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Research article

The modified extended tanh technique ruled to exploration of soliton solutions and fractional effects to the time fractional couple Drinfel'd–Sokolov–Wilson equation

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ABSTRACT

The modified extended tanh technique is used to investigate the conformable time fractional Drinfel'd-Sokolov-Wilson (DSW) equation and integrate some precise and explicit solutions in this survey. The DSW equation was invented in fluid dynamics. The modified extended tanh technique executes to integrate the nonlinear DSW equation for achieve diverse solitonic and traveling wave envelops. Because of this, trigonometric, hyperbolic and rational solutions have been found with a few acceptable parameters. The dynamical behaviors of the obtained solutions in the pattern of the kink, bell, multi-wave, kinky lump, periodic lump, interaction lump, and kink wave types have been illustrated with 3D and density plots for arbitrary chose of the permitted parameters. By characterizing the particular benefits of the exemplified boundaries by the portrayal of sketches and by deciphering the actual events, we have laid out acceptable soliton plans and managed the actual significance of the acquired courses of action. New precise voyaging wave arrangements are unambiguously gained with the aid of symbolic computation using the procedures that have been announced. Therefore, the obtained outcomes expose that the projected schemes are very operative, easier and efficient on realizing natures of waves and also introducing new wave strategies to a diversity of NLEEs that occur within the engineering sector.

1. Introduction

Nonlinear evolution equations (NLEEs) have recently taken center stage in a few areas of the nonlinear sciences. The amazing and an extraordinary variety of explanatory methods which are growing more important for NLEEs may be used to demonstrate the excessive miracles that have emerged in the domains of diagram and mathematical component science. Numerous disciplines, including as physics of solid state, mathematical physics, optical fiber, oceanography, communication systems, mathematical biology, fluid mechanics, geochemistry, plasma physics, and chemical physics [1–12], make advantage of the wave phenomena of NLEEs. The NLEEs equation has not yet been solved using such a technique. Due to this, several researchers have created a variety of trustworthy,

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effective, and simple methods for solving NLEEs equations like as the procedure of enhanced (G ′/G)-expansion [13], the manner of modified Kudryashov [14], the procedure of modified simple equation [15], the procedure of sine-Gordon equation expansion [16], the procedure of extended sinh-Gordon equation expansion [17], the manner of exp $(-\varphi(\xi))$-expansion [18], the new auxiliary equation and modified Kudryashov scheme [19], MSE scheme [20], the trial solution formula [21], the procedure of Frobenius integrable decomposition [22], $(\frac{G'}{G},\frac{1}{G})$– expansion and Sine-Gordon-expansion methods [23], the manner of multiple simplest equation [24], the manner of solitary wave ansatzes [25], the manner of simple equation [26], the manner of extended simplest equation [27], the scheme of modified extended tanh-function [28], the manner of Hirota [29–32], new extended direct algebraic method [33], MSE method [34], the tanh-coth method [35], Riemann-Hilbert problems [36–39] and so on. As a result, nonlinear science is emerging as one of the fundamental areas of research in the growing wave system.

We have implemented the modified extended tanh technique [40] which finds out innovative traveling wave solution of the conformable time fractional DSW equation. The conformable time fractional DSW equation is as following form:

$$\left.\begin{aligned} D_t^{\delta}(u_t)+\alpha v v_x=0, \\ D_t^{\delta}(v_t)+\beta u v_x+s u_x v+\eta v_{xxx}=0 \end{aligned}\right\}. \tag{1}$$

Where α,β,s,η are all non-zero wave modes and u,v are the functions of space and time. The DSW condition is initially presented by Drinfel'd and Sokolov [41] and Wilson [42]. The DSW condition has been viewed as in late expositions [43–49], where a number of doubly periodic and soliton solutions are introduced. The DSW equation belongs to the Kadomtsev–Petviashvili (KP) hierarchy [50], as demonstrated by Jimbo and Miwa. The DSW condition can be gotten from a six-decrease of the KP ordered progression in Ref. [43]. Using the direct algebra method [44,45], recent traveling wave authorizations to the DSW equation have been viewed. Fan used the algebraic manner [46] a year later to precisely solve the DSW equation. The DSW equation was solved precisely using the manner of generalized Jacobi elliptic function [47], which was demonstrated to yield kink, bell, singular, and periodic solutions. Inc [48] has tracked down the around doubly occasional wave arrangements of the DSW condition by utilizing the Adomain decomposition technique. Using the improved F-expansion technique, Zhao and Zhi [49] discovered the periodic and rational solution to DSW equation. The DSW equation is subjected to the Darboux transformation [51], which results in a reduction and precise solutions. In addition, references [52–54] discuss the DSW equation's discussion of the conservation law and lie symmetry analysis. We additionally examined the nonlocal evenness and acquired the unequivocal arrangements of the DSW condition [55].

The purpose of our study is to build up the precise traveling wave solutions in mathematical physics for NEEs applying the modified extended tanh approach and the DSW equation. We learned the new type of trigonometric, exponential and rational solitary wave condition in different way. We also compare our solutions to other solutions. In recent years, many researchers have worked on our proposed model, which is mentioned in the paragraph above. The paper is manufactured as: in section 2, the CFD and its properties are described; in section 3, the suggested technique is discussed. We have used methods to the nonlinear evolution equation that was mentioned earlier in section 4. Section 5 presents our findings and related debates. in section 6 presents the similarity and difference with other published paper. Conclusions are provided in the final part.

2. Preliminaries and methods

2.1. Definition and a few features of conformable fractional derivative

Khalil et al. [56] first proposed the conformable fractional derivative with a limit operator.

Definition: $f:(0,\infty)\rightarrow\mathbb{R}$ then, the conformable fractional derivative of f with order δ is stated to be

$$D_t^{\delta}f(t)=\lim_{\varepsilon\rightarrow}\left(\frac{f(t+\varepsilon t^{1-\delta})-f(t)}{\varepsilon}\right)\textit{for all } t>0, 0<\delta\leq 1.$$

Afterward, Abdeljawad [57] made fractional versions of conformable derivatives for Taylor power series expansions, chain rule, Laplace transform, Gronwalls inequality, integration by parts, and exponential functions. The shortcomings of the current modified Riemann-Liouville derivative definition can be easily overcome with the definition of a conformable fractional derivative.

Theorem 1. *Assume $\delta\in(0,1]$, and $f=f(t), g=g(t)$ be δ-conformable differentiable* [57] *at a point $t>0$, then*:

(i) $D_t^{\delta}(cf+dg)=cD_t^{\delta}f+dD_t^{\delta}g$, *for all* $c,d\in\mathbb{R}$.

(ii) $D_t^{\delta}(t^{\gamma})=\gamma t^{\gamma-\delta}$, *for all* $\gamma\in\mathbb{R}$.

(iii) $D_t^{\delta}(fg)=gD_t^{\delta}(f)+fD_t^{\delta}(g)$..

(iv) $D_t^{\delta}(f/g)=\frac{gD_t^{\delta}(f)-fD_t^{\delta}(g)}{g^2}$..

