\alpha_j, j = 1, 2, 3$ for each group in a way that $\alpha_1 > \alpha_2 > \alpha_3$. Therefore, the first objective is proposed to maximize total score of assigning SKUs to each class. This objective is formulated as:

$$\text{Max} \sum_{i=1}^{N} \sum_{j=1}^{3} F_i \hat{d}_i x_{ij} \alpha_j \tag{3}$$

Hereafter, $\forall i \in \{1, 2, \ldots, N\}$ and $\forall j \in \{1, 2, 3\}$,

$$x_{ij} = \begin{cases} 1 & \text{if SKU}_i \text{ is assigned to inventory class } j \\ 0 & \text{otherwise} \end{cases}$$

On the other hand, considering stochastic nature of SKUs demand, the overall value obtained in Eq. (3) will have a variance. The aim of the second objective is to minimize $\mathrm{Var}\left(\sum_{i=1}^{N}\sum_{j=1}^{3}F_i\hat{d}_iX_{ij}\alpha_j\right)$. This objective is formulated as:

$$\mathrm{Min}\sum_{i=1}^{N}\sum_{j=1}^{3}\left(F_i\sigma_i\alpha_j\right)^2x_{ij}^2 \tag{4}$$

Also, since assigning each SKU to each class imposes some overhead costs due to different inventory control policies; the third objective is formulated to minimize the total overhead cost of SKUs assignment to inventory classes as below:

$$\mathrm{Min}\sum_{j=1}^{3}\tilde{\omega}_j\left(\sum_{i=1}^{N}x_{ij}\right) \tag{5}$$

where $\tilde{\omega}_j,\quad j=1,2,3$ is the fixed overhead management cost of inventory group j. Equations (3)–(5) are three considered objectives of the problem.

The first constraint is a rational constraint imposed to assign each SKU to just one class:

$$\sum_{j=1}^{3}x_{ij}=1,\forall i \tag{6}$$

Second constraint is to assure that if SKU_i is assigned to class j, then its inventory level should always be greater than its lead time demand $\hat{d}_iL_i$ with at least probability α_j. This constraint can be illustrated as:

$$\Pr\left(\hat{d}_iL_ix_{ij}\le v_i\right)\ge\alpha_j,\quad\forall i,j \tag{7}$$

In Eq. (7), $v_i,\quad i=1,2,\ldots,N$ is the minimum required inventory level of $\mathrm{SKU}_i,\quad\forall i$. This constraint is transformed to an equivalent constraint that can be handled efficiently. To achieve this aim, it is notable that $E\left[d_iL_ix_{ij}\right]=\bar{d}_iL_ix_{ij}$ and $\mathrm{Var}\left[d_iL_ix_{ij}\right]=(\sigma_iL_i)^2x_i^2$. Then, the constraint is reformulated as:

$$\Pr\left(\frac{d_iL_ix_{ij}-\bar{d}_iL_ix_{ij}}{\sigma_iL_ix_{ij}}\le\frac{v_i-\bar{d}_iL_ix_{ij}}{\sigma_iL_ix_{ij}}\right)\ge\alpha_j,\quad\forall i,j \tag{8}$$

If $\phi^{-1}(\alpha_j)$ is defined as the converse cumulative standard normal distribution at satisfaction level α_j, Eq. (8) is transformed as below:

$$\frac{v_i-\bar{d}_iL_ix_{ij}}{\sigma_iL_ix_{ij}}\ge\phi^{-1}(\alpha_j)\rightarrow v_i-\left(\bar{d}_i+\sigma_i\phi^{-1}(\alpha_j)\right)x_{ij}\ge0,\forall i,j \tag{9}$$

Third constraint is to limit the expenditure amount of SKUs inventory levels.

$$\sum_{i=1}^{N}\tilde{c}_iv_i\le B,\forall i \tag{10}$$

This constraint is a fuzzy type constraint. If $\tilde{c}_i=(c_{i1},c_{i2},c_{i3},c_{i4})$ is a trapezoidal fuzzy number, Jimenez et al. (2007) proposed the below equivalent constraint at a satisfaction level of α, based on the notion of expected interval and expected value of fuzzy numbers (Heilpern 1992) and the concept of the degree in which a fuzzy number is greater than another, introduced by Jimenez (1996).

$$\sum_{i=1}^{N}\left[(1-\alpha)\frac{c_{i1}+c_{i2}}{2}+\alpha\frac{c_{i3}+c_{i4}}{2}\right]v_i\le B,\quad\forall i \tag{11}$$

