Original Article

Estimation of distance error by fuzzy set theory required for strength determination of HDR ^{192}Ir brachytherapy sources

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ABSTRACT

Verification of the strength of high dose rate (HDR) ^{192}Ir brachytherapy sources on receipt from the vendor is an important component of institutional quality assurance program. Either reference air-kerma rate (RAKR) or air-kerma strength (AKS) is the recommended quantity to specify the strength of gamma-emitting brachytherapy sources. The use of Farmer-type cylindrical ionization chamber of sensitive volume 0.6 cm^3 is one of the recommended methods for measuring RAKR of HDR ^{192}Ir brachytherapy sources. While using the cylindrical chamber method, it is required to determine the positioning error of the ionization chamber with respect to the source which is called the distance error. An attempt has been made to apply the fuzzy set theory to estimate the subjective uncertainty associated with the distance error. A simplified approach of applying this fuzzy set theory has been proposed in the quantification of uncertainty associated with the distance error. In order to express the uncertainty in the framework of fuzzy sets, the uncertainty index was estimated and was found to be within 2.5%, which further indicates that the possibility of error in measuring such distance may be of this order. It is observed that the relative distance l_i estimated by analytical method and fuzzy set theoretic approach are consistent with each other. The crisp values of l_i estimated using analytical method lie within the bounds computed using fuzzy set theory. This indicates that l_i values estimated using analytical methods are within 2.5% uncertainty. This value of uncertainty in distance measurement should be incorporated in the uncertainty budget, while estimating the expanded uncertainty in HDR ^{192}Ir source strength measurement.

Key words: Brachytherapy, farmer-type ionization chamber, fuzzy set theory, HDR ^{192}Ir source

Introduction

The use of high dose rate (HDR) remote afterloading brachytherapy units are rapidly increasing in many countries around the world. Verifying the strength of HDR ^{192}Ir brachytherapy sources on receipt from the vendor is an important component of institutional quality assurance program.[1,2] The recommended quantity to specify the strength of gamma-emitting brachytherapy sources is either reference air-kerma rate (RAKR) or air-kerma strength (AKS). RAKR is the AKR to air, in air, at a reference distance of 1 m, corrected for attenuation and scattering; and refers to the quantity determined along the transverse bisector of the source. AKS is the AKR in air at a given distance corrected for attenuation and scattering and that is multiplied by the square of the given distance.[1-6] Calibration of the ^{192}Ir sources used in HDR remote afterloading brachytherapy units is carried out either by using a thimble ionization chamber (in-air jig method) or by using a well-type ionization chamber. A Farmer-type cylindrical ionization chamber of nominal sensitive volume of 0.6 cm^3 is frequently used for in-air calibration of HDR ^{192}Ir brachytherapy sources in addition to a suitable well-type ionization chamber.[7-9] The European Society for Therapeutic Radiology and Oncology (ESTRO) also recommends the use of thimble ionization chamber for calibration of HDR ^{192}Ir brachytherapy sources at hospitals.[10] Although well-type

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ionization chambers are preferred over cylindrical chambers for calibration of HDR ^{192}Ir brachytherapy sources due to ease in its use and reproducibility of source positioning, the Farmer-type cylindrical ionization chamber is also used for RAKR or AKS measurement of HDR ^{192}Ir brachytherapy sources.[11-14] This is due to the fact that cylindrical ionization chambers are readily available in the hospitals and in case of nonavailability of a well-type ionization chamber, the use of cylindrical ionization chamber is an obvious choice. It has also been demonstrated by Stump *et al.*,[15] that the RAKR measured by Farmer type and well-type ionization chambers for different HDR sources are comparable within 0.5%.

