Special Feature

Leonardo da Vinci: Cause, effect, linearity, and memory

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HIGHLIGHTS

- Leonardo da Vinci's physics concepts were analyzed.
- The principle of causality introducing a system impression was discussed.
- Leonardo hypothesized a general law on linearity (pyramidal law).
- Leonardo conciliated the Aristotle's and the Newton's positions.
- The dynamics of Leonardo was framed within the modern linear response theory.

GRAPHICAL ABSTRACT
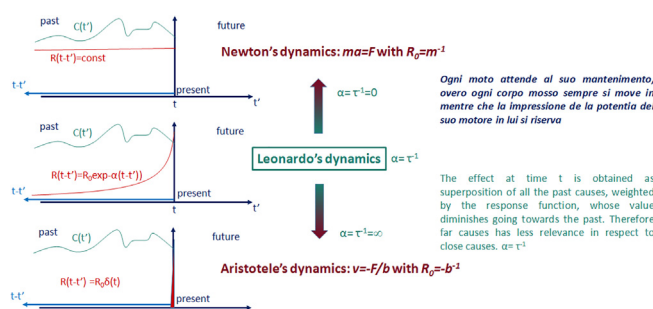
ABSTRACT

In this contribution, some textual portions of the Leonardo da Vinci's work were analyzed with the aim to highlight how, moving from Aristotle and going beyond him, he combines the intermediate positions that, from the Greek philosopher, passing through Buridan, arrive to Newton. This has been performed following a path that passes through the formulation of the principle of causality, the use of the concept of linear relationship (pyramidal law) between cause and effect and the introduction of a duration of the impression (memory) of mechanical systems. In the framework of the studies aimed to a valorization of Leonardo as a scientist, which is a crucial aspect in the analysis of the Leonardo genius, the present work sheds a new light on his intuitions about some fundamental physics concepts as well as about the conceptual model that, several centuries later, will be formalized in the modern linear response theory.

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Introduction

In approaching some text portions which compose the "scientific production" of Leonardo, who used to fix with excelled attitude and pictorial vehemence even the most rigorous theories of dynamics, the following epistemological considerations are assumed as fundamental:

(i) At the time of Leonardo physics was not an autonomous field of investigation characterized by independent investigation methods and hence it was not separable from philosophy, being by definition *physicus* someone dealing in a general way with the *physis*, i.e. the vast and complex science (in the primordial and authentic sense of knowledge) of Nature. In a broader and also more respectful meaning of the term, the *physicus* was the "philosopher of nature".

Furthermore, the so-called *scientia* should be interpreted as a form of universal knowledge and, when this is not the case, it aspires to be *Sapientia* (as in the case of hermetic-alchemic

writings, where operations on the material world have as their own purpose the operations on the human mind).

Following the Aristotelian pattern, knowing the "things of Nature" means: (i) to investigate the causes responsible for what happens (*efficient cause*), (ii) to explain which material agent provides the basis of the investigated process (*material cause*), (iii) to investigate to which form the matter tends (*formal cause*) and finally (iv) to understand the *final cause*, that is the purpose of the process.

In the effort of investigating Nature, the *tetrapartita* division is not far from the Leonardo view, who used the four fundamental variables – that, in his dynamic theories, are the motion, the weight, the force and the percussion [1] – and the four constitutive elements of the sublunar world according to the Aristotelian cosmology, also in agreement with the neoplatonic models and the philosophy coming from the Arabs (we know that Leonardo knew, for example, Rhasis).

(ii) For the philosophers of Nature, which can be considered as the direct emanation of the *Platonic One* or the *Christian God*, the laws of the material world (i.e. immanent as part of the visible world) must correspond to the laws of a transcendent (i.e. invisible and superior) world, that, albeit in a way susceptible to constant investigations (all metaphysical and cosmological questions are constantly subjected to comments, elucidations, corrections or refutations over the centuries), do not aspire to "separate" themselves from the general questions of human knowledge: medicine, architecture, astrology, music, etc.

In this sense, our approach to the Leonardo's work aims to fill the epistemic gap between the method of today's exact sciences, which is often separated from the human sciences one, and the widely used analogical method adopted by Leonardo. Leonardo, in fact, had an approach to knowledge which today can be defined as "systemic": he possessed the notion of complexity of interdependent phenomena and therefore he contemplated them to discover their secret dynamics. However, his culture did not conceive the use of analysis as a discriminating or separation criterion, enslaved to a single domain of knowledge; therefore his admirable analytical efforts were always at the service of the profound and metaphysically justified reasons of the *synthesis*.

Up to the eighteenth century, the speculations on the material world were in many cases originated by a "qualitative" interest rather than by a "quantitative" one, and to such speculations the analogical method applied powerful syntheses that interweaved mythology, astrology, natural sciences, medicine, mathematics, etc. Leonardo interpreted the analysis instances basing his approach on the measurable quantities, on the experiments and on the replicability of the phenomena, all these features characterizing the future science in the sense conferred by the post-Newtonian episteme; however, Leonardo blended tradition and innovation, analytical-quantitative spirit and analogical-qualitative method.

(iii) Within the culture of Leonardo the laws of physical motion were not always separable from their ethical consequences, nor from their metaphysical roots since, thanks to the prevailing Aristotelian *auctoritas*, the motion was local (i.e. *locomotion*); furthermore, it was also alteration of quantity and change of quality, i.e. the motion regulated, for example, the flow of the humors in the human body and determined its physiology and character, illness and health. The same laws of sublunar nature (the "second nature" that regulates the elementary world) were not wholly autonomous from the celestial world.