Using the above relations, the multi-objective HFSMCIC problem is constructed as below:

$$\begin{array}{ll}
\mathrm{Max}\sum_{i=1}^{N}\sum_{j=1}^{3}f_i\hat{d}_ix_{ij}\alpha_j & \text{(a)}\\
\mathrm{Min}\sum_{i=1}^{N}\sum_{j=1}^{3}\sigma_i\left(f_ix_{ij}\alpha_j\right)^2 & \text{(b)}\\
\mathrm{Min}\sum_{j=1}^{3}\tilde{\omega}_j\left(\sum_{i=1}^{N}x_{ij}\right) & \text{(c)}\\
\text{S.T.}\quad\sum_{j=1}^{3}X_{ij}=1,\quad\forall i & \text{(d)}\\
v_i-\left(\bar{d}_i+\sigma_i\phi^{-1}(\alpha_j)\right)x_{ij}\ge0,\quad\forall i,j & \text{(e)}\\
\sum_{i=1}^{N}\left[(1-\alpha)\frac{c_{i1}+c_{i2}}{2}+\alpha\frac{c_{i3}+c_{i4}}{2}\right]v_i\le B,\quad\forall i & \text{(f)}\\
x_{ij}\in\{0,1\},\quad\forall i,j & \text{(g)}\\
v_i\ge0,\quad\forall i & \text{(h)}
\end{array} \tag{12}$$

4 Solving approach

The model proposed in Eq. (12) is a hybrid multi-objective fuzzy-stochastic problem. Considering the second objective, the above model is a multi-objective constrained quadratic binary programming problem. In this section, a hybrid approach is proposed to solve the above problem, inspiring from chance-constrained programming to deal with stochastic constraints and objectives, and possibilistic programming to handle fuzzy objectives and constraints.

According to Abdelaziz et al. (2007) and Ekhtiari and Ghoseiri (2013), to transform this stochastic objective into an equivalent deterministic inequality, an ideal value is required for the first objective. To this aim, ideal value of the first objective is determined by letting $j=1$ and $x_{i1}=1$. The ideal value is determined as $F_1^*=\sum_{i=1}^{N}f_i\alpha_1(\bar{d}_i+3\sigma_i)$. Now, let

$$\mathrm{Prob}\left(\sum_{i=1}^{N}\sum_{j=1}^{3}f_i\hat{d}_ix_{ij}\alpha_j\le F_1^*\right)\le\alpha \tag{13}$$

Now, the first objective is restated as:

$$\begin{aligned}&\text{Min } y\\&\text{Prob}\left(\sum_{i=1}^{N}\sum_{j=1}^{3} f_i\hat{d}_i x_{ij}\alpha_j + y \geq F_1^*\right) \geq 1-\alpha\end{aligned} \tag{14}$$

Then,

$$\begin{aligned}&\text{Min } y\\&\text{Prob}\left(F_1^* - \sum_{i=1}^{N}\sum_{j=1}^{3} f_i\hat{d}_i x_{ij}\alpha_j \leq y\right) \geq 1-\alpha\end{aligned} \tag{15}$$

Since $\hat{d}_i$ is normally distributed, $F_1^* - \sum_{i=1}^{N}\sum_{j=1}^{3} f_i\hat{d}_i x_{ij}\alpha_j$ also normally distributed with mean of $F_1^* - \sum_{i=1}^{N}\sum_{j=1}^{3} f_i\bar{d}_i\alpha_j x_{ij}$ and standard deviation of $\sum_{i=1}^{N}\sum_{j=1}^{3} f_i\sigma_i\alpha_j x_{ij}$. To achieve the equivalent form of Eq. (15), considering $\phi(1-\alpha) = \text{Prob}(z \leq 1-\alpha)$, then it can be concluded that:

$$\frac{y - \left(F_1^* - \sum_{i=1}^{N}\sum_{j=1}^{3} f_i\bar{d}_i\alpha_j x_{ij}\right)}{\sum_{i=1}^{N}\sum_{j=1}^{3} f_i\sigma_i\alpha_j x_{ij}} \geq \phi^{-1}(1-\alpha) \tag{16}$$

And therefore,

$$y \geq \phi^{-1}(1-\alpha)\sum_{i=1}^{N}\sum_{j=1}^{3} f_i\sigma_i\alpha_j x_{ij} + F_1^* - \sum_{i=1}^{N}\sum_{j=1}^{3} f_i\bar{d}_i\alpha_j x_{ij} \tag{17}$$

On the other hand, since for $x_{ij} \in \{0,1\}$, $x_{ij}^2 = x_{ij}$, the third objective, Eq. (11c), is transformed into a multi-objective mixed binary programming model. In this case, the second objective can be stated as $\sum_{i=1}^{N}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij}$.

Using these modifications, the problem in Eq. (13) is transformed into the following problem which is a multi-objective linear mixed binary programming model.

$$\begin{aligned}&\text{Min} y\\&\text{Min}\sum_{i=1}^{N}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij}\\&\text{Min}\sum_{j=1}^{3}\tilde{\omega}_j\left(\sum_{i=1}^{N} x_{ij}\right)\\&\text{S.T.}\quad \sum_{j=1}^{3} X_{ij} = 1, \quad \forall i\\&y \geq \phi^{-1}(1-\alpha)\sum_{i=1}^{N}\sum_{j=1}^{3} f_i\sigma_i\alpha_j x_{ij} + F_1^* - \sum_{i=1}^{N}\sum_{j=1}^{3} f_i\bar{d}_i\alpha_j x_{ij}\\&v_i - \left(\bar{d}_i + \sigma_i\phi^{-1}(\alpha_j)\right)x_{ij} \geq 0, \quad \forall i,j\\&\sum_{i=1}^{N}\left[(1-\alpha)\frac{c_{i1}+c_{i2}}{2} + \alpha\frac{c_{i3}+c_{i4}}{2}\right]v_i \leq B,\\&x_{ij} \in \{0,1\}, \quad \forall i,j\\&v_i \geq 0, \quad \forall i\end{aligned} \tag{18}$$