A 370 GBq (10 Ci) ^{192}Ir source provides an ionization current of only about 1×10^{-11} A in a 1.0 cm^3 ionization chamber at a distance of 20 cm.[16] It is true that very near to a brachytherapy source, the radiation intensity changes very rapidly due to inverse-square law. A 0.1 cm error in a 10 cm distance causes a 2% error in calibration.[17] Small errors in positioning the chamber can translate into large errors in the estimation of source strength. Increasing the separation between centers of the chamber and the source will improve the measurement accuracy. However, this will result in proportionate reduction in the current, leading to larger percentage contributions by leakage current and gamma-ray scattering from the room surroundings and poor reproducibility. Getting closer of course worsens the distance error and requires a large geometric correction[18-20] for the size and shape of the ionization chamber.

The 7 distance method is recommended as a standard method to maximize the accuracy in measurement of the strength of HDR brachytherapy sources by using cylindrical ionization chamber.[8-10,15,16] While using the cylindrical chamber method, it is required to determine the positioning error of the ionization chamber with respect to the source, which is commonly called as the distance error. Earlier, we have developed the analytical methods to estimate the distance error required to determine the source strength using 7 distance method by cylindrical ionization chamber.[21,22] As further research in this work, an attempt has been made to apply the fuzzy set theory to estimate the subjective uncertainty associated with the distance error, which is the subject matter of this paper.

Fuzzy set theory has been applied for risk analysis[23] and image analysis[24] in the domain of medical dosimetry. In view of these applications, we have proposed an approach of applying this fuzzy set theory in the quantification of uncertainty associated with the distance error required for the measurement of strength of HDR ^{192}Ir brachytherapy sources. While using, Farmer-type cylindrical ionization chamber to measure the strength of a brachytherapy source, distance has to be measured accurately. Since the distance measured possesses some error during the measurement and the input components are imprecise, fuzzy set theory is an appropriate tool to quantify the uncertainty due to such ambiguity present in input component.[25] The fuzzy set theory is strictly applicable where there is insufficient information in the measured data.

Materials and Methods

Multiple distance measurement technique

The microSelectron-HDR unit from Nucletron was used in this work. This unit uses an old design micro Selectron ^{192}Ir HDR brachytherapy source (Nucletron B V, Veenendaal, Netherlands) with 370 GBq (10 Ci) nominal activity to treat brachytherapy patients with HDR comparable to teletherapy. The old microSelectron HDR source is cylindrical in geometry with 0.6 mm active diameter and 3.5 mm active length. PTW 30001 0.6 cm^3 Farmer-type ionization chamber (PTW, Freiburg, Germany) was used in this work. Further details about the old microSelectron HDR source and the PTW 30001 ionization chamber are available elsewhere.[26,27]

To determine experimentally, the RAKR of an HDR ^{192}Ir brachytherapy source using a Farmer-type cylindrical ionization chamber, a multiple distance measurement technique was used. This measurement has historically been made at seven separate distances. Thus the technique has been termed the '7 distance' measurement (7DM). While using cylindrical ionization chamber (0.6 cm^3) for measurement of the strength of HDR ^{192}Ir brachytherapy sources, it is necessary to estimate three items, viz. (i) The positioning error of the ionization chamber with respect to the source which is commonly called 'distance error (±)', (ii) the contribution of scatter radiation (M_s) from the floor, walls ceiling, and other material in the treatment room, and (iii) a proportionality constant. The 7DM was suggested to determine these parameters and thereafter the strength of HDR ^{192}Ir brachytherapy sources.[8,9,15,16] Although Kumar *et al.*,[21,22] has described in detail the procedure for measuring the RAKR of HDR ^{192}Ir brachytherapy sources, a brief review of the method is given below for the sake of completeness in description.