In the question of motion, in fact, Leonardo expressed an appreciative "apostrophe" towards the real causative agent of the motion on Earth, i.e. the *Primo mobile* of Aristotelian origin, saying: "O mirabile giustizia di te, primo motore, tu non hai voluto mancare a nessuna potenzia l'ordine e qualità dei suoi necessari effetti!", i.e "O admirable justice of you, first engine, you did not want to miss at any power the order and quality of its necessary effects!" (A 24r.). Leonardo falls within "a great conception of magic-metaphysics relative to Nature" [1].

In our analysis we will attempt an exegetical path that compares the homologies and the differences among the passionate and often elusive observations of the Leonardo *physicus* and the clear assumptions of the contemporary scientific dictate. The sources that we have taken into account are: (i) sources of which Leonardo had direct knowledge and which intersect almost all the fields of his time knowledge, without having the pretension of an integral, systematic and less than ever exhaustive screening [2]; (ii) sources contemporary to Leonardo from which he may have viewed the unmistakable philosophical profile of the Medicis' Florence; we know that Leonardo read, for example, the Platonic Theology of Ficino [3]; (iii) indirect sources, i.e. the ancient and modern critical studies that, with variously modulated critical purposes and aims, have collected, preserved and thought about the work of Leonardo; such sources are the expression of the work of historians of art, scientists, philologists or philosophers; (iv) the assessment of the fundamental contribution, but certainly and unfortunately less traceable, constituted by the corporative knowledge transmitted orally to Leonardo in his training workshop ("bottega").

The cognitive corpus handed down by a corporative way constituted *sensu stricto* the so-called "secrets of the trade" [4]. The foundational contribution of the tradition of "bottega" to the Leonardo's hermeneutics also seems to have been today revalued [5]. It is important to remind that the "pittore" belongs to the same guild of "medici e speziali" and in this sense Leonardo can be placed in the philosophical context of Dante [6].

In the following, some phrases transcribed by the Leonardo's notebooks are presented, together with their translation, with the aim to clarify the meanings attributed to the introduced terms and quantities by Leonardo. Our analysis proposes a new interpretation of these statements in the light of the principle of causality, of the relations of linearity existing between cause and effect and of the concept of memory expressed, in the terms appropriate to him and to his time, by Leonardo.

The complex principle of causality in Leonardo

"Nessuno effetto è in natura sanza ragione; intendi la ragione e non ti bisogna sperienza" (Codex Atlanticus, 147 r. a.)

No effect is in Nature without cause; you understand the cause and you do not need any experience.

This statement represents a formulation of the principle of causality, according to which each effect is linked to its own cause. The knowledge, through experience, of the cause that has determined a given effect allows to identify phenomenological relationships between the defined quantities making possible the prediction of the behavior of the system and hence making the experience no longer necessary.

"La gravità, la forza insieme col moto materiale e lla percussione sono le quattro potentie accidentali colle quali l'umana spetie, nelle sue mirabili e varie operationi pare in questo mondo dimostrarsi una seconda natura. Imperoché con tali potentie tutte l'evidenti opere de' mortali anno loro essere e lloro morte" (Codex Arundel, P 12v, 151v.)

Weight, force and casual impulse, together with resistance, are the four external powers in which all the visible actions of mortals have their being and their end.

Weight, force, casual impulse and resistance play the role of causes, while the actions play the role of effects.

"La forza da carestia o dovizia è generata. Questa è figliola del moto materiale e nepote del moto spirituale e madre e origine del peso" (Codex Arundel 151 r.)

Force arises from dearth or abundance; it is the child of physical motion, and the grand-child of spiritual motion, and the mother and origin of gravity.

The force derives by a defect or by an excess, i.e. by an imbalance of other forces; today we would say that every time a body changes its momentum, there is at least one force responsible for that change. The force is generated by the physical motion which in turn is generated by the spiritual motion. The Aristotelian kinetics (accepted with emphasis by Leonardo in his apostrophe to the *Primo mobile*) prescribes that in the universe there is a first engine (*Primo mobile*) that, through a paradoxical immobility, impresses the motion to the circular spheres – the heavens. In turn, as in a cascade effect, the celestial movement is imparted to the sublunar world, marked, unlike the first, by space, time and processes of generation and corruption [2].

Since the spiritual motion is often indicated by Leonardo as the effect (or cause) of an "invisible and spiritual force", we could finally define this force, as it has been already done for centuries, as an "unknown force", a *virtus occulta*, which from time to time was considered responsible for the mineral action of the stones, of the action of the magnets, as well as of the percussive action of the power of the eye – and the examples could multiply -, that is, ultimately as a synonym of action directed in an invisible and immaterial way from the top of the superlunar world and engraved in the terrestrial world [7].

This spiritual force is transmitted to the living body that generates a physical motion from which in turn the material force is generated: "Adunque il moto materiale nasce dallo spirituale" (Codex Arundel 151 r. v.).

"La forza dal moto spirituale ha origine; il quale moto, scorrendo per le membra degli animali sensibili, ingrossa i muscoli di quelle; onde, ingrossati, essi muscoli si vengano a raccortare, e tiransi dirieto i nerbi che con essi sono congiunti, e di qui si causa la forza per le membra umane" (Codex Arundel, 151 r.)