Yet, $D_t^{\delta}(f(t))=t^{1-\delta\frac{df}{dt}}$, *where f is differentiable.*

Theorem 2. *Assume $f:(0,\propto)\rightarrow R$ is a function like that f is differentiable and δ-conformable differentiable* [58]. *Further, expect g is a differentiable capability characterized in the scope of f. Then,*

$$D_t^{\delta}(fog)(t) = t^{1-\delta} f'(g(t)) g'(t).$$

Where prime indicate the classical derivatives dependent on t.

3. Procedures of modified extended tanh method

In this segment, we tell in details the modified extended tanh scheme for discovering traveling wave equations of nonlinear equations. Assume the partial differential equation:

$$\Re(U, U_t, U_x, U_{tt}, U_{xx}, U_{xt}\ldots\ldots\ldots) = 0. \tag{2}$$

R is a polynomial of $u(x,t)$ and its partial derivatives, including the highest order derivatives and nonlinear terms, where $U = u(x,t)$ is an unfamiliar function. Recognition the arrangement of the nonlinear equation (7) is linked to the advancements listed below employing this technique.

Step-1: The definite PDE (2) can be transferred into ODE by utilizing the change of $= u(x,t) = u(\zeta);\ \zeta = x \pm \omega \frac{t^{\delta}}{\delta}$, where ω is the velocity of traveling wave like that $\omega \in R - \{0\}$. The mentioned wave variable turns the NLPDE eq. (2) into an ODE as

$$\Re(u, u', u'', \ldots\ldots\ldots\ldots\ldots\ldots) = 0. \tag{3}$$

Where $u'(\zeta) = \frac{du}{d\zeta}, u''(\zeta) = \frac{d^2u}{d\zeta^2}$, and so on.

Step-2: Assume the solution of eq. (3) can be defined by a polynomial in $\Theta(\zeta)$:

$$U = u(\zeta) = \Omega_0 + \sum_{j=1}^{m} \left(\Omega_j(\Theta(\zeta))^j + \Delta_j(\Theta(\zeta))^{-j}\right). \tag{4}$$

and $\Theta(\zeta)$ satisfies the ODE in

$$\Theta'(\zeta) = E + \Theta^2(\zeta). \tag{5}$$

Equation (5) has the accompanying solutions:

Case-I: Whereas $\mathrm{E} < 0$, the following hyperbolic solutions are earned:
$\Theta(\xi) = -\sqrt{-E}tanh(\sqrt{-E}\zeta)$.And $\Theta(\xi) = -\sqrt{-E}coth(\sqrt{-E}\zeta)$.

Case-II: Whereas $\mathrm{E} > 0$, the following trigonometric solutions are earned:
$\Theta(\xi) = \sqrt{E}tan(\sqrt{E}\zeta)$.
And $\Theta(\xi) = -\sqrt{E}cot(\sqrt{E}\zeta)$.

Case-III: Whereas $\mathrm{E} = 0$, we get the following solution:
$\Theta(\xi) = -\frac{1}{\zeta}$.

Step-3: Balance the highest order derivatives terms with the highest order nonlinear terms in eq. (4) to get the standard of the positive integer. Whether the degree of $u(\zeta)$ is $D[u(\zeta)] = m$, then the degree of the other expressions will be as follows:
$D\left[\frac{d^p u(\zeta)}{d\zeta^p}\right] = m + p, D\left[u^p\left(\frac{d^q u(\zeta)}{d\zeta^q}\right)^s\right] = mp + s(m+q)$.

Step-4: Concerning the replacement of eq. (4) into eq. (3) and by applying eq. (5), in which we perform a polynomial form of $(\Theta(\xi))$ after bringing together all of the same order of $(\Theta(\xi))$. An algebraic system can be constructed by reducing all polynomial's coefficient to zero.

Step-5: Measure the mathematical terms that were established in step 4 to alter the approximation of the constants. Replacing the approximations of the constants organized with the preparations of eq. (5), we will be able to obtain new precise and extensive traveling wave arrangements for the nonlinear development eq. (2).

4. Applications

By rule we started, the fractional complex transformation [59] is as follow:

$$u(x,t) = U(\xi), v(x,t) = V(\xi), \xi = k\left(x - \omega\frac{t^{\delta}}{\delta}\right). \tag{6}$$

Put equation (6) into equation (1), the ODE form of equation (1) is as:

$$\left.\begin{array}{r} -k\omega U' + \alpha k V V' = 0 \\ -k\omega V' + \beta k U V' + s k U' V + \eta k^3 V''' = 0 \end{array}\right\}. \tag{7}$$

From 1st equation in eq. (7) we have

$$U = \frac{\alpha}{2\omega} V^2. \tag{8}$$

Using eq. (8) into the 2nd equation of eq. (7), we gain under mentioned ODE:

$$-6\omega^2 V+\alpha(\beta+2s)V^3+6\eta\omega k^2 V''=0. \tag{9}$$

According to Modified Extended tanh Method by balancing V'' and V^3 of eq. (9) we have following solution form:

$$V(\xi)=\Omega_0+\Omega_1\Theta(\zeta)+\frac{\Delta_1}{\Theta(\zeta)}. \tag{10}$$

Substituting eq. (10) with (5) into (9) and afterward setting the coefficients of $\Theta(\zeta)$ to zero, which can be tackled by Maple, we can get the accompanying sets:

Set 1: $k=k,\omega=2\mathrm{E}\eta k^2,\Omega_0=0,\Omega_1=\pm2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta,\Delta_1=0.$

Set 2: $k=\pm\left(\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{48\mathrm{E}^3\eta^2}\right)^{\frac14},\omega=-\frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{48\mathrm{E}^3\eta^2}}},\Omega_0=0,\Omega_1=\frac{\Delta_1}{\mathrm{E}},\Delta_1=\Delta_1.$

Set 3: $k=\pm I\left(\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{48\mathrm{E}^3\eta^2}\right)^{\frac14},\omega=\frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{48\mathrm{E}^3\eta^2}}},\Omega_0=0,\Omega_1=\frac{\Delta_1}{\mathrm{E}},\Delta_1=\Delta_1.$

Set 4: $k=\pm\left(\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}\right)^{\frac14},\omega=-\frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}}},\Omega_0=0,\Omega_1=\frac{\Delta_1}{\mathrm{E}},\Delta_1=\Delta_1.$

Set 5: $k=\pm I\left(\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}\right)^{\frac14},\omega=\frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}}},\Omega_0=0,\Omega_1=\frac{\Delta_1}{\mathrm{E}},\Delta_1=\Delta_1.$

Hyperbolic solution because of $\mathbf{E<0}$.

Family 1:

$$V_{1,2}=\mp2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{-\mathrm{E}}\tanh\left(\sqrt{-\mathrm{E}}\,\zeta\right).$$

$$U_{1,2}=\frac{\alpha}{2\omega}\left(\mp2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{-\mathrm{E}}\tanh\left(\sqrt{-\mathrm{E}}\,\zeta\right)\right)^2.$$

$$V_{3,4}=\mp2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{-\mathrm{E}}\coth\left(\sqrt{-\mathrm{E}}\,\zeta\right).$$

$$U_{3,4}=\frac{\alpha}{2\omega}\left(\mp2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{-\mathrm{E}}\coth\left(\sqrt{-\mathrm{E}}\,\zeta\right)\right)^2.$$

Where $k=k,\omega=2\mathrm{E}\eta k^2$ and $\xi=k\left(x-\omega\frac{t^\delta}{\delta}\right)$.