In 7DM method, the output of the source in air is measured at seven different distances each corresponding to a meter reading M_d, which is the sum of primary and scattered radiation:

$$M_d = M_p + M_s \qquad .....(1)$$

where M_p is the meter reading due to primary radiation only and M_s is the meter reading due to scattered radiation only; and this is assumed to be independent of the distance. As the primary radiation follows inverse-square law, Equation 1 can be written as

$$M_p = \left(M_{d_i} - M_s\right) = \frac{f}{\left(d_i + c\right)^2} \quad \text{where i} = 0 \ldots 6 \qquad .....(2)$$

where d_i is the apparent distance between source and the chamber centers, c is the offset error in the measurement of the distance and commonly known as 'distance error'; *f* is a proportionality constant which is independent of distance. On solving Equation 2, one may obtain the following functional form for relative distance l_i between the successive measurement points (i = 1, 2,..., 6)[22]

$$l_i = \left(d_i - d_0\right) = f^{1/2}\left\{\left(\frac{1}{M_{d_i}^{\ 1/2}}\right) - \left(\frac{1}{M_{d_0}^{\ 1/2}}\right)\right\} + \frac{M_s f^{1/2}}{2}\left\{\left(\frac{1}{M_{d_i}^{\ 3/2}}\right) - \left(\frac{1}{M_{d_0}^{\ 3/2}}\right)\right\} \quad .....(3)$$

where M_{di} is the meter reading at distance d_i from the source.

Equation 3 has two unknowns which were determined by regressing over six points (instead of 7 points) by introducing the notations

$$\left(\frac{1}{M_i^{1/2}}\right) - \left(\frac{1}{M_0^{1/2}}\right) = x_i \quad .....(4)$$

$$\left(\frac{1}{M_i^{3/2}}\right) - \left(\frac{1}{M_0^{1/2}}\right) = y_i \quad .....(5)$$

$$f^{1/2} = a \quad .....(6)$$

$$M_s f^{1/2} = b \quad .....(7)$$

Equation 3 can now be written as:

$$l_i = ax_i + by_i, i = 1,2, ...6, \quad .....(8)$$

Given the set of values (l_i, x_i, y_i) from each set of six experimental data sets, we evaluated the coefficients *a* and *b* (hence *f* and M_s) by bivariate linear regression analysis by adopting the least square method to determine a and b. The *f* and M_s are given by the following equations

$$f = \left\{\frac{\left(\sum_{i=1}^{6} l_i x_i \sum_{i=1}^{6} y_i^2\right) - \left(\sum_{i=1}^{6} l_i y_i \sum_{i=1}^{6} x_i y_i\right)}{\left(\sum_{i=1}^{6} x_i^2 \sum_{i=1}^{6} y_i^2\right) - \left(\sum_{i=1}^{6} x_i y_i\right)}\right\}^2 \quad .....(9)$$

$$M_s = \frac{2\left\{\left(\sum_{i=1}^{6} x_i^2 \sum_{i=1}^{6} l_i y_i\right) - \left(\sum_{i=1}^{6} x_i y_i \sum_{i=1}^{6} l_i x_i\right)\right\}}{\left(\sum_{i=1}^{6} l_i x_i \sum_{i=1}^{6} y_i^2\right) - \left(\sum_{i=1}^{6} l_i y_i \sum_{i=1}^{6} x_i y_i\right)} \quad .....(10)$$

The values of *f* and M_s, thus obtained using Equations 9 and 10, respectively, were used to estimate the value of '*c*'. Having determined the value of *f* using Equation 9, the AKR (Gys^{-1}) can be calculated using the formula

$$AKR = \frac{N_K f}{(d+c)^2 \Delta t} \quad .....(11)$$

where N_K, is the interpolated air-kerma calibration coefficient of the chamber for HDR ^{192}Ir brachytherapy source and Δ t is the time interval of the measurement.

RAKR can then be determined using the following equation

$$RAKR = AKR\left(\frac{d+c}{d_{ref}}\right)^2 \quad .....(12)$$

where d_{ref} is the reference distance of 1 m.[28]

Fuzzy set theory

Zadeh[25,29] introduced the fuzzy set as a class of object with a continuum of grades of membership. In contrast to classical crisp sets where a set is defined by either membership or non-membership, the fuzzy approach relates to a grade of membership between [0,1], defined in terms of the membership function of a fuzzy number. Hence, the classical notion of binary membership has been modified for the representation of uncertainty in data. The details about fuzzy set may be found elsewhere,[29] however, for the sake of completeness, a brief description of the definition of a fuzzy set and its fundamental properties pertaining to the topic of the present work is described here. This is a paradigm shift in which the crisp variable is fuzzified through a membership function or a linguistic variable depending upon the specific problem. Strictly speaking, alpha-cut theory of the fuzzy set[29] is adopted to compute the uncertainty associated with the distance.