Now, suppose that the solution space of the above problem at satisfaction levels $\alpha_1, \alpha_2, \alpha_3$ and α is illustrated as $S_{(\alpha_1,\alpha_2,\alpha_3,\alpha)}$.

At the first step of the proposed solving approach, the below problems are solved:

$$\begin{aligned}&\text{Min} y\\&\text{S.T.}\quad (x,v) \in S_{(\alpha_1,\alpha_2,\alpha_3,\alpha)}\end{aligned} \tag{19}$$

$$\begin{aligned}&\text{Min}\sum_{i=1}^{N}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij}\\&\text{S.T.}\quad (x,v) \in S_{(\alpha_1,\alpha_2,\alpha_3,\alpha)}\end{aligned} \tag{20}$$

$$\begin{aligned}&\text{Min}\sum_{j=1}^{3}\tilde{\omega}_j\left(\sum_{i=1}^{N} x_{ij}\right)\\&\text{S.T.}\quad (x,v) \in S_{(\alpha_1,\alpha_2,\alpha_3,\alpha)}\end{aligned} \tag{21}$$

To handle the fuzzy objective of the third model in Eq. (21), according to Jimenez et al. (2007), the expected value of the objective function, when $w_j = \left(w_{j1}, w_{j2}, w_{j3}, w_{j4}\right), j = 1,2,3$ is a trapezoidal fuzzy number, is represented as below:

$$\begin{aligned}&\text{Min}\sum_{j=1}^{3}\left(\frac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{N} x_{ij}\right)\\&\text{S.T.}\quad (x,v) \in S_{(\alpha_1,\alpha_2,\alpha_3,\alpha)}\end{aligned} \tag{22}$$

Solving these problems, the anti-ideal value of the first objective, i.e., F_{1*}, and the ideal values of second and third objectives are obtained as F_2^*, and F_3^*, respectively. Then, the anti-ideal values of second and third objectives are obtained reversing their objectives from Min to Max. These anti-ideal objectives are illustrated as F_{2*}, and F_{3*}. Now, following Zimmermann (1978, 1983), Tiwari et al. (1986), Sadeghi et al. (2013), Razavi Hajiagha et al. (2014), and the membership function of the first objective function can be constructed as below:

$$\mu_1(x,v) = \begin{cases} 1 & \text{if}\quad y \geq F_1^* \\ \dfrac{y - F_{1*}}{F_1^* - F_{1*}}, & \text{if}\ F_1^* \leq y \leq F_{1*} \\ 0 & \text{if}\quad F_{1*} \geq y \end{cases} \tag{23}$$

Similarly, the membership functions of the second and third objectives are formulated as

$$\mu_2(x,v) = \begin{cases} 1, & \text{if}\ \sum_{i=1}^{N}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij} \leq F_2^* \\ \dfrac{F_{2*} - \sum_{i=1}^{N}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij}}{F_{2*} - F_2^*}, & \text{if}\ F_2^* \leq \sum_{i=1}^{N}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij} \leq F_{2*} \\ 0, & \text{if}\ F_{2*} \leq \sum_{i=1}^{N}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij} \end{cases} \tag{24}$$

$$\mu_3(x,v) = \begin{cases} 1, & \text{if } \sum_{j=1}^{3}\left(\frac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{N} x_{ij}\right) \le F_3^* \\ \frac{F_{3*} - \sum_{j=1}^{3}\left(\frac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{N} x_{ij}\right)}{F_{3*} - F_3^*}, & \text{if } F_3^* \le \sum_{j=1}^{3}\left(\frac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{N} x_{ij}\right) \le F_{3*} \\ 0, & \text{if } F_{3*} \le \sum_{j=1}^{3}\left(\frac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{N} x_{ij}\right) \end{cases} \quad (25)$$