Family 2

$$V_{5,6}=-\frac{\Delta_1\,\mathrm{sech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\sinh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}.$$

$$U_{5,6}=\frac{\alpha}{2\omega}\left(-\frac{\Delta_1\,\mathrm{sech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\sinh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}\right)^2.$$

$$V_{7,8}=\frac{\Delta_1\,\mathrm{cosech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\cosh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}.$$

$$U_{7,8}=\frac{\alpha}{2\omega}\left(\frac{\Delta_1\,\mathrm{cosech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\cosh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}\right)^2.$$

Where, $k=\pm\left(\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{48\mathrm{E}^3\eta^2}\right)^{\frac14},\omega=-\frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{48\mathrm{E}^3\eta^2}}}$ and $\xi=k\left(x-\omega\frac{t^\delta}{\delta}\right)$.

Family 3:

$$V_{9,10} = -\frac{\Delta_1 \operatorname{sech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\sinh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}.$$

$$U_{9,10} = \frac{\alpha}{2\omega}\left(-\frac{\Delta_1 \operatorname{sech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\sinh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}\right)^2.$$

$$V_{11,12} = \frac{\Delta_1 \operatorname{cosech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\cosh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}.$$

$$U_{11,12} = \frac{\alpha}{2\omega}\left(\frac{\Delta_1 \operatorname{cosech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\cosh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}\right)^2.$$

Where, $k = \pm I\left(\frac{\alpha\beta{\Delta_1}^2 + 2\alpha s{\Delta_1}^2}{48\mathrm{E}^3\eta^2}\right)^{\frac{1}{4}}, \omega = \frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{48\mathrm{E}^3\eta^2}}}$ and $\xi = k\left(x - \omega\frac{t^\delta}{\delta}\right)$.

Family 4:

$$V_{13,14} = -\frac{\Delta_1 \operatorname{sech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\sinh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}.$$

$$U_{13,14} = \frac{\alpha}{2\omega}\left(-\frac{\Delta_1 \operatorname{sech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\sinh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}\right)^2.$$

$$V_{15,16} = \frac{\Delta_1 \operatorname{cosech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\cosh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}.$$

$$U_{15,16} = \frac{\alpha}{2\omega}\left(\frac{\Delta_1 \operatorname{cosech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\cosh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}\right)^2.$$

Where, $k = \pm\left(\frac{\alpha\beta{\Delta_1}^2 + 2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}\right)^{\frac{1}{4}}, \omega = -\frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}}}$ and $\xi = k\left(x - \omega\frac{t^\delta}{\delta}\right)$.

Family 5:

$$V_{17,18} = -\frac{\Delta_1 \operatorname{sech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\sinh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}.$$

$$U_{17,18} = \frac{\alpha}{2\omega}\left(-\frac{\Delta_1 \operatorname{sech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\sinh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}\right)^2.$$

$$V_{19,20} = \frac{\Delta_1 \operatorname{cosech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\cosh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}.$$

$$U_{19,20} = \frac{\alpha}{2\omega}\left(\frac{\Delta_1 \operatorname{cosech}\left(\sqrt{-\mathrm{E}}\,\zeta\right)}{\sqrt{-\mathrm{E}}\cosh\left(\sqrt{-\mathrm{E}}\,\zeta\right)}\right)^2.$$

Where, $k = \pm I\left(\frac{\alpha\beta{\Delta_1}^2 + 2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}\right)^{\frac{1}{4}}, \omega = \frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}}}$ and $\xi = k\left(x - \omega\frac{t^\delta}{\delta}\right)$.

Trigonometric solution because of $E > 0$

Family 6:

$$V_{21,22} = \mp 2\sqrt{-\frac{6E}{\alpha\beta + 2\alpha s}}k^2\eta\sqrt{E}\tan\left(\sqrt{E}\zeta\right).$$

$$U_{21,22} = \frac{\alpha}{2\omega}\left(\mp 2\sqrt{-\frac{6E}{\alpha\beta + 2\alpha s}}k^2\eta\sqrt{E}\ \tan\left(\sqrt{E}\zeta\right)\right)^2.$$

$$V_{23,24} = \pm 2\sqrt{-\frac{6E}{\alpha\beta + 2\alpha s}}k^2\eta\sqrt{E}\cot\left(\sqrt{E}\zeta\right).$$

$$U_{23,24} = \frac{\alpha}{2\omega}\left(\pm 2\sqrt{-\frac{6E}{\alpha\beta + 2\alpha s}}k^2\eta\sqrt{E}\ \cot\left(\sqrt{E}\zeta\right)\right)^2.$$

Where, $k = k, \omega = 2E\eta k^2$ and $\xi = k\left(x - \omega\frac{t^\delta}{\delta}\right)$.

Family 7:

$$V_{25,26} = \frac{\Delta_1\ \sec\left(\sqrt{E}\zeta\right)}{\sin\left(\sqrt{E}\zeta\right)}.$$

$$U_{25,26} = \frac{\alpha}{2\omega}\left(\frac{\Delta_1\ \sec\left(\sqrt{E}\zeta\right)}{\sin\left(\sqrt{E}\zeta\right)}\right)^2.$$

$$V_{27,28} = -\frac{\Delta_1 \mathrm{cosec}\left(\sqrt{E}\zeta\right)}{\cos\left(\sqrt{E}\zeta\right)}.$$

$$U_{27,28} = \frac{\alpha}{2\omega}\left(-\frac{\Delta_1 \mathrm{cosec}\left(\sqrt{E}\zeta\right)}{\cos\left(\sqrt{E}\zeta\right)}\right)^2.$$

Where, $k = \pm\left(\frac{\alpha\beta{\Delta_1}^2 + 2\alpha s{\Delta_1}^2}{48E^3\eta^2}\right)^{\frac{1}{4}}, \omega = -\frac{\alpha{\Delta_1}^2(\beta + 2s)}{12E^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2 + 2\alpha s{\Delta_1}^2}{48E^3\eta^2}}}$ and $\xi = k\left(x - \omega\frac{t^\delta}{\delta}\right)$.