Basic concept

A fuzzy set A is denoted by an ordered set of pairs (x, μ (x)), where, the element x μ X (crisp value) of a specific universe and μ (x) denotes the degree of membership, μ (x) μ [0,1]. A membership function can be of any shape depending on the type of a fuzzy set it belongs to. The only condition a membership function must satisfy is it should vary between 0 and 1. The membership function of a fuzzy set, A is defined in the form of a triangular or trapezoidal fuzzy number as shown in Figure 1a and b. The analytical form of the triangular membership function is depicted in Equation 13

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ \frac{c-x}{c-b}, & b \le x \le c \\ 0 & \text{otherwise } (x \le a, x \ge c) \end{cases} \quad .....(13)$$

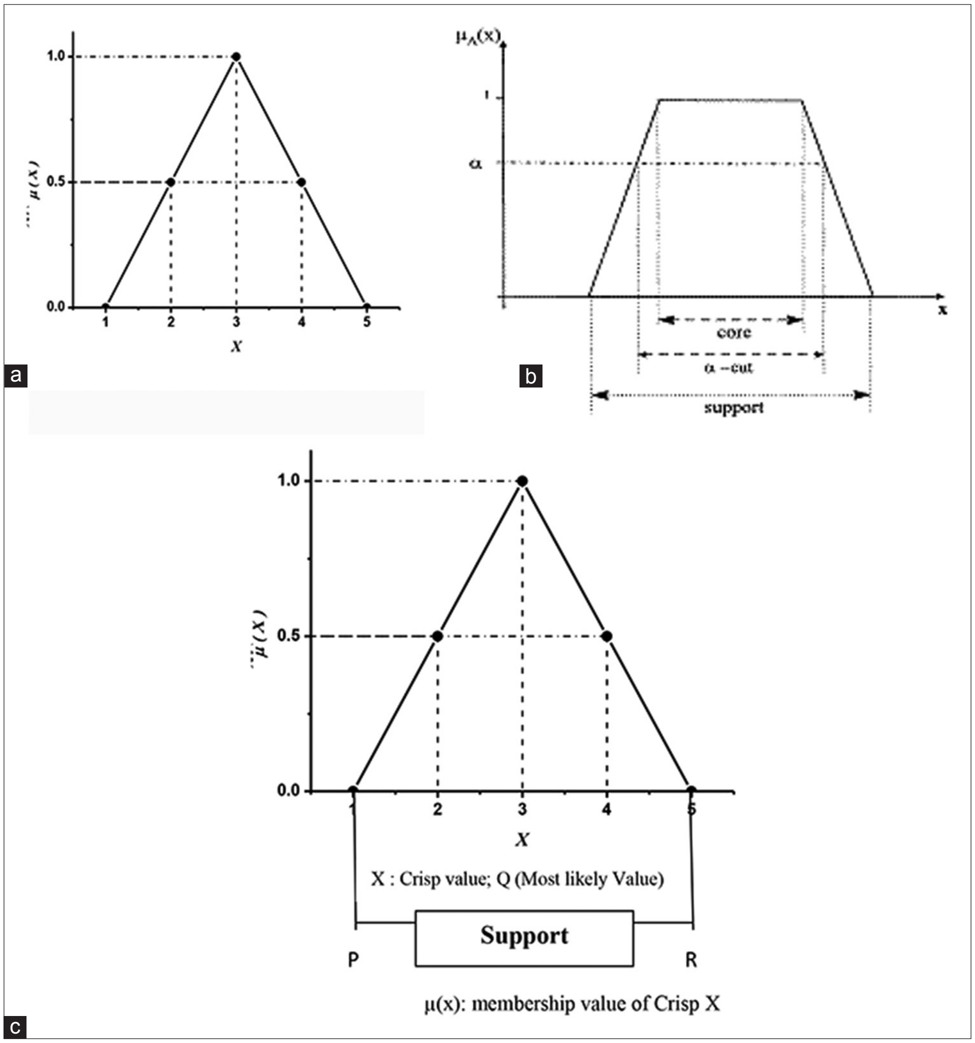

Figure 1: Pictorial representation of the membership function of a fuzzy set (a) triangular, (b) trapezoidal, and (c) support of a triangular fuzzy number