Force has its origin in spiritual motion; and this motion, flowing through the limbs of sentient animals, enlarges their muscles. Being enlarged by this current the muscles are shrunk in length and contract the tendons which are connected with them, and this is the cause of the force of the limbs in man.

The force originates from the spiritual motion that flows into the muscles of sensitive beings and creates their physical strength.

"Il moto è causa d'ogni vita" (Codex Trivulzianus, 36)
The motive power is the cause of all life.
A motion is necessary for life.

Based on the above reported phrases, a first conceptual map can be formulated (Fig. 1).

In the light of the current language of physics, the principle of causality exposed by Leonardo can be summarized as it follows: if *E* is a physical quantity that plays the role of "effect" and *C* is a physical quantity representing the respective "cause", then

(i) effect *E* is a function of the cause *C*, i.e. $E = E(C)$;
(ii) if the cause is null, the effect will also be null, i.e. $C = 0 \rightarrow E = 0$.

Initial motion and derivative motion

"Il moto primitivo ecquel cheffatto dal mobile nel tenpo chelli e chongiunto al suo motore. Il moto diriuativo ecquel cheffa il mobile infràìlaria poi chelli e separato dal suo motore. Il moto dirivativo adiri a derivatione dal moto primitivo enmai eppotenie velocità o potentia simile alla velocità o potentia desso primitivo. Il chorso di quel mobile ara conformità chol corso del sito motore la rectitudine che a il corso del suo motore quando tutte le sue parte desso mobile fien di moto equale al moto primitivo del suo motore. Se tutte le parte del moto che fatto dalla parte duntutto saran di moto equale allora tal mobile non sara circhunvolubile, ecquessto tal moto se ricievera la intera potentia del suo motore e osservera la debita lunghezza chessinchiede al suo moto esendo il peso del mobile proportionalo a la potentia del suo motore" (G 87 r.)

The initial motion is that of the object that moves together with its motor. The derivative motion is that of the object moving in the air, after it has separated itself from its motor. The derivative motion takes its origin from the initial motion and has never velocity or power equal to the velocity or power of the initial motion. The running of this moving object is coherent with the direction of its motor when all parts of the moving object have a motion equal to the initial motion of the motor. If all parts of the motion carried out by the part of an all are equal, the moving object will not rotate; this motion will undergo the entire power of its motor, and will observe the required length, the weight being proportional to the power of the motor.

The initial motion is the motion of the object that moves along with its motor, while the derivative (from the initial motion) motion is the motion of the object after it is removed from its motor; it is less powerful than the initial motion due to the loss of energy. Its direction is kept in accordance with that of its motor if the body does not rotate, that is when all its points have the same velocity and, in this case, it will undergo the entire power of its motor which is proportional to its mass (weight).

"Quel corpo sarà di più veloce corso che da più veloce motore sarà sospinto" (K 110 (30) r.)

The object moves a lot when separated from its motor, if it is moved by a larger power.

The motion of the moving object, i.e. the effect, is proportional to the cause.

"De' moti si trova di 2 spezie, cioè semplici e composti. De' semplici nessuno fia più tardo o veloce che la tardità o velocità del suo motore; de' composti possano essere più tardi e più veloci infinitamente più che il suo motore, ed etiam possano essere equali a esso motore" (K 107 (27) r.)

There are two types of motion that is simple and composed. About the simple motion, it does not exceed its motor in slowness or velocity.

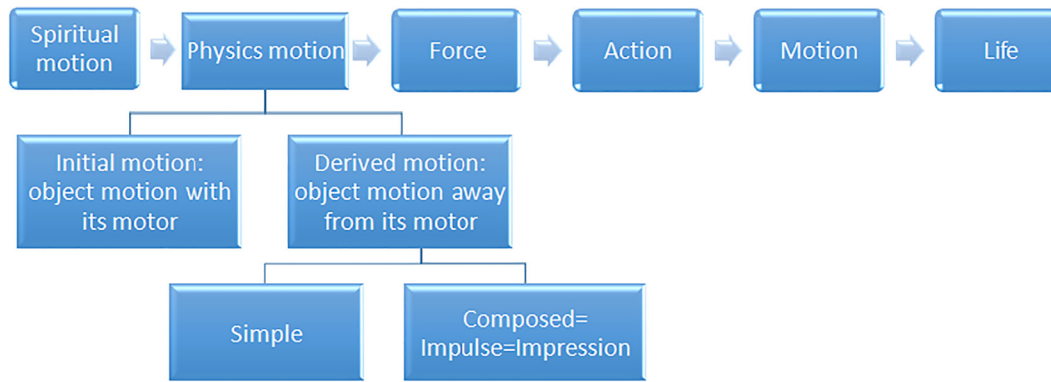**Fig. 1.** First conceptual map related to the principle of causality.

Fig. 2. Second conceptual map including motions.

Composed motions can be infinitely slower or more rapid than their motor; and also be equal to it.

The simple motion is the one that characterizes the moving object without the intervention of other forces that induce changes of the motion, as it happens in the composed motion.