At this stage, an additional constraint is added to the model to assure the distribution of items between three classes. Therefore, if $p_j, j = 1,2,3$ is defined as the percentage (of number) of items in jth class, a constraint of the following type is added to the model:

$$\sum_{i=1}^{N} x_{ij} \le p_j, \quad j = 1,2,3 \quad (26)$$

Based on these membership functions, the solution of the main multi-objective problem is obtained by solving the following single-objective binary programming problem using the max–min operator of Zimmermann (1978):

$$\begin{aligned} &\text{Max Min}\{\mu_1(x,v), \mu_2(x,v), \mu_3(x,v)\} \\ &\text{S.T.} \quad \sum_{j=1}^{3} X_{ij} = 1, \quad \forall i \\ &\quad 0 \le \mu_1(x,v) \le 1 \\ &\quad 0 \le \mu_2(x,v) \le 1 \\ &\quad 0 \le \mu_3(x,v) \le 1 \\ &\quad y \ge \phi^{-1}(1-\alpha)\sum_{i=1}^{N}\sum_{j=1}^{3} f_i\sigma_i\alpha_j x_{ij} + F_1^* - \sum_{i=1}^{N}\sum_{j=1}^{3} f_i\bar{d}_i\alpha_j x_{ij} \\ &\quad v_i - \left(\bar{d}_i + \sigma_i\phi^{-1}(\alpha_j)\right)x_{ij} \ge 0, \quad \forall i,j \\ &\quad \sum_{i=1}^{N}\left[(1-\alpha)\frac{c_{i1}+c_{i2}}{2} + \alpha\frac{c_{i3}+c_{i4}}{2}\right]v_i \le B, \quad \forall i \\ &\quad \sum_{i=1}^{N} x_{ij} \le p_j, \quad j = 1,2,3 \\ &\quad x_{ij} \in \{0,1\}, \quad \forall i,j \\ &\quad v_i \ge 0, \quad \forall i \end{aligned} \quad (27)$$

Or using additive model of Tiwari et al. (1986):

$$\begin{aligned} &\text{Max } \mu_1(x,v) + \mu_2(x,v) + \mu_3(x,v) \\ &\text{S.T.} \sum_{j=1}^{3} X_{ij} = 1, \quad \forall i \\ &\quad 0 \le \mu_1(x,v) \le 1 \\ &\quad 0 \le \mu_2(x,v) \le 1 \\ &\quad 0 \le \mu_3(x,v) \le 1 \\ &y \ge \phi^{-1}(1-\alpha)\sum_{i=1}^{N}\sum_{j=1}^{3} f_i\sigma_i\alpha_j x_{ij} + F_1^* - \sum_{i=1}^{N}\sum_{j=1}^{3} f_i\bar{d}_i\alpha_j x_{ij} \\ &\quad v_i - \left(\bar{d}_i + \sigma_i\phi^{-1}(\alpha_j)\right)x_{ij} \ge 0, \quad \forall i,j \\ &\quad \sum_{i=1}^{N}\left[(1-\alpha)\frac{c_{i1}+c_{i2}}{2} + \alpha\frac{c_{i3}+c_{i4}}{2}\right]v_i \le B, \quad \forall i \\ &\quad \sum_{i=1}^{N} x_{ij} \le p_j, \quad j = 1,2,3 \\ &\quad x_{ij} \in \{0,1\}, \quad \forall i,j \\ &\quad v_i \ge 0, \quad \forall i \end{aligned} \quad (28)$$

The proposed methodology to solve the model is illustrated in Fig. 3.

Both models in Eqs. (27) and (28) are binary programming ones. Both of these approaches can be used separately or comparably to determine the problem solution. The model in Eq. (27) is used the Zimmermann (1978) min–max approach. This approach can be considered as a cautious and non-compensatory approach to solving the problem. On the other hand, the problem in Eq. (28) illustrated a compensatory approach that allows the compensation among objectives. Both methods can be applied to solve the problem, and there is no guarantee that one of them can be considered as the more preferred one in all situations. The main suggestion is to use both approaches in each occasion and to compare and choose the better one. This problem can be solved using ordinary optimization packages, e.g., Lingo or GAMS. A real-world case study along with its corresponding discussion is presented in the next section.