Family 8:

$$V_{29,30} = \frac{\Delta_1\ \sec\left(\sqrt{E}\zeta\right)}{\sin\left(\sqrt{E}\zeta\right)}.$$

$$U_{29,30} = \frac{\alpha}{2\omega}\left(\frac{\Delta_1\ \sec\left(\sqrt{E}\zeta\right)}{\sin\left(\sqrt{E}\zeta\right)}\right)^2.$$

$$V_{31,32} = -\frac{\Delta_1 \mathrm{cosec}\left(\sqrt{E}\zeta\right)}{\cos\left(\sqrt{E}\zeta\right)}.$$

$$U_{31,32} = \frac{\alpha}{2\omega}\left(-\frac{\Delta_1 \mathrm{cosec}\left(\sqrt{E}\zeta\right)}{\cos\left(\sqrt{E}\zeta\right)}\right)^2.$$

Where, $k = \pm I\left(\frac{\alpha\beta{\Delta_1}^2 + 2\alpha s{\Delta_1}^2}{48E^3\eta^2}\right)^{\frac{1}{4}}, \omega = \frac{\alpha{\Delta_1}^2(\beta + 2s)}{12E^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2 + 2\alpha s{\Delta_1}^2}{48E^3\eta^2}}}$ and $\xi = k\left(x - \omega\frac{t^\delta}{\delta}\right)$.

Family 9:

$$V_{33,34} = \frac{\Delta_1 \sec\left(\sqrt{\mathrm{E}}\,\zeta\right)}{\sin\left(\sqrt{\mathrm{E}}\,\zeta\right)}.$$

$$U_{33,34} = \frac{\alpha}{2\omega}\left(\frac{\Delta_1 \sec\left(\sqrt{\mathrm{E}}\,\zeta\right)}{\sin\left(\sqrt{\mathrm{E}}\,\zeta\right)}\right)^2.$$

$$V_{35,36} = -\frac{\Delta_1 \mathrm{cosec}\left(\sqrt{\mathrm{E}}\,\zeta\right)}{\cos\left(\sqrt{\mathrm{E}}\,\zeta\right)}.$$

$$U_{35,36} = \frac{\alpha}{2\omega}\left(-\frac{\Delta_1 \mathrm{cosec}\left(\sqrt{\mathrm{E}}\,\zeta\right)}{\cos\left(\sqrt{\mathrm{E}}\,\zeta\right)}\right)^2.$$

Where, $k = \pm\left(\frac{\alpha\beta{\Delta_1}^2 + 2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}\right)^{\frac{1}{4}}, \omega = -\frac{\alpha{\Delta_1}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta{\Delta_1}^2+2\alpha s{\Delta_1}^2}{-96\mathrm{E}^3\eta^2}}}$ and $\xi = k\left(x - \omega\frac{t^\delta}{\delta}\right)$.

Family 10:

$$V_{37,38} = \frac{\Delta_1 \sec\left(\sqrt{\mathrm{E}}\,\zeta\right)}{\sin\left(\sqrt{\mathrm{E}}\,\zeta\right)}.$$

$$U_{37,38} = \frac{\alpha}{2\omega}\left(\frac{\Delta_1 \sec\left(\sqrt{\mathrm{E}}\,\zeta\right)}{\sin\left(\sqrt{\mathrm{E}}\,\zeta\right)}\right)^2.$$

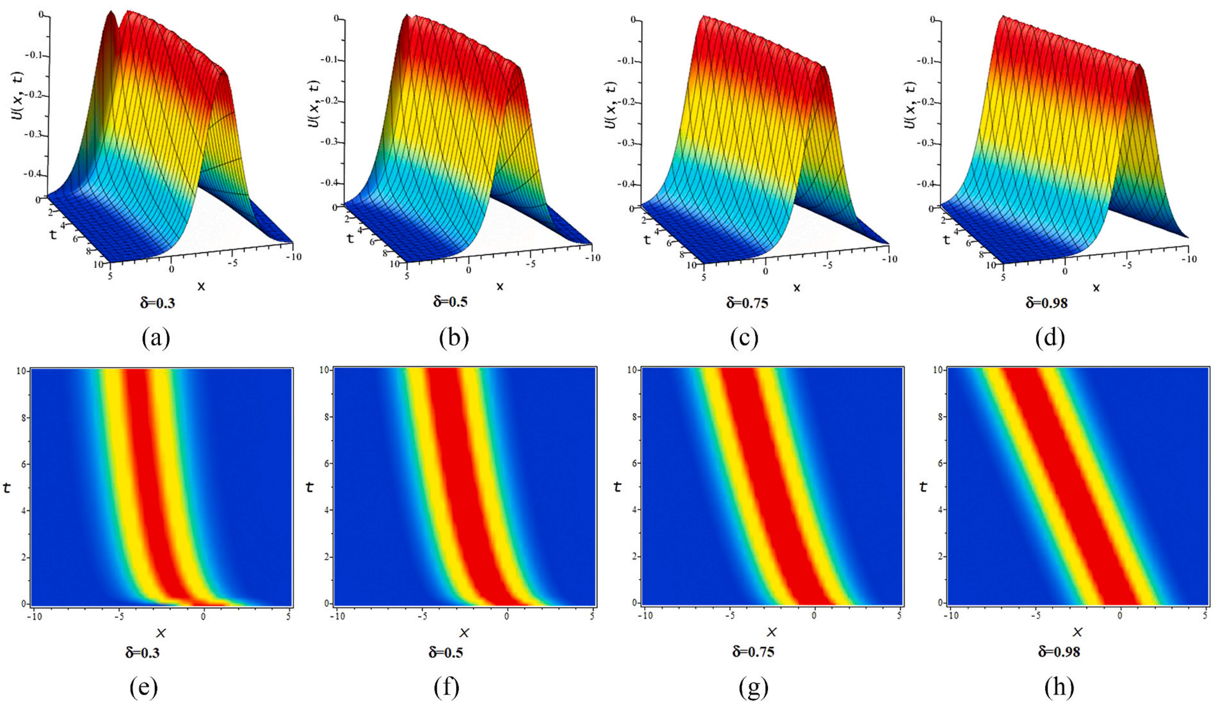

Fig. 1. Profile of bell shape of the solution $U_1(x,t)$ for the standard of parameters $E=-1, \eta=\alpha=s=Z=1, \beta=2, k=0.55, \Omega_0=\Delta_1=0$ at the fractional parameter values $\delta=0.3, \delta=0.5, \delta=0.75$ and $\delta=0.98$ respectively. (a), (b), (c), (d) display 3D plots while (e), (f), (g), (h) display density plots.

$$V_{39,40} = -\frac{\Delta_1 \mathrm{cosec}\left(\sqrt{\mathrm{E}}\zeta\right)}{\cos\left(\sqrt{\mathrm{E}}\zeta\right)}.$$

$$U_{39,40} = \frac{\alpha}{2\omega}\left(-\frac{\Delta_1 \mathrm{cosec}\left(\sqrt{\mathrm{E}}\zeta\right)}{\cos\left(\sqrt{\mathrm{E}}\zeta\right)}\right)^2.$$

Where, $k = \pm I\left(\frac{\alpha\beta\Delta_1{}^2 + 2\alpha s\Delta_1{}^2}{-96\mathrm{E}^3\eta^2}\right)^{\frac{1}{4}}, \omega = \frac{\alpha\Delta_1{}^2(\beta+2s)}{12\mathrm{E}^2\eta\sqrt{\frac{\alpha\beta\Delta_1{}^2+2\alpha s\Delta_1{}^2}{-96\mathrm{E}^3\eta^2}}}$ and $\xi = k\left(x - \omega\frac{t^\delta}{\delta}\right)$.