"Della violentia dicho oni chorpo mosso o perchosso ritiene in sé per alquanto spatio la natura dessa violenza ettura perchussione o monimento e // ritiella tanto piu o meno quanto sara magiore omminore la potentia alla forza desso colpo o moto vimento. Esempio: vedi un colpo dato in una campana quanto riserva in se il romore della perchussione. Vedi una pietra uscita dalla bombarda quanto riserva la natura del movimento. Il corpo dato in un corpo denso durerà più il sono che in chorporaro. Il ecquello arà più durata cheffia in corpo sospeso essettile. Lochio riserva in se lemagini de chorpi luminosi per alquanto spatio" (Codex Trivulzianus, 73 a)

I say that every moved or affected body preserves during some time the nature of the shot or motion, and this time will be proportional to the force of the shot or motion. Example: observe a shot given on a bell, as it retains the noise of the percussion. Observe a stone projected by a bombardier, as it preserves the nature of the motion. The body settled on a thick body will retain the sound for longer time than a thin body, and this shall have longer duration if it will be produced on a suspended body and of small thickness. The eye preserves for some time the pictures of the luminous bodies.

Every system moved or beaten by a "violence" restrains it in itself for a certain space, which is proportional to the applied force (linearity between cause and effect). For example, the motion of a body moved or hit, the noise generated by a percussion, the motion of a stone projected by a bombardier, the sound produced by a more or less thick body, the image of a luminous body, and, in general, the effects of an applied solicitation (cause) to a system extend for a certain time (duration of memory) and, consequently, for a certain space.

Based on the above reported phrases, a second conceptual map can be formulated (Fig. 2).

Linear dependence of effect on cause

"Perché il moto naturale delle cose gravi in ogni grado di discenso acquista un grado di velocità e per questo tal moto si figura, nell'acquistare di potenzia, di figura piramidale, perché la piramide acquista similmente in ogni grado della sua lunghezza un grado di larghezza; e così tale proporzione d'acquisto si trova in proporzione aritmetica, perché li eccessi sempre sono equali" (M 59 v.)

Why the natural motion of heavy things at each degree of descent acquires a degree of velocity and for that such a motion is shown, in acquiring power, as a pyramidal figure, because the pyramid similarly is acquiring at each degree of its length a degree of width; and such a proportion of the gain respects an arithmetic ratio, since excesses are always equal.

"Prova della proporzione del tenpo e del moto insieme colla velocità fatta nel discendere de corpi gravi colla figura piramidale peiche le predette potentie son tutte piramidali perche cominciano niente e vanno ere sdendo a gradi di proporzione arìtmetricha. Settu tagli la pramide in quanlunche grado di della sua alteza chon linia equidistante alla sua basa tu troverrai che quelle proportione che a lo spatio chee da tale taglio insino alla baso con tutta lalteza di tale piramide. Tale proportione ara la largeza di tal taglio colla largeza di tutta labasa" (M 44 r.)

Evidence of the proportion of time and motion together with the velocity which is found in the descent of heavy bodies with the pyramidal figure, because the aforementioned powers are all pyramidal, since they begin null and go increasing by degrees of arithmetical proportion. If you cut the pyramid at any degree of its height with an equidistant line at its base, you will find that such proportion has the space that there is in this section at its base with the whole height of this pyramid. This proportion will have the width of such section with the width of the whole base.

"Sella corda del balestro dopo la fuga chessa dette alla saetta resta incurvata cierto e chella sua potentia in ogni grado di moto acquista gradi di tardita e debolezza; finita onde tal potentia direno essere piramidale che comincia in basa e finiscie in punto. Ancora essendo la saetta sosspinta dalla corda della balesstra essa e piramidale perche in ogni grado di moto acquista gradi di tardità he deboleza, ma perche tal piramide epiv lunga che quela del suo motore la saetta siparti dalla corda prima che tal corda si fermassi anzi quandera in potentia, magiore il suo motore" (M 90 r. e v.)

If the rope of the crossbow, after the leak it has given to the arrow, remains bent, it is certain that its power at each degree of motion has acquired degrees of slowness and infinite weakness; then we say that such a power is pyramidal, which begins in a base and ends in a point. The arrow, being still pushed by the rope of the crossbow, is pyramidal, because at every degree of motion it acquires degrees of slowness and weakness, but, because this pyramid is longer than that of its motor, the arrow left the rope before this rope would be arrested; much more, when its motor was in the greatest power.

"Una medesima virtù è tanto più potente, quanto ella occupa minore loco. Questa s'intende pel caldo e per la percussione e pel peso e forza e molte altre cose. E diren prima del caldo del sole, che s'imprieme nello specchio concavo e refrette di quello in figura piramidale, la qual piramide quanto più si restrigne, tanto proporzionevolmente acquista di potenzia coe sella piramide percote col lobietto colla metà della sua lungheza essa risstrignie la meta de la sua grosseza dappiedi essella percote nelli novantanove centesimi della sua lungheza essa di siresstringe li 99 centessimi della sua basa cere sce li 99 centesimi del chaldo che riceve essa basa del detto caldo del sole o del focho. Ancora la percussione du ferro

piramidale p neterra tanto più la cosa dallui penetrabile dalla sua punta percossa quanto essa punta sarà più stretta. Ancora il grave ridotto imen locho e di magior peso perche men quantità d'aria lire siste del moto efforza direno altrove (G 89 v.)

The same virtue is the more powerful the more it is concentrated. This is the case of warmth, percussion, weight, force, and many other things. First of all we talk about the warmth of the sun that is set in a concave mirror, which reflects it in a pyramidal figure, whose power increases in the extent that it (the pyramid) shrinks. In other words, if the pyramid hits the object with half its length, it reduces its base by half its thickness and if it hits it at 99% of its length, it reduces its base by 99% and increases by 99% the heat that this base receives from the sun or from the fire. Moreover, the percussion of the iron in the form of pyramid will penetrate the body touching its toe the more deeply the more it is more subtle. Even the heavy matter, once confined in a minimum space, increases in weight, understood that it is opposed to a minimum amount of air, of motion and force we will talk elsewhere.