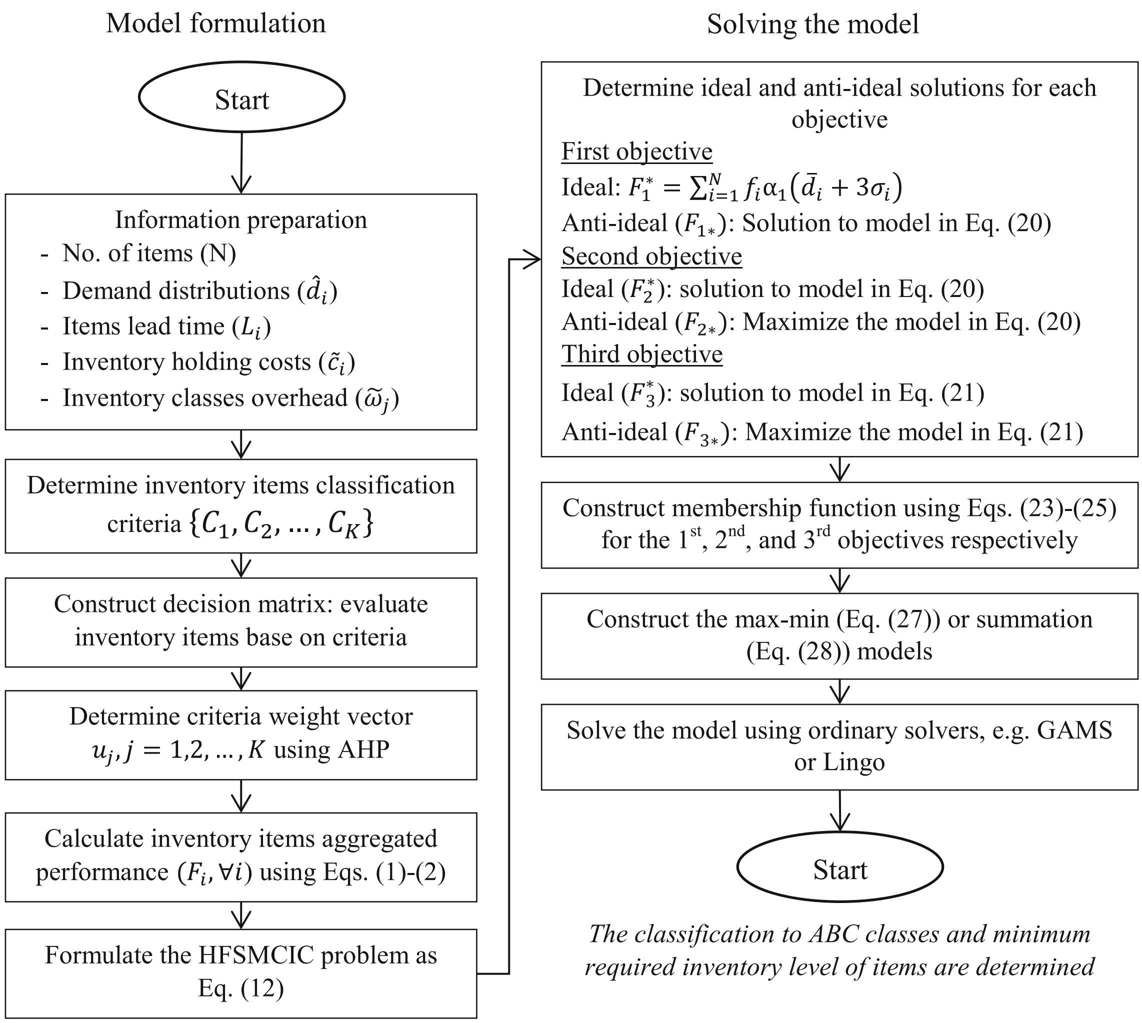

Fig. 3 Flowchart of solving methodology

5 Case study

Dastgireh Iran Tolerance (DIT) is a manufacturer of door handle and plaques in Iran with more than 30 years of relevant background. The advent of new competitors caused DIT to become more sensitive to its expenditures to maintain and enhance its competitive advantage and market share. According to the impact of inventory management systems costs on the company's overall cost, the necessity of monitoring and improving the performance of this system became evident. Therefore, the company decided to restudy its inventory management system status.

This company carries 51 types of inventory items and parts in its warehouse. A large percentage of these items can be classified as easy-to-acquire items that do not need to be controlled through rigorous and exact inventory control methods. Therefore, the managers of DIT decided to classify their inventory items using a multi-criteria ABC classification analysis.

In the first step, the classification criteria are identified based on the literature. Considering Fig. 2, four criteria are identified to be:

- Unit price (C_1);
- Current stock value (C_2);
- Lead time (C_3);
- Criticality factor of the part (C_4);

In this stage, using pairwise comparisons, the importance of the above criteria is identified. To this aim, a group of three experts completed their pairwise matrices and using geometric average (Saaty (1989)), the group pairwise matrix is constructed as Table 4.

Using MATLAB, the largest eigenvalue of the above pairwise matrix is obtained as $\lambda_{\max} = 4.0145$ with the corresponding eigenvector of $(-0.8060, -0.4251, -0.1713, -0.3745)$. The consistency ratio of this matrix is calculated to be 0.0054 that is less than 0.1. Therefore, normalizing the eigenvector, the weight vectors of criteria are obtained to be $(0.4536, 0.2392, 0.0964, 0.2107)$.

Raw data for inventory items are represented in Table 5. The demand for each item is approximated using a normal distribution, where the numbers in column $\hat{d}_j$ of Table 5 indicate its mean and variance that are approximated to be equal numbers. Also, the last column of the table, titled f_i, is calculated using Eq. (1).

The HFSMCIC model is formulated based on the above information. Solving the models in Eqs. (19)–(21), the ideal and anti-ideal objective values, i.e., F_r^* and F_{r*}, respectively, are determined as shown in Table 6. In these models, $(\alpha_1, \alpha_2, \alpha_3, \alpha) = (0.8, 0.6, 0.5, 0.1)$.