The functional form of the trapezoidal membership function is given by

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0 & \text{otherwise } (x \le a,\ x \ge d) \end{cases} \quad .....(14)$$

Alpha-cut of a fuzzy set

Alpha-cut of a fuzzy set A is defined as the set of values of x, for which the membership value, $\mu(x) \geq \alpha$ and is given by

$$A^{\alpha} = \{x \in X \mid \mu_L(x) \geq \alpha\}$$

Basically, alpha-cut is an interval and in practice, interval arithmetic operation[30] is carried out for obtaining the membership value of the output of a model containing the fuzzy input. The present paper, applies the alpha-cut value of the fuzzy set.

Implementation of alpha-cut of a fuzzy set

In the present problem, fuzzy set theory has been applied to estimate the relative distance, l_i as shown in Equation 3. Each parameter in Equation 3 was treated as triangular fuzzy number because, the experimental determination provides the most likely value with the two extreme bounds scattered by the error obtained during measurement. The computation scheme of the membership value of the relative distance l_i is as follows:

The parameters, f, M_{di}, M_0, and M_s are taken into account as triangular fuzzy number. Alpha-cut representation ranged from 0 to 1 with an increment of 0.1 of these parameters was applied in Equation 3. In order to derive the membership value of the relative distance, l_i using the alpha-cut representation of fuzzy parameters as mentioned above, we used the interval arithmetic operations and the algorithm for computation. The details of this computation are given in the Appendix.

Support and uncertainty of a fuzzy set

A fuzzy set having triangular membership function is always characterized by its support and height. If height of a fuzzy set is 1 and if that fuzzy set is bounded by two extremes, then it is called a triangular fuzzy number. Support of such a triangular fuzzy number is defined as the range of the extremes at alpha-cut = 0 as shown in Figure 1c. From Figure 1c, we can write the support of a fuzzy set as S = (R−P), where, R and *P* are the two extreme bounds. In order to express the uncertainty in the framework of fuzzy sets, we define uncertainty index[30] as the ratio of the support to the most likely value (crisp value at membership equal to 1). Again, from Figure 1c, uncertainty index of the given fuzzy set is written as U = (S/Q), where, Q is the most likely value. The uncertainty index for each relative distance measured experimentally was estimated.

Results and Discussion

We have estimated the uncertainty of the relative distance, l_i [= $(d_i$-$d_0)$] for experimentally measured distances such as 5, 10, 15, 20, 25, and 30 cm and the corresponding membership functions are shown in Figure 2a-f. It can be interpreted from Figure 2a-f that the membership function μ (l_i) of the distance (l_i) for each measurement distance is turned out to be a triangular in shape because the initial consideration of the subjective-based uncertain parameters are taken into consideration as "around the measured value". On the contrary, had this consideration been within the phrase of "approximately lying between two different distances", we would have obtained the shape of the membership function of the output as trapezoidal. Since the