Leonardo formulated the connection between the cause and the effect through linear functions which he called "pyramidal" because of their geometric depictions and he often used pyramids (and isosceles triangles) for quantitative evaluations. In fact, similarly to many other scholars, he used geometrical figures to represent algebraic relationships. Leonardo believed that linear relationships were universal in Nature and he extensively used linear proportions also in the pictorial perspective [8].

The Leonardo's use of the term "pyramid" in referring to a triangle is due to the translation of the Ptolemy's works on astronomy and optics into Arabic and then into Latin by Eugene of Sicily, where the Greek word "konos" was rendered in "piramis" [8].

Leonardo formulated a general law on linearity: "We will be telling the truth by affirming that it is possible to imagine all powers capable of infinite augmentation or diminution. Consequently, all powers are pyramidal because they can grow from nothing to infinite greatness by equal degrees. And by similar degrees they decrease to infinity by diminution ending in nothing. Therefore nothingness borders on infinity" (Madrid I 128 v); "All natural powers have or are to be called pyramidal inasmuch as they have degrees in continuous proportion towards their diminution as towards their increase. Observe the weight which in each degree of its free descent is in continuous [arithmetical] geometrical proportion, and similarly for the force of levity" (Codex Atlanticus). It should be noticed that the word "arithmetical" is substituted by the term "geometrical", which makes reference to the 'pyramidal' figure used in the notes.

Leonardo's linearity law was represented in different cases in five drawings present in the Codex Atlanticus, 151 ra; here starting from the left one encounters:

(i) A balance suspended on a fulcrum with a weight suspended at one unit length on the right arm and on the left side a longer arm with eight possible unit lengths and a triangle whose basis increases linearly with the number of unit lengths; from the relation $l_x p_1 = l_1 p_x$ it follows $p_x = (p_1/l_1)l_x$, i.e. at the equilibrium a linear relation between the weight p_x and the length l_x;

(ii) A weight thrown upwards ("accidental motion") to show the "diminution by degrees in continuous proportion", i.e. the linear relation between velocity variation and time $\Delta v = -gt$.

(iii) A falling weight with the "pyramidal" increase of velocity versus time of, i.e. $\Delta v = gt$.

(iv) A horizontal motion.

(v) The evaluation of the percussion of falling water.

(vi) The linear narrowing of a falling stream of water.

The adopted pyramidal protocol was also used for many measurement checks which Leonardo often performed. It should be

taken into account that at that time the measurement processes were not easy for both the lack of agreed units for lengths or weights, and for the roughness of the available instruments; this circumstance often led Leonardo to substitute measurements with proportionality evaluations.

Based on the above reported phrases and in the light of the current language of physics, the principle of linearity exposed by Leonardo can be summarized as follows: if E is a physical quantity that plays the role of "effect" and C is a physical quantity representing the respective "cause", the principle of linearity between cause and effect can be formulated as $E = R*C$, where R is the system response function.

Force as cause and effect

"La forza è causa del moto, el moto è causa della forza" (A 34 v.)
The force is the cause of the motion; the motion is the cause of force.

"Forza dicho essere una potentia spirituale, inchorpora, inpalpabile, invisibile, la quale con breve vita sichausa inqueli chorpi, che peracidentale violenza, stano fori di loro, naturale essere e riposo, spirituale, dissi, perché, inessa forza evita, attiva, inchorpoera e invisibile, dicho perché il corpo dove nascie, non crescie, in peso ne informa, dipocha vita perché sempre desidera vincere la sua chagione ecquella vita se occide" (B 63 r.)
I define force as a spiritual, intangible and invisible power, characterized by a short life that manifests itself in the bodies that, following an accidental violence, are found out of their natural state or inertia. I say spiritual because an active, intangible life resides in this force, and I call it invisible because the body in which it manifests does not increase in weight or volume; and short-lived, because it constantly seeks to win the cause that produced it, and this won, dies.

"La forza in corpi non si può creare sanza forza" (Codex Atlanticus, 314 v. b)
Force in bodies cannot be created without any force.

"Peso, forza, colpo e impeto son figlioli del moto, perché da quello nascono. Il peso e la forza sempre desiderano lor morte e ciascun da violenzia è mantenuto. L'impeto è molte volte causa che 'l moto prolunga il desiderio della cosa mossa" (C. A. 123 r. a.)
Weight, force, percussion and impulse are children of the motion, being originated by it. Weight and force always tend towards their death and each is maintained through violence. The impulse is often cause that the motion prolongs the desire of the moved thing.

Leonardo defines the "force" as a "virtue" that cannot have its genesis in the elementary world except as a reflection and derivation of the sublunar, celestial, "spiritual" and "invisible" world. The texts on the plants that Leonardo has read are teeming with similar "virtues"; in other terms, every virtue of the elementary world was for the ancients an "emanation" of the supra-elementary, astral world. Let us observe that the term *potentia* is complementary to "act" (*atto*). The Latin virtue is the corresponding of the Greek *dynamis* and *energheia*: active force that pushes to become act.