Based on these values, the membership functions can be developed as follows:

$$\mu_2(x,v) = \begin{cases} 1, & \text{if } \sum_{i=1}^{51}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij} \le 87431.327 \\ \dfrac{223824.197 - \sum_{i=1}^{51}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij}}{136392.9}, & \text{if } 87431.327 \le \sum_{i=1}^{51}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij} \le 223824.197 \\ 0, & \text{if } 223824.197 \le \sum_{i=1}^{51}\sum_{j=1}^{3}\left(f_i\alpha_j\sigma_i\right)^2 x_{ij} \end{cases}$$

$$\mu_3(x,v) = \begin{cases} 1, & \text{if } \sum_{j=1}^{3}\left(\dfrac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{51} x_{ij}\right) \le 446.25 \\ \dfrac{6477 - \sum_{j=1}^{3}\left(\dfrac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{N} x_{ij}\right)}{6030.75}, & \text{if } 446.25 \le \sum_{j=1}^{3}\left(\dfrac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{51} x_{ij}\right) \le 6477 \\ 0, & \text{if } 6477 \le \sum_{j=1}^{3}\left(\dfrac{w_{j1}+w_{j2}+w_{j3}+w_{j4}}{4}\right)\left(\sum_{i=1}^{51} x_{ij}\right) \end{cases}$$

Inventory management department determines that service levels of 80%, 60%, and 50% are required for inventory classes A, B, and C, respectively. The overhead cost associated with developing and maintaining an exact computerized system for inventory items of class A is approximated to be (80\$, 120\$, 140\$, 168\$) per item. An overhead cost for developing and controlling an inter-company-based system for inventory class B is approximated to be (25\$, 40\$, 55\$, 77\$) per item and the human-based inventory control system for items in class C is approximated to have an overhead cost of (5\$, 8\$, 10\$, 12\$) per item.

Table 4 Group pairwise matrix of classification criteria

	C_1	C_2	C_3	C_4
C_1	1.00	1.714	5.60	1.980
C_2	0.583	1.000	2.289	1.120
C_3	0.178	0.437	1.00	0.500
C_5	0.505	0.892	2.00	1.00

$$\mu_1(x,v) = \begin{cases} 0, & \text{if } y \le 26153900000 \\ \dfrac{y - 26153900000}{1425089}, & \text{if } 26153900000 \le y \le 26155325089 \\ 1, & \text{if } 26155325089 \le y \end{cases}$$

Now, solving the model in Eq. (27),—considering 11 items in class A, 20 items in class B, and 20 items in class C—the optimal objective value is obtained as 2.647. On the other hand, the optimal objective value of Eq. (28) is obtained to be 0.65. The membership and objective values of three objectives are demonstrated in Table 7.

According to Table 7, the results obtained from solving the model in Eq. (27) outperformed the results of Eq. (28). That is to say since the obtained membership value for the first and third objective are equal in both models, and the max–min model attained a higher membership value from the additive model. Therefore, the results from Eq. (27) are preferred to Eq. (28). However, it is not a general case. The main advantage of the maxi-min approach against the additive model, in this case, is that it improves the variance of inventory items classification about 47% regard to additive model, meaning a more stable classification result in the uncertain context of the study.