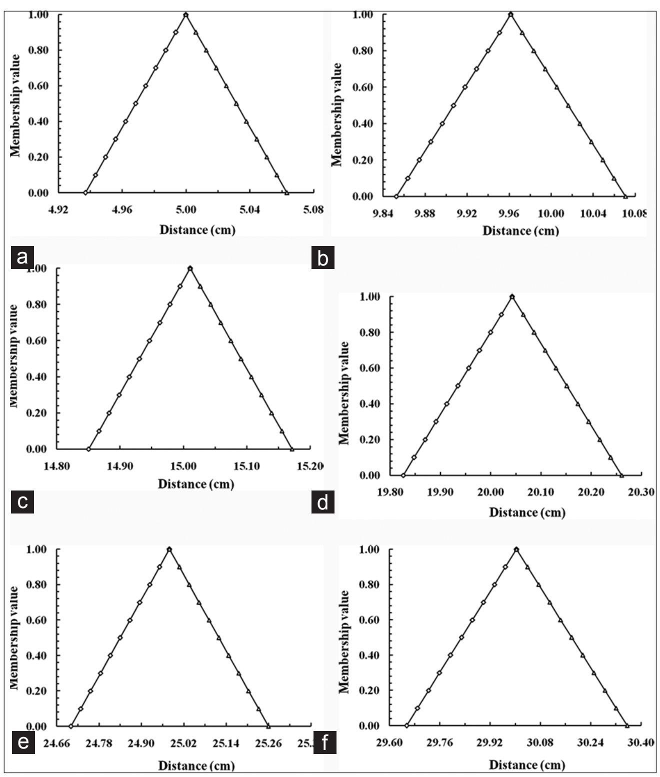

Figure 2: Pictorial representation of membership value μ (l_i) and relative distance l_i (cm) [= (d_i − d_0)] for the distance (a) 5, (b) 10, (c) 15, (d) 20, (e) 25, and (f) 30 cm. Here, ◊ is lower bound and ∆ is upper bound

measurement uncertainty is always quoted at one sigma level, fuzzy set theory-based approach of uncertainty quantification is also quoted at an equivalent level, and here this is 0.5 alpha-cut value of the fuzzy set. That is why, in this work, 0.5 alpha-cut of the output fuzzy set is considered to express the subjective uncertainty. Results of alpha-cut = 0.5 of the fuzzy distance (l_i) along with experimental and analytical values of (l_i) are given in Table 1 and it can be seen that analytical value as well as the experimentally measured relative distance lie within the bounds of the subjective uncertainty of the l_i. Support of each triangular membership values corresponding to each experimental distance and the associated uncertainty index are further shown in Table 2. It can be seen from Table 2 that the uncertainty indices remain same for all the experimentally-measured distances indicating that each and every triangular fuzzy membership function is normalized and convex. Maximum value of the uncertainty index is found to be within 2.5%, which further indicates that the possibility of error in measuring such distance may be of this order. It is worth mentioning here that in absence of a quantified value of uncertainty associated with distance measurement, the overall uncertainty in source strength measurement has been estimated earlier by ignoring the uncertainty associated with distance error.[15] This work reveals that the uncertainty associated with distance error should not be ignored while preparing the uncertainty budget in the determination of HDR source strength.

Conclusion

Uncertainty of the positioning error, the so called "distance error", of the ionization chamber with respect to the source was evaluated. Fuzzy set theory was applied

Table 1: Comparison of experimentally recorded, analytically calculated, and fuzzy set theory computed values of l_i [= (d_i-d_0)]

Experimental	*Analytical*	*Fuzzy set theory*	
		$(l_i)^L$	*$(l_i)^U$*
5	4.999	4.968	5.031
10	9.961	9.907	9.961
15	15.009	14.931	15.091
20	20.041	19.934	20.152
25	24.977	24.840	25.119
30	30.001	29.829	30.180

Table 2: Support and uncertainty index of relative distance l_i

Experimental	*Support*	*Uncertainty index*
5	0.126	0.025
10	0.218	0.022
15	0.321	0.021
20	0.435	0.022
25	0.559	0.022
30	0.702	0.023

for this evaluation due to the subjectivity involved in the experimental facility. Uncertainty in the possible input parameters was addressed as triangular fuzzy number. Propagation of uncertainty of the input parameters is carried out on the basis of the model described in this work (see subsection: Algorithm to compute alpha-cut distance representation of distance) via the alpha-cut of a fuzzy set. The crisp values of l_i estimated using analytical method lie within the bounds computed using fuzzy set theory. This indicates that l_i values estimated using analytical methods are within 2.5% uncertainty. This value of uncertainty in distance measurement should be incorporated in the uncertainty budget, while estimating the expanded uncertainty in HDR ^{192}Ir source strength measurement.