For Leonardo the force is placed, as if it were a "substance", and infused, as if it were a "flow", into the bodies. This vision, which seems quite animistic when compared with the aseptic operations of a dynamics devoid of metaphorical impulses, is actually the basis of the theories within which Leonardo works; these theories see the force as a "liquid" and therefore it can be instilled from the motor to the moving object. In this sense there is certainly the influence of Albertus Magnus, one of the largest *auctoritates* between the Middle Ages and the Renaissance.

In the words of the Doctor of the Church: «Motum esse sicut fluxum quendam» [9] and «Ad modum liquidi elementi emanat» [10], i.e. the motion is a kind of flow which, among other things, emanates as if it were a liquid element. Albertus Magnus talks in

this sense not only about the qualitative and quantitative motion, but also about the local motion. He also says that in the motion two "things" are acting: *actio* and *passio*. The *actio* is responsible for the agent of the motion and the *passio* (which translates Aristotelian *pathos*) is the "passion", that is the act of "patire" the action, so that «pati moveri est», i.e being moved is suffering.

Leonardo refers to the motion of the bodies in the presence of dissipation effects, which lead to a decrease in the memory of the motion, and to the arrest of the body in dependence to the extent of the causes that intervene.

"Forza dico essere una virtù spirituale, una potenza invisibile, la quale per accidentale esterna violenza è causata dal moto e collocata e infusa ne' corpi, i quali sono dal naturale uso retratti e piegati dando a quelli vita attiva di meravigliosa potenzia; costrigne tutte le create cose a mutazione di forma e di sito, corre con furia alla sua desiderata morte e vassi diversificando secondo le cagioni. Tardità la fa grande e prestezza la fa debole. Nasce dalla violenza e muore di libertà; e più è grande più velocemente si consuma. Scaccia con furia ciò che si oppone a sua disfatione; desidera vinciere, occidere la sua cagione, il suo contrasto e, vinciendo, se stessa occide; fassi più potente dove truova maggior contrasto. Ogni cosa volentieri fugge alla sua morte. Essendo costretta ogni cosa costrigne. Nessuna cosa senza di lei si move. Il corpo dove nascie non cresce ne' in peso ne' in forma. Nessuno moto fatto da lei fia durabile. Aumenta con lo sforzo e sparisce con il riposo. Il corpo dove essa è confinata è privo di libertà. Spesso, anche, attraverso il suo movimento, essa genera una forza nuova" (A 34 v.)

I say that force is a spiritual virtue, an invisible power which, by means of accidental violence, is caused by the motion, introduced and infused into the bodies, which are drawn and diverted from their natural habit, giving to them an active life of a wonderful power; it compels all the created things to change shape and place, it runs with fury to its desired death, and will diversify according to the causes. The slowness makes it great and velocity makes it weak. It is born by violence and dies of freedom. And the bigger it is, the faster it consumes itself. It hunts with fury what opposes its destruction, it desires to defeat, to kill its cause, what hinders it, and winning, it kills itself; it becomes more powerful by finding larger obstacles. Everything escapes with fury to its death. Being coerced it constrains everything. Nothing moves without it. Where the body is born, it does not grow and its weight and its shape are not modified. No motion made by it is sustainable. It grows in fatigues and disappears by rest. The body where it is confined has no freedom. And often, through motion, it generates a new force.

In a shock interaction process bodies lose their velocity by providing velocity and acceleration to the impacted bodies. The bodies are diverted from their natural trajectory and therefore driven by the violent motion that is precisely configured as a very powerful and worthy of amazement (*di meravigliosa potentia*) *actio* (active life).

Some key-terms, which have not yet received the necessary prominence, are here introduced: *furia* and *desiderio*. The term "fury" (*furia*) could ultimately be connected, through Ficino, to the four types of *furor* that Plato describes in several places alluding to the possibility of a motion of return of the soul to its origin. The reference is not far from Leonardo when we think of the words that he uses about desire – *desire* is an Aristotelian term – of the human soul to return "to its representative": "Ma questo desidero ène in quella quintessenza spirito degli elementi, che, trovandosi rinchiusa per anima dello umano corpo, desidera sempre tornare al suo madatario" (Codex Arundel, P iv: f. 156v), i.e. but this desire lays in that quintessential spirit of the elements, which, finding itself locked up for the soul of the human body, always wishes to return to its representative.

For Aristotle, in many places, *desire* expresses the yearning of matter and object to return to its natural place and regain its state

of quiet. When the things of the sublunary world lose their quiet wish to find it again, that is, in the Leonardo's impressionism, they wish to "die".

Virtue or invisible power is infused into the bodies and varies their state; it dissipates with fury towards the death diversifying from case to case: the slowness makes its effects great and the rapidity makes them weak.

Here we find the concept of a "first" connected to the cause and an "later" connected to an effect that extinguishes over time. There is, finally, a reference to the rapidity of decay: the greater it is the faster it consumes; we would say today that the response of the system is, in general, of exponential type.

Here a reference to the aspects related to the duration of the memory is made. Systems with long memory, at the infinite limit, obey to the law $a = F/m$. Systems with short memory, at the zero limit, obey to the law $v = -b^{-1}F$ (where b is the viscous friction constant).

The text of Leonardo points out how the desire put in place by the motion and "infused" in the body has as an end (as an *entelechy*) its extinction, that, with a meaningful and very recurring term in the language of Leonardo, is called *disfatione*, that is "decay". The force that moves the body and which is put into the body by the violent motion has as an end its own dissipation.

Because of the inertia and memory of their motion, the bodies tend to persist in their state of motion, being able to counteract what opposes to the motion itself, that is the causes of dissipation if they are present; in this latter case, that is the case of a finite memory, the motion dies; it could be inferred that the effect would be null in the absence of dissipation.