Rational solution is rejected for their predefined condition.

N.B. all solution has been satisfied in the proposed equation with the favor of computational programing software maple and Mathematica.

5. Results and discussion

In this section, using the modified extended tanh approach, We will investigate whether the DSW equation's explicit and analytic traveling wave solutions are physically feasible. For the particular parameter's values in the solution that was obtained, the impact of time fractional derivative is recorded for the standards of the fractional parameters $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ with 3D graphical and corresponding density plots. Form the obtained solution, we get bell shape solution, periodic soliton solution, periodic lump wave solution, multi-wave solution, kinky periodic wave, linked-lump wave, interaction of kink and lump wave etc.

For the value of parameters $E = -1, \eta = \alpha = s = Z = 1, \beta = 2, k = 0.55, \Omega_0 = \Delta_1 = 0$, Fig. 1 illustrates the bell shape of the solution $U_1(x,t)$ at the values of the fractional parameters $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. In Fig. 2, the periodic wave solution illustrates from the solution $U_{21}(x,t)$ for the value of parameters $E = 0.1, \eta = -s = Z = k = 1, \beta = 0.5, \alpha = 2, \Omega_0 = \Delta_1 = 0$ where the standards of the fractional parameters $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. In Fig. 3, the profile of the periodic rouge wave obtains from the solution $U_{29}(x,t)$ for the standard of the parameters $\alpha = z = \beta = \eta = 1, \Omega_0 = 0, \Delta_1 = -3, E =, s = -1$ at the fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. The solution $U_7(x,t)$ illustrated the multi type wave of the solution for the standard of the parameters $E = \alpha = z = 1, \beta = \eta = -1, \Omega_0 = 0, \Delta_1 = -0.1, s = -6$ at the fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively, that shows in Fig. 4. For the value of the parameters $E = -3$, $\alpha = z = \beta = \eta = s = 1, \Omega_0 = 0, \Delta_1 = -3$, Fig. 5 represents the profile of the kinky periodic wave of the solution $U_{19}(x,t)$ at the

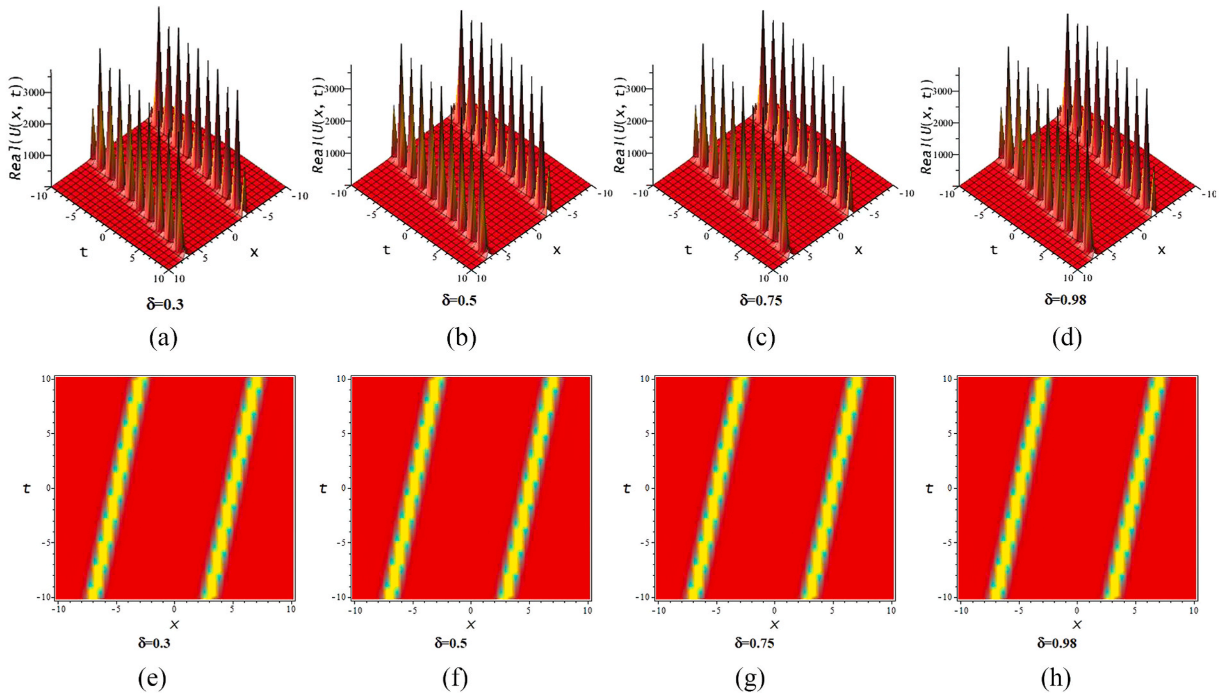

Fig. 2. Profile periodic wave of the solution $U_{21}(x,t)$ for the standard of parameters $E = 0.1, \eta = -s = Z = k = 1, \beta = 0.5, \alpha = 2, \Omega_0 = \Delta_1 = 0$ at the fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. (a), (b), (c), (d) display 3D plots while (e), (f), (g), (h) display density plots.