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Appendix

Algorithm to compute alpha-cut representation of distance

1. Given a fuzzy parameter, say, constant of proportionality, f (see Equation (2)) as a triangular fuzzy number: $\langle f\rangle = \langle f_{LB}, f^{most\ likely}, f_{UB}\rangle = \langle 1945, 1972, 1999\rangle$, we have the alpha-cut representation as $\left[f_\alpha^{LB}, f_\alpha^{UB}\right] = [1950.4\ ,\ 1993.6]_{\alpha=0.2}$
2. In a similar way, alpha-cut representations of all other fuzzy parameters are constructed.
3. Alpha-cut representation being an interval number, we use the interval arithmetic operation of

$$[f]^{1/2}\times\left\{\left[\left(\frac{1}{M_{d_i}}\right)\right]^{1/2}-\left[\left(\frac{1}{M_{d_0}}\right)\right]^{1/2}\right\}\quad \text{as:}$$

$$\left[f_\alpha^{LB},f_\alpha^{UB}\right]^{1/2}\times\left\{\left[\left(\frac{1}{M_{d_i}}\right)_\alpha^{UB},\ \left(\frac{1}{M_{d_i}}\right)_\alpha^{LB}\right]^{1/2}-\left[\left(\frac{1}{M_{d_0}}\right)_\alpha^{UB},\ \left(\frac{1}{M_{d_0}}\right)_\alpha^{LB}\right]^{1/2}\right\}$$

$$=\left[f_\alpha^{LB},f_\alpha^{UB}\right]^{1/2}\times\left\{\left[\left(\frac{1}{M_{d_i}^{1/2}}\right)_\alpha^{UB}-\left(\frac{1}{M_{d_0}^{1/2}}\right)_\alpha^{LB}\right],\ \left[\left(\frac{1}{M_{d_i}^{1/2}}\right)_\alpha^{LB}-\left(\frac{1}{M_{d_0}^{1/2}}\right)_\alpha^{UB}\right]\right\}$$

4. $$[A,B]=\left[f_\alpha^{LB},f_\alpha^{UB}\right]^{1/2}\times[\eta\ ,\ \lambda\ ]=$$

$$\left\{\begin{matrix}\min\begin{bmatrix}(f_\alpha^{LB})^{1/2}\eta,(f_\alpha^{UB})^{1./2}\eta,\\(f_\alpha^{LB})^{1./2}\lambda,(f_\alpha^{UB})^{1./2}\lambda\end{bmatrix},\\ \max\begin{bmatrix}(f_\alpha^{LB})^{1/2}\eta,(f_\alpha^{UB})^{1./2}\eta,\\(f_\alpha^{LB})^{1./2}\lambda,(f_\alpha^{UB})^{1./2}\lambda\end{bmatrix}\end{matrix}\right\}$$

where

$$\eta=\left(\frac{1}{M_{d_i}^{1/2}}\right)_\alpha^{UB}-\left(\frac{1}{M_{d_0}^{1/2}}\right)_\alpha^{LB},$$

$$\lambda=\left(\frac{1}{M_{d_i}^{1/2}}\right)_\alpha^{LB}-\left(\frac{1}{M_{d_0}^{1/2}}\right)_\alpha^{UB}$$