Table 5 Data for inventory items

Code	Inventory item	Unit	Criteria						Holding cost
			$\hat{d}_i$	Unit price	Criticality degree	Lead time	Inventory on hand	f_i	
1	Zamak	kg	89,000	0.01	9	2	7000	0.4578	(0.15, 0.29, 0.44, 0.59)
2	Seamless aluminum pipe	Beam	160	0.02	7	1	13	0.4089	(0.09, 0.17, 0.26, 0.35)
3	Seamless brass pipe	Beam	380	0.03	7	1	30	0.4187	(0.16, 0.32, 0.48, 0.64)
4	Corrugated brass pipe	Beam	1150	0.04	7	1	80	0.4221	(0.18, 0.37, 0.55, 0.74)
5	Oily flat sheet 25.1 mm	m^2	420	0.05	7	2	30	0.4049	(0.03, 0.07, 0.1, 0.13)
6	Oily flat sheet 1 mm	m^2	2800	0.06	8	2	200	0.4314	(0.04, 0.07, 0.11, 0.15)
7	Galvanized flat sheet 4.0 mm	m^2	390	0.07	7	1	30	0.4018	(0.03, 0.07, 0.1, 0.14)
8	Galvanized flat sheet 5.1 mm	m^2	390	0.08	7	1	30	0.4028	(0.04, 0.08, 0.12, 0.17)
38	Lubricant spray	Quantity	596	0.09	1	1	48	0.2461	(0.05, 0.1, 0.16, 0.21)
39	Stabilizer	Liter	170	0.1	2	2	10	0.2766	(0.06, 0.12, 0.17, 0.23)
45	Acetic acid (vinegar)	Liter	740	0.11	5	1	60	0.3499	(0.04, 0.08, 0.12, 0.16)
48	Chromic acid	kg	1500	0.12	5	1	100	0.5477	(1.5, 3, 4.5, 6)
51	Laboratorial nitric acid	Liter	76	0.13	5	1	4	0.3727	(0.21, 0.42, 0.62, 0.83)
60	Cyanide Copper plater A	Liter	760	0.14	3	2	60	0.3623	(0.5, 0.99, 1.49, 1.98)
61	Cyanide Copper plater B	Liter	760	0.15	3	2	60	0.3936	(0.73, 1.45, 2.18, 2.91)
161	10*1000 m Bolt	Quantity	1450	0.16	6	3	110	0.3775	(0.002, 0.003, 0.005, 0.006)
182	8*1000 m Bolt	Quantity	1450	0.17	6	3	110	0.3774	(0.001, 0.003, 0.004, 0.005)
274	Rosette fix bolt	Quantity	360,000	0.18	6	3	25,000	0.3232	(0, 0.001, 0.001, 0.001)
275	Yellow fix bolt	Quantity	1,380,000	0.19	6	3	110,000	0.1383	(0, 0.001, 0.001, 0.001)
277	White fix bolt	Quantity	1,380,000	0.2	6	3	110,000	0.1383	(0, 0.001, 0.001, 0.001)
282	6*6 Set screw	Quantity	1,310,000	0.21	6	3	105,000	0.1492	(0, 0.001, 0.001, 0.001)
283	6*8 Set screw	Quantity	1,310,000	0.22	6	3	105,000	0.1492	(0, 0.001, 0.001, 0.001)
338	Thiocyanate	kg	200	0.23	2	2	15	0.2970	(0.21, 0.42, 0.62, 0.83)
365	Electrical oil skimmer	kg	3800	0.24	1	2	300	0.2520	(0.08, 0.15, 0.23, 0.3)
366	Hot oil skimmer	kg	900	0.25	1	2	50	0.2493	(0.05, 0.1, 0.15, 0.2)
375	Solvent	Liter	320	0.26	5	4	20	0.4202	(0.48, 0.97, 1.45, 1.94)
398	Resin 413	kg	530	0.27	7	30	40	0.6499	(1.15, 2.31, 3.46, 4.62)
399	Resin 418	kg	2110	0.28	7	30	150	0.6653	(1.27, 2.54, 3.81, 5.08)
401	Electrophoretic yellow color	Cc	25,000	0.29	1	5	2000	0.3596	(0.82, 1.64, 2.47, 3.29)
402	Electrophoretic red color	Cc	6800	0.3	1	5	500	0.7050	(3.35, 6.69, 10.04, 13.38)
426	Caustic soda	kg	1400	0.31	3	4	100	0.3046	(0.02, 0.04, 0.07, 0.09)
429	Ammonium crystal sulfate	kg	1400	0.32	4	3	100	0.3457	(0.16, 0.31, 0.47, 0.62)
430	Zinc sulfate	kg	1400	0.33	4	3	100	0.3340	(0.07, 0.14, 0.21, 0.28)
431	Nickel sulfate	kg	1400	0.34	4	3	100	0.3509	(0.19, 0.39, 0.58, 0.78)
433	Cyanide sodium	kg	900	0.35	4	6	50	0.3597	(0.18, 0.37, 0.55, 0.74)
434	Cyanide copper	kg	394	0.36	4	6	30	0.3989	(0.47, 0.95, 1.42, 1.89)
447	Spring 3	Quantity	720,000	0.37	4	26	50,000	0.2926	(0, 0.001, 0.001, 0.002)
486	Barium carbonate	kg	700	0.38	3	2	50	0.3005	(0.04, 0.08, 0.12, 0.16)

Table 5 (continued)

Code	Inventory item	Unit	Criteria						Holding cost
			$\hat{d}_i$	Unit price	Criticality degree	Lead time	Inventory on hand	f_i	
488	Sodium carbonate	kg	1450	0.39	3	2	100	0.2993	(0.03, 0.06, 0.1, 0.13)
495	Ammonium chloride	kg	750	0.4	3	1	50	0.2956	(0.03, 0.06, 0.09, 0.12)
496	Nickel chloride	kg	750	0.41	3	1	50	0.3262	(0.25, 0.51, 0.76, 1.02)
511	Polish skimmer	kg	640	0.42	2	7	40	0.2955	(0.08, 0.15, 0.23, 0.3)
515	Satin/opaque polish (Turkey)	Liter	220	0.43	2	7	10	0.4481	(1.2, 2.4, 3.6, 4.8)
559	Rosette fix nut	Quantity	384,000	0.44	6	5	30,000	0.3191	(0.001, 0.002, 0.003, 0.005)
563	24 white fix nut	Quantity	1,460,000	0.45	6	5	110,000	0.1451	(0.001, 0.002, 0.003, 0.005)
574	Silver	Gr	19,800	0.46	7	4	1500	0.4076	(0.03, 0.05, 0.08, 0.11)
576	Rochelle salt	kg	350	0.47	2	2	20	0.2852	(0.12, 0.24, 0.36, 0.48)
665	Hex key 4	Quantity	94,000	0.48	1	3	7000	0.2308	(0.001, 0.003, 0.004, 0.005)
965	Nanomat	kg	320	0.49	4	4	20	0.5236	(1.44, 2.88, 4.33, 5.77)
966	Potassium cyanide	kg	260	0.5	4	5	10	0.4097	(0.58, 1.15, 1.73, 2.31)
561	24 yellow fix nut	Quantity	1,460,000	0.51	6	5	110,000	0.1451	(0.001, 0.002, 0.003, 0.004)

Table 6 Objective functions ideal and anti-ideal values

	Ideal value (F_r^*)	Anti-ideal value (F_{r*})
First objective	26,155,325,089	26,153,900,000
Second objective	87,431.327	223,824.197
Third objective	446.250	6477.000

The classification of inventory items with their required inventory level is illustrated in Table 8. In this table, the classifications of inventory items are determined and the minimum required inventories to assure meeting the corresponding satisfaction levels are determined. The above models are solved using the GAMS optimization package by Cplex solver.