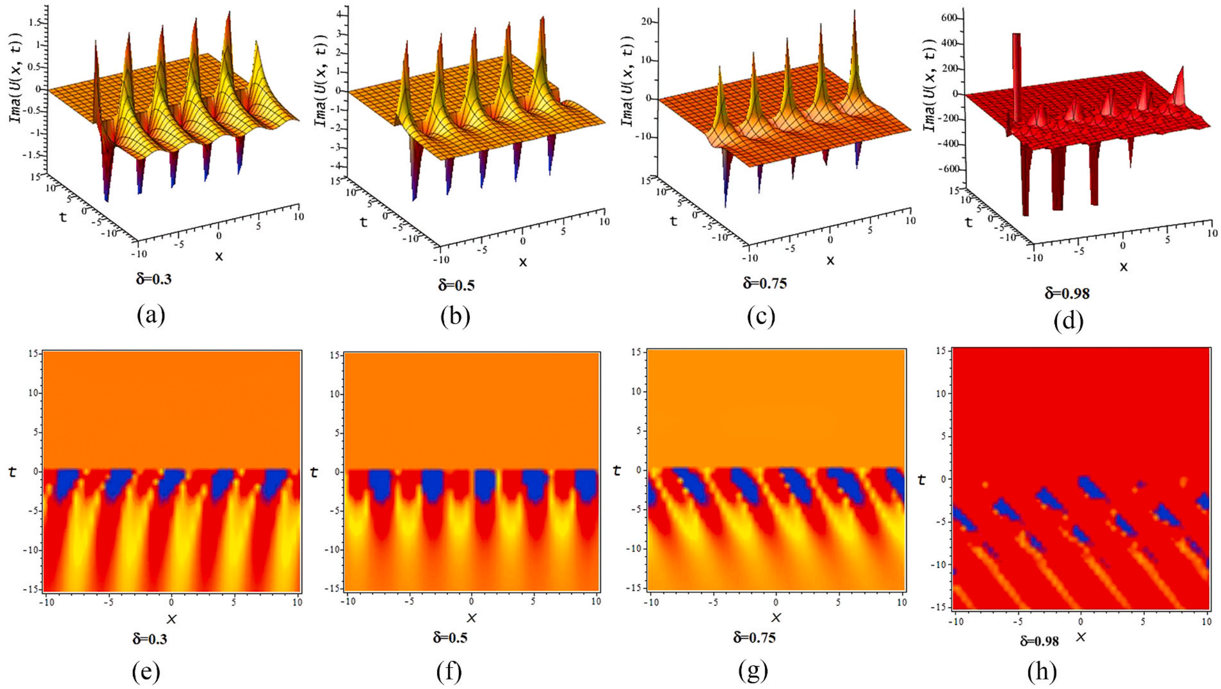

Fig. 3. Profile of the periodic rouge wave of the solution eq. $U_{29}(x,t)$ for the standard of the parameters $\alpha = z = \beta = \eta = 1, \Omega_0 = 0, \Delta_1 = -3, E = s = -1$ at the fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. (a), (b), (c), (d) display 3D plots while (e), (f), (g), (h) display density plots.

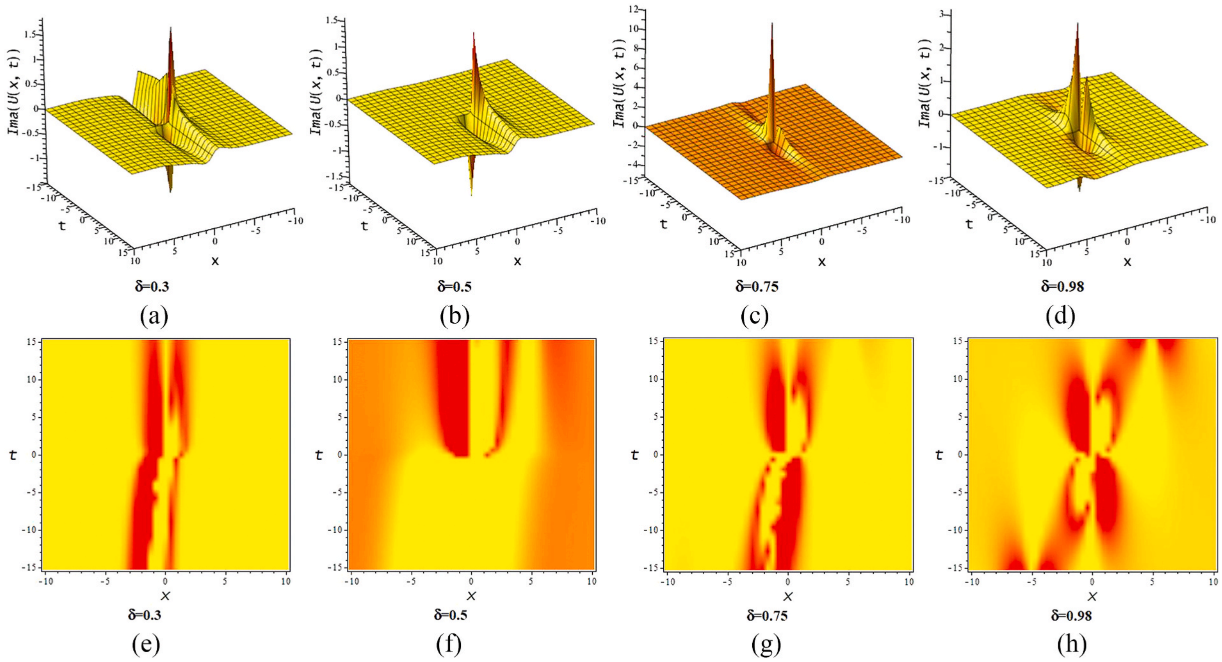

Fig. 4. Profile of the multi-wave of the solution eq. $U_7(x,t)$ for the standard of the parameters $E = \alpha = z = 1, \beta = \eta = -1, \Omega_0 = 0, \Delta_1 = -0.1, s = -6$ at the fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. (a), (b), (c), (d) display 3D plots while (e), (f), (g), (h) display density plots.

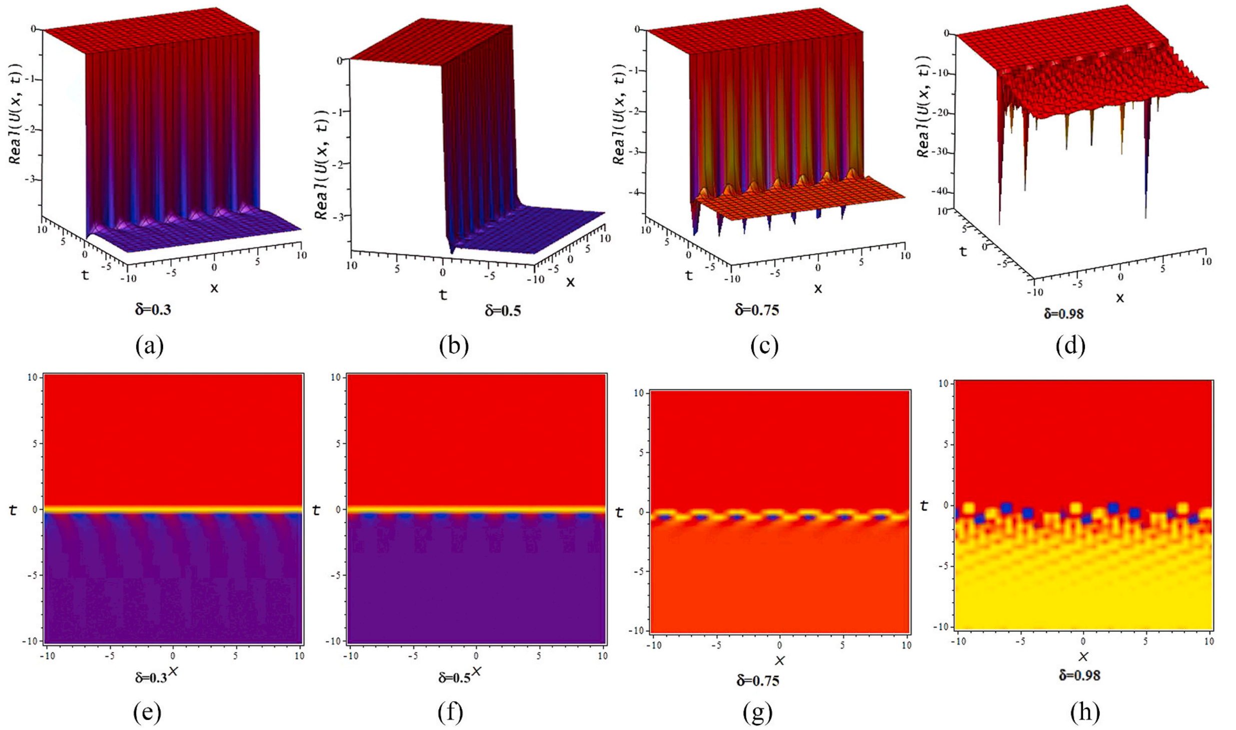

Fig. 5. Profile of the kinky periodic wave of the solution eq. $U_{19}(x,t)$ for the standard of the parameters $E=-3, \alpha=z=\beta=\eta=s=1, \Omega_0=0, \Delta_1=-3$ at the fractional parameter values $\delta=0.3, \delta=0.5, \delta=0.75$ and $\delta=0.98$ respectively. (a), (b), (c), (d) display 3D plots while (e), (f), (g), (h) display density plots.