5. Alpha-cut representation being an interval number, we use the interval arithmetic operation of

$$\frac{1}{2}[(M_s)]\ \times[f]^{1/2}\times\left\{\left[\left(\frac{1}{M_{d_i}^{3/2}}\right)-\left(\frac{1}{M_{d_i}^{3/2}}\right)\right]\right\}\quad \text{as:}$$

$$[C,D]=\frac{1}{2}\left[(M_s)_\alpha^{LB},(M_s)_\alpha^{UB}\right]\ \times\left[f_\alpha^{LB},f_\alpha^{UB}\right]^{1/2}\times$$

$$\left\{\begin{matrix}\left[\left(\frac{1}{M_{d_i}^{3/2}}\right)_\alpha^{UB}-\left(\frac{1}{M_{d_0}^{3/2}}\right)_\alpha^{LB}\right],\\ \left[\left(\frac{1}{M_{d_i}^{3/2}}\right)_\alpha^{LB}-\left(\frac{1}{M_{d_0}^{3/2}}\right)_\alpha^{UB}\right]\end{matrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix}\min\begin{Bmatrix}(M_s)_\alpha^{LB}(f_\alpha{}^{LB})^{1/2},(M_s)_\alpha^{LB}(f_\alpha{}^{UB})^{1/2},\\(M_s)_\alpha^{UB}(f_\alpha{}^{LB})^{1/2},(M_s)_\alpha^{UB}(f_\alpha{}^{UB})^{1/2}\end{Bmatrix},\\ \max\begin{Bmatrix}(M_s)_\alpha^{LB}(f_\alpha{}^{LB})^{1/2},(M_s)_\alpha^{LB}(f_\alpha{}^{UB})^{1/2},\\(M_s)_\alpha^{UB}(f_\alpha{}^{LB})^{1/2},(M_s)_\alpha^{UB}(f_\alpha{}^{UB})^{1/2}\end{Bmatrix}\end{bmatrix}\times[\tau,\zeta]$$

$$=\frac{1}{2}[\delta,\ \ \gamma]\times[\tau,\ \ \zeta]\frac{1}{2}\begin{bmatrix}\min(\delta\tau,\ \ \gamma\tau,\ \ \delta\zeta,\ \ \gamma\zeta)\ ,\\ \max(\delta\tau,\ \ \gamma\tau,\ \ \delta\zeta,\ \ \gamma\zeta)\end{bmatrix},$$

where

$$\tau=\left(\frac{1}{M_{d_i}{}^{3/2}}\right)_\alpha^{UB}-\left(\frac{1}{M_{d_0}{}^{3/2}}\right)_\alpha^{LB},$$

$$\zeta=\left(\frac{1}{M_{d_i}{}^{3/2}}\right)_\alpha^{LB}-\left(\frac{1}{M_{d_0}{}^{3/2}}\right)_\alpha^{UB}$$

$$\delta=\min\begin{Bmatrix}(M_s)_\alpha^{LB}(f_\alpha{}^{LB})^{1/2},(M_s)_\alpha^{LB}(f_\alpha{}^{UB})^{1/2},\\(M_s)_\alpha^{UB}(f_\alpha{}^{LB})^{1/2},(M_s)_\alpha^{UB}(f_\alpha{}^{UB})^{1/2}\end{Bmatrix},$$

$$\gamma=\max\begin{Bmatrix}(M_s)_\alpha^{LB}(f_\alpha{}^{LB})^{1/2},(M_s)_\alpha^{LB}(f_\alpha{}^{UB})^{1/2},\\(M_s)_\alpha^{UB}(f_\alpha{}^{LB})^{1/2},(M_s)_\alpha^{UB}(f_\alpha{}^{UB})^{1/2}\end{Bmatrix}$$

6. Finally, $l_i^{\alpha}=\{[A,B]+[C,D]\}=[A+C\ ,\ B+D]$

The 0.5 alpha-cut values of a fuzzy set was used to quote the bounds of the uncertainty of the imprecise or vague information applied to any physical quantity because the uncertainty bounds of the input triangular fuzzy parameters are taken as one sigma level, that is, $f_{LB,\,UB} = (f^{\text{most likely}} \pm \sigma)$ according to the principle of measurement uncertainty. Hence, l_i values were chosen for 0.5 alpha-cut value[29] and compared with the analytically estimated values. Here, in this case, the bounds are positive numbers and hence in case of multiplication operation of two intervals, we have applied restricted Dong, Shah and Wang (DSW) algorithm.[25,30]

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