According to Table 8, 11 items (20%) are classified as extremely important (i.e., A), 20 items (40%) are classified as moderately important (i.e., B), and 20 items (40%) are assigned to a relatively unimportant class (i.e., C). The obtained result is consistent with the traditional 80-20 rule. On the other hand, the optimal inventory levels obtained can be used as a determinant of an inventory control policy for each class.

6 Conclusions

In this paper, a model is proposed for multi-criteria ABC inventory classification when the uncertainties of data have a hybrid form of stochastic and fuzzy information.

Generally speaking, ABC classification is a fundamental decision in the field on inventory control that is considered by many researchers and a planetary of approaches are proposed to handle this problem. The main consideration in this problem is to determine the classification of inventory items to develop various inventory control policies for each category. The multi-criteria and uncertain nature of this problem seems inevitable. Therefore, this paper can be considered as an extension of this path of researches.

A part of previous studies proposed ABC classification under crisp and exact data, while another part considered uncertain data as stochastic or fuzzy. However, the type of uncertainty in information related to ABC classification is usually different, and considering a unique type of uncertainty seems challenging. *Theoretically,* this paper extended the context of uncertain ABC classification into a heterogeneous environment in which information follow

Table 7 Comparison of results from Eqs. (27) and (28)

	Objective function value obtained by solving …		Membership value obtained by solving … (%)	
	Equation (27)	Equation (28)	Equation (27)	Equation (28)
First objective	26,155,300,000	26,155,300,000	98.24	98.24
Second objective	87,826.26	129,200.5	99.71	69.38
Third objective	2557	2557	64.67	64.67

Table 8 ABC classification and the minimum required inventory level

Code	Inventory class	Minimum required inventory (v_i)	Code	Inventory class	Minimum required inventory (v_i)
1	C	89,000	398	C	530
2	A	170.646	399	C	2110
3	B	384.939	401	C	25,000
4	B	1158.591	402	C	6800
5	B	425.192	426	B	1409.479
6	C	2800	429	B	1409.479
7	B	395.003	430	B	1409.479
8	B	395.003	431	B	1409.479
38	A	616.547	433	B	907.600
39	A	180.973	434	B	399.029
45	B	746.892	447	C	720,000
48	C	1500	486	B	706.703
51	A	83.337	488	B	1459.647
60	B	766.984	495	B	756.938
61	B	766.984	496	B	756.938
161	C	1450	511	A	661.292
182	B	1459.647	515	A	232.483
274	C	360,000	559	C	384,000
275	C	1,380,000	563	C	1,460,000
277	C	1,380,000	574	C	19,800
282	C	1,310,000	576	A	365.745
283	C	1,310,000	665	C	94,000
338	A	211.902	965	B	324.532
365	C	3800	966	A	273.571
366	A	925.249	561	C	1,460,000
375	A	335.055			

different behavioral patterns. Considering the heterogeneity of information, a hybrid fuzzy-stochastic model is developed dealing with a different types of uncertainty. With an overview of previous studies in this field, no other research is found to investigate this common heterogeneous information in ABC classification. The model is formulated as a hybrid fuzzy-stochastic multi-objective model and a method is developed based on possibilistic programming and chance-constrained programming to solve the problem.

Practically, the application of the proposed method is illustrated in a real-world case, consisting of 51 inventory items. Formulating and solving the problem, the items are classified into three classes of A, B, and C. Also, the minimum required inventory levels of items are determined. It seems that the importance of inventory items classification will be rethinking considering the recently experienced disorder of supply chains due to the Coronavirus pandemic. However, especially, the magnitude of oscillation observed in demand information required a more powerful framework to analyze the MCIC problem in a noisy environment. The varieties of information ambiguity necessitate the application of flexible frameworks to analyze the inventory classification framework. The proposed method in this paper can be considered as a methodology to respond to this need.

Future researchers can focus on extending the proposed model in two directions. First of all, one of the limitations of the proposed model was to assume that the item demands follow a normal distribution. Future researchers can extend some algorithms to solve the model under general statistical distribution using different multi-objective stochastic programming approaches like Abdelaziz (2012) and Amoozad Mahdiraji et al. (2018). Also, since in some cases, the number of inventory items might be very large, e.g., more than one hundred thousand, using ordinal binary programming approaches might be inefficient. Therefore, researchers can use population-based (e.g., genetic algorithm), or single solution-based (e.g., simulated annealing) metaheuristics to solve the proposed problem.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies on human participants or animals performed by any of the authors.

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