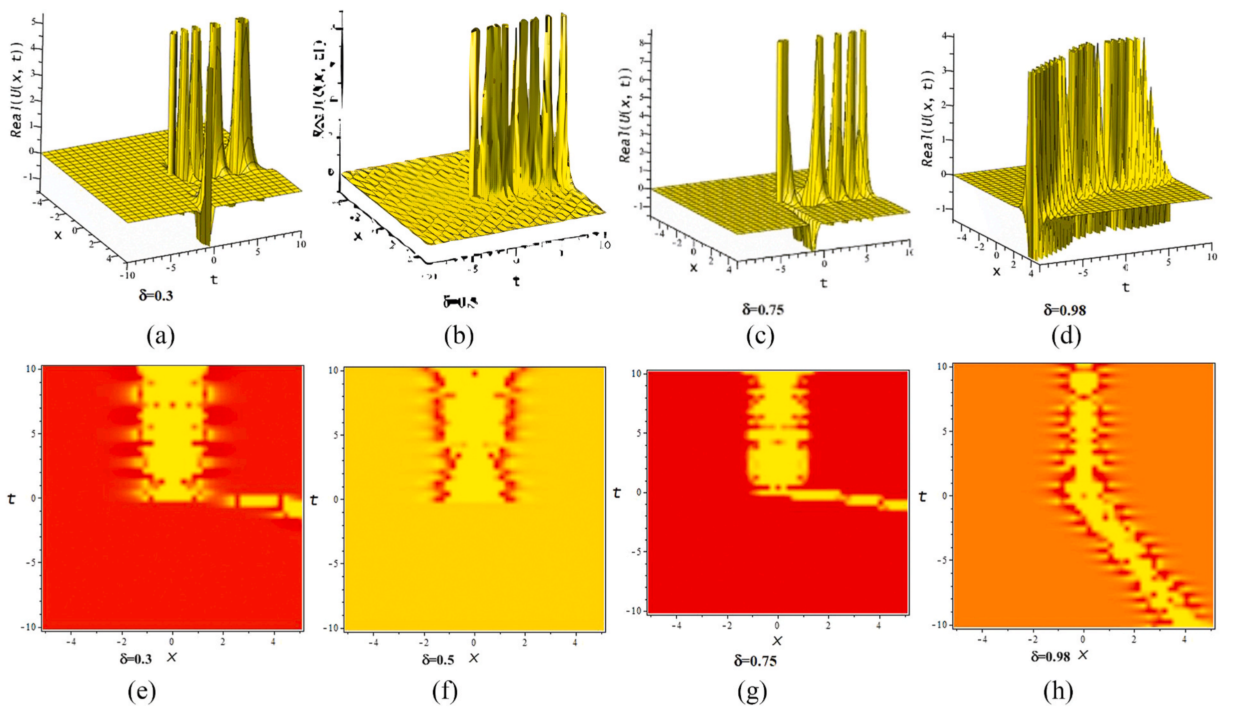

Fig. 6. Profile of the linked-lump wave of the solution eq. $U_{39}(x,t)$ for the standard of the parameters $E=0.3, \alpha=-2, z=1, \beta=1, \eta=-1, \Omega_0=0, \Delta_1=1, s=-1$ at the fractional parameter values $\delta=0.3, \delta=0.5, \delta=0.75$ and $\delta=0.98$ respectively. (a), (b), (c), (d) display 3D plots while (e), (f), (g), (h) display density plots.

fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. In Fig. 6, the profile of the linked-lump wave of the real portion of solution $U_{39}(x,t)$ for the standard of the parameters $E = 0.3, \alpha = -2, z = 1, \beta = 1, \eta = -1, \Omega_0 = 0, \Delta_1 = 1, s = -1$ at the fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. In Fig. 7, the profile of the interaction of kink and lump wave of the imaginary portion of solution $U_{39}(x,t)$ for the standard of the parameters $E = 0.3, \alpha = -2, z = 1, \beta = 1, \eta = -1, \Omega_0 = 0, \Delta_1 = 1, s = -1$ at the fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively.

6. Resemblances and comparisons of this manuscript

In this portion, we will evaluate the obtained solutions to those of Arnous et al. [60] that was using the Riccati equation's backlund transformation and the trial function approaches and also to find resemblances of our obtained solution with Bashar et al. [61] that was using the manner of new auxiliary equation.

6.1. For Backlund transformation of Riccati equation method

With the assist of the backlund transformation of the Riccati equation, Arnous et al. [60] investigated the soliton solutions to Eq. (1) and discovered four pairs of solutions (Kindly see in Ref. [60]). On the other hand, in this article, using the manner of modified extended tanh, we have discovered forty precise solutions to Eq. (1). For both approaches, the Riccati equation is not the same.

6.2. For trail function method

With the assist of the trail function method, Arnous et al. [60] investigated the soliton solutions to Eq. (1) and discovered eight sets of arrangements (Kindly see Ref. [60]). On the other hand, in this article, using the technique of modified extended tanh, we have discovered forty-four precise solutions to Eq. (1). For both approaches, the Riccati equation is not the same.

It is clear that the approach is strong, practical, and easy to use and that it can be used with all NLEEs. Many solutions, including hyperbolic function solutions, trigonometric function solutions, and rational function solutions, have been found for this work. The results are expressed as wave profiles with kink shapes, bell shapes, bright shapes, dark shapes, and unique stage shapes. We have talked about some of these wave profiles' consequences, which make quite evident. There are numerous potential applications for these wave characteristics.

6.3. For new auxiliary equation method

We discovered several solutions to the provided equation by the assist of the modified extended tanh method. Bashar et al. [61] used new auxiliary equation scheme and acquired forty four solutions (Kindly see Ref. [61]) to the DSW equation.

Bashar et al. [61] solutions

Placing $p\sqrt{-rp} = \sqrt{-\mathrm{E}},\ V_{27,28} = V_{1,2},\ V_{29,30} = V_{3,4},\ U_{27,28} = U_{1,2}$, and $U_{29,30} = U_{3,4}$ in solution set 8 then it's turns to similar of our solution (see in family 1).

$$V_{1,2} = \mp 2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{-\mathrm{E}}\tanh(\sqrt{-\mathrm{E}}\,\zeta).$$

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$$V_{3,4} = \mp 2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{-\mathrm{E}}\coth(\sqrt{-\mathrm{E}}\,\zeta).$$

$$U_{3,4} = \frac{\alpha}{2\omega}\left(\mp 2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{-\mathrm{E}}\ \coth\left(\sqrt{-\mathrm{E}}\,\zeta\right)\right)^2.$$

Placing $p = \sqrt{\mathrm{E}},\ V_{43,44} = V_{21,22}$, and $U_{43,44} = U_{21,22}$ in solution set 19 then its turns to similar of our solution (see in family 6).

$$V_{21,22} = \mp 2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{\mathrm{E}}\ \tan\left(\sqrt{\mathrm{E}}\,\zeta\right).$$

$$U_{21,22} = \frac{\alpha}{2\omega}\left(\mp 2\sqrt{-\frac{6\mathrm{E}}{\alpha\beta+2\alpha s}}k^2\eta\sqrt{\mathrm{E}}\ \tan\left(\sqrt{\mathrm{E}}\,\zeta\right)\right)^2.$$

It should have noted that, we got some of our solution of this is as same as previous published paper. Bashar et al. [61] used new auxiliary equation are seen in Appendix.