The body interacts with everything of the environment that surrounds it and suffers its effects; similarly, the environment with which the body interacts suffers its effects. The force generates motion.

"Ogni moto attende al suo mantenimento, overo ogni corpo mosso sempre si move in mentre che la impressione de la potentia del suo motore in lui si riserva" (F, 13 r)

Every motion tends to its maintenance, that is everybody always moves during the impression of the power of its motor is preserving in it.

The body will always continue to move as long as the impression of the power of its motion, that is the memory of the motion itself, is conserved in the body. Examples: (i) the motion of a body that is characterized by a constant memory, or a memory whose decay time is infinite, will persist indefinitely in its state; (ii) conversely, in the motion of a body with a finite memory, or with a finite time of decay, the impression of the power of the motion will be lost over time.

The Leonardo impulse is the system memory

"L'impeto ecquel che peraltro nome eddecto moto derivativo, il quale nasscie dal moto primitivo cioè quando esso mobile era chongiunto chol suo motore. Mai in nessuna parte del moto dirivativo si troverà alchuna velocità equale acquella del moto primitivo, provasi perche, in ogni grado del moto, che a la chorda dellarcho si perde dellacquista ta potentia congiuntali dal suo motore e perché ogni effecto participa della sua chavsa il moto derivativo della saetta va diminuendo a gradi la sua potentia echo si participa della potentia dellar cho laqual sichomella fu generata a gradi cosi si desstruggie ec. L'impeto impresso dal motore nel mobile e infuso in tutte le parte unite desso mobile ecquesto si manifesta perché ogni parte desso mobile cho si intrinsicha chome superfitiale son dequal moto eccietto nel moto circhunvolubile perchè in quello sempre la parte più inpetuosa siragira intor no alla meno inpetuosa cioè il quelle chesson più vicine al cientro del mobile. E cquella pa

te cheprima simosse ressta senpre più distante dal principio del suo moto sella non inpedita ecquesto si crede [concede] per che ella è piu potente in essa circhunvolubilità. E se per lo avversario si diciessi l'impeto che move il mobile è nell'aria che lo circunda dal mezzo indirieto, questo si niega, perché l'aria che seguita il mobile è tirata da esso mobile per riempiere il vacuo da lui lasciato, e, ancora, l'aria, che si condensa dinanzi al mobile, si fugge indirieto, in contrario corso. E se l'aria ritorna in dirieto, elli è manifesto segno ch'ella si percote in quella che 'l mobile si tira dirieto e, quando due cose si percotano, è nasce il moto riflesso di ciascuna, li quali si convertano in opposti moti revertiginosi, li quali son portati dall'aria riempitrice del vacuo che di sé lascia il mobile, e impossibile è che 'l moto del motore sia aumentato dal moto del mobile un medesimo tempo, perché sempre è più potente il motore che 'l mobile" (G 85 v.)

The impulse which, under another name, is called "derivative motion", arises from the initial motion of the moving object when it was joined to its initial motor. At every moment of the derivative motion, you will not find a velocity equal to that of the initial motion. It is proved because at every degree of the motion, as for the chord of the bow, there is a loss of the power that its motor transmitted to it. And, since every effect is participating in its cause, the derivative motion of the arrow gradually decreases in power and thus participates in the power of the bow which is destroyed little by little as it is produced. The impulse that the motor impresses to the moving object is infused into all the related parts of this object. And this is proved with the fact that all the parts – both inferior and superficial – are of equal motion, except the motion of revolution, where the most impetuous parts always revolve around the less impetuous ones, that is, those that are closest to the center of the moving object. And always the first moved part remains more removed from the principle of its motion, if it is not impeded; this is possible because it has a greater rotational force. If my adversarial said that the impulse that animates this moving object is in the air that surrounds it from the middle to the back part, this would be invented; because the moving object drags the air that follows it, to fill the void it has left, and also because the compressed air in front of it escapes in the opposite direction. If the air comes back, it is the manifest proof that it hurls against what the moving object drags to its retinue; or, when two things collide, each one assumes a reflex motion, and these reflex motions convert into motions that rotate on themselves carried by the air that fills the void left by the moving object; it is therefore impossible that the motion of the motor is increased by the motion of the object moving at the same time, assumed that the motor is always more powerful than the object that moves.

"Impeto è impressione di moto trasmutato dal motore nel mobile. Ogni impressione attende alla permanenza over desidera permanenza. Che ogni impressione desidera permanenza provasi nella impressione fatta dal sole nell'occhio d'esso sguardatore, e nella impression del sono, fatto dal martello di tal campana percussore" (G 73 r.)

Impetus is the impression of motion transmitted from the motor into the moved object. Every impression tends and wishes to stay. That every impression tends to the permanence is proved in the impression made by the sun in the eye of its observer, and in the impression of the sound, made by the hammer of the bell.

The impulse is defined as a derivative motion, which is seen as an impression of the motion transmitted by the motor to the moving object. The impulse tends to be maintained for a certain time.

A fundamental contribution to the question of the impression of the motion in medieval physics is given by the Avicennian theory of *fayḍ* or *fluxus* for which a fundamental role must therefore be attributed "to the causal mechanism of the impression or influence that presides to the 'appropriate preparation' of the matter to receive the different forms [*omitted* ...]. If the idea of flowing represents, in fact, the principle of a "descending" causality that is

explication of the action of the First Principle and of each of the celestial substances on everything that is inferior, the idea of influencing represents the expression of a causality which is in the first place 'impression' of something in something else" [11].