7. Conclusions

In this work, the bell shape solution, periodic wave solution, periodic lump wave solution, multi-wave solution, kinky periodic wave, linked-lump wave, interaction of kink and lump wave solution have been presented in this work to observe fluid dynamics problems by applying the modified extended tanh techniques. Additionally, these methods can be computerized by utilizing a personal computer and well-known applications like Maple, Matlab, and Mathematica, among others. This gives us permission to perform tedious and baffling arithmetic counting. The traveling wave transformation formulae were used to find the required solutions, and visual representations were used to interpret these solutions. Last but not least, we have emphasized that the proposed approach outperforms all other expansion techniques for nonlinear evolution equations and related mathematical physics models in terms of

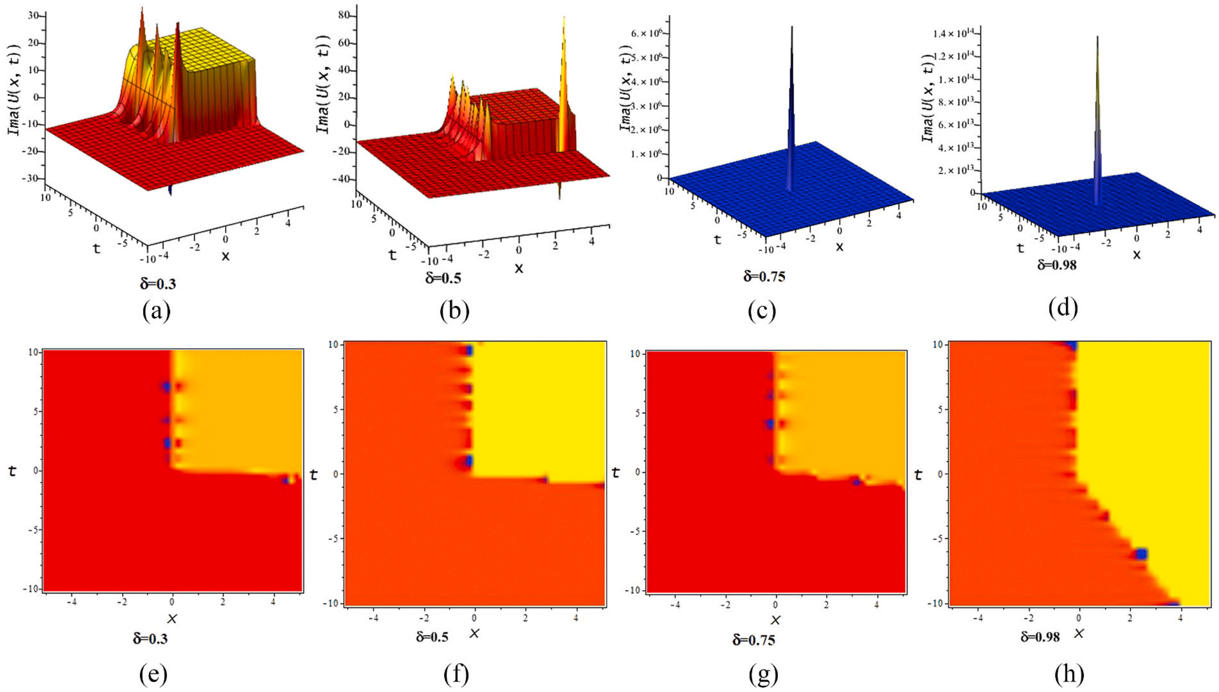

Fig. 7. Profile of the interaction of kink and lump wave of the solution eq. $U_{39}(x,t)$ for the standard of the parameters $E = 0.3, \alpha = -2, z = 1, \beta = 1, \eta = -1, \Omega_0 = 0, \Delta_1 = 1, s = -1$ at the fractional parameter values $\delta = 0.3, \delta = 0.5, \delta = 0.75$ and $\delta = 0.98$ respectively. (a), (b), (c), (d) display 3D plots while (e), (f), (g), (h) display density plots.

ease of use, efficiency, power, and output. In future, we will investigate the non-autonomous soliton solutions that would be engendered by diverse NLEEs when their coefficients are variables.

Author contribution statement

Md. Habibul Bashar: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper; HZ Mawa, Anita Biswas: Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper; M. M. Rahman, Md. Mamunur Roshid, Jahedul Islam: Analyzed and interpreted the data; Contributed analysis tools or data.

Data availability statement

Data included in article /supplementary material/ referenced in article.

Declaration of interest's statement

The authors declare no conflict of interest.

Appendix

The solutions of Bashar et al. [61] are listed below
Solutions set 8: when $pr < 0, q = 0$ and $r \neq 0,$.

$$V_{27,28} = \mp\left(-\sqrt{\frac{-p}{r}}\tanh\left(p\sqrt{-rp}\,\xi\right)\right)\sqrt{-\frac{24p^2}{\alpha\beta+2\alpha s}rk^2\eta}$$

$$U_{27,28} = \frac{\alpha}{2\omega}\left(\mp\left(-\sqrt{\frac{-p}{r}}\tanh\left(p\sqrt{-rp}\,\xi\right)\right)\sqrt{-\frac{24p^2}{\alpha\beta+2\alpha s}rk^2\eta}\right)^2$$

Or

$$V_{29,30} = \mp\left(-\sqrt{\frac{-p}{r}}\coth\left(p\sqrt{-rp}\,\xi\right)\right)\sqrt{-\frac{24p^2}{\alpha\beta+2\alpha s}rk^2\eta}$$

$$U_{29,30} = \frac{\alpha}{2\omega}\left(\mp\left(-\sqrt{\frac{-p}{r}}\coth\left(p\sqrt{-rp}\,\xi\right)\right)\sqrt{-\frac{24p^2}{\alpha\beta+2\alpha s}rk^2\eta}\right)^2$$

Solutions set 19:when $r = p$ and $q = 0,$.

$$V_{43,44} = \pm 2\tan(p\xi)\sqrt{-\frac{6p^2}{\alpha\beta+2\alpha s}pk^2\eta}$$

$$U_{43,44} = \frac{\alpha}{2\omega}\left(\pm 2\tan(p\xi)\sqrt{-\frac{6p^2}{\alpha\beta+2\alpha s}pk^2\eta}\right)^2$$

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