The impression of the power whom Leonardo talks about, with reference to the embryonic principle of inertia, has as a substrate the philosophical Aristotelian-Avicennian-Albertian theses. Albertus Magnus, in fact, describes the motion of the point "flowing" in the line, but he is careful to reiterate that the motion is customary to the point and to the line [9], so that the motion is not an *ens permanens*, but a street, a road in which the object that travels is identified with the road itself. The motion is in fact a continuous flow that does not conceive any distance within the trajectory between what moves and what is moved; in other terms, the motion remains, as in Leonardo, something that in the Aristotelian view is still inside the body (placed and infused) even when it is moved from the outside and, as for Albertus Magnus, it is by definition "continuous" [12].

About the impression of memory, reference is made to the Plato's Theaetetus [13] in which the soul is a block of wax on which those that today we would call the *mnestic traces* are imprinted. It should also be taken into account the explanation of Aristotle in the treatise of Remembrance and Reminiscence [14] in which memory is described as the motion that is produced in the object and which impresses a kind of figure of the perceived object which is not different from the fingerprint made with a ring.

"Ogni moto ha terminata lunghezza, secondo la potenzia che lo move; e sopra questo si fa regola. Ogni mobile che acquista velocità nel moto, fia mosso al moto suo naturale; e così del converso, quando perde, si move di moto accidentale" (Forster II 141 v.)

Each motion has finished length, according to the power that move it; and on this basis the rule is made. Every moved object gaining velocity during the motion is moved to its natural motion; and so on the contrary, when it loses velocity, it is moved by accidental motion.

The impulse or imprinted force changes the natural position – or state – of the object. When the object increases its velocity, it moves of natural motion, while when it slows down, it moves of accidental motion.

Here Leonardo makes a clear reference to the Aristotelian difference between natural motion and violent motion. All the things that move in their own element are characterized by a natural motion; when they are diverted by this motion, which belongs to them *naturaliter*, this is called "violent motion". At this point the *focus* changes and is put on what is *accidental*, that is no longer an internal motion (the one that makes sure that the smoke rises upward by its natural disposition, pushed essentially by itself), but an "external violence", as Leonardo says, who finds the cause of such a motion precisely in the force that pushes the object from the outside, that is, in the *impetus* that is impressed to it.

"Il moto violento quanto più s'esercita più s'indeboliscie. Il naturale fa l'oposito" (Codex Trivulzianus, 26 r.)

The more the violent motion is exerted the more is weakened. The natural one makes the opposite.

The natural motion is characterized by an increasing velocity; the violent motion is therefore the accidental motion, in which the velocity decreases.

According to the theory of the linear response, this corresponds to the application of an impulsive force that leads the system out of the original equilibrium and whose subsequent dynamics, with a predominant dissipation character, leads to a weakening of the motion with a velocity decrease.

"Tutti i moti violenti quanto più si partano dalla lor chausa più indebolisscano" (H 77 (29) v.)

Every violent motion the more it is separated by the cause the more it is weakened.

According to the linear response theory, an external impulsive force applied to a system at a certain instant generates a dynamics dominated by dissipation effects leading to a subsequent weakening of the motion; the greater this weakening is the greater the interval of time elapsed from the application of the impulsive force on the system is.

"Quanto il moto naturale si parte dalla sua causa tanto più si fa veloce" (H 78 (30) v.)

The more the natural motion separates from its cause the more it gets fast.

The natural motion is characterized by an increasing velocity. In the case, for example, of a falling body, the external force generates an increase of the velocity which is greater the greater is the interval of time elapsed from the release of the body which gave rise to its fall.

"Onni impressione è per alquanto tempo riservata nel suo obietto sensibile; e quella fia piùriservata nel suo obietto, la quale fu di maggior potenzia, ecosì meno delle men potente. In questo caso io domando sensibile quelloobietto, il quale per alcuna impressione si move di quel che prima era; obbietto insensibile è quello che, ancora che si mova del suo primo essere, essonon riserva in sé alcuna impressione della cosa che lo mosse" (Codex Atlanticus, 360 r. a)

Each impression is maintained during a certain time in its sensitive object: the impression whose power was high will exist longer in its object; and the least powerful one will exist for less time. In this regard, I apply the word "sensible" to the object which, subject to an impression, loses its initial character of insensitive object; this latter is the object that, when changing its primitive state, does not maintain in itself any impression of the thing that moved it.

The impression, that is the memory, is maintained for a certain time. The impression (memory) characterized by a long time produces more lasting effects, while a short memory gives effects that occur in shorter times. The object sensitivity is proportional to its inertia.

The theory of the *impetus* is already present *in nuce* in the Aristotelian texts, in particular as a theory of the moved objects [15,16] and as a theory of proportionality [17].

A theory is found in the Giovanni Filopono's *De opificio mundi*, an Alexandrian commentator of Aristotle [18]. Then there will be, among others, the authoritative interventions of Thomas Aquinas, Roger Bacon (scholar and theoretician of alchemic art), Bradwardine, Albert of Saxony, Albertus Magnus, up to the valuable contributions of Francesco De Marchia (Francesco Rossi di Appignano) with his theory of the two motions and Jean Buridan who deals with the *impetus* in his *Questiones* [19].

The importance of the theory of *impetus* in its Arab declination through the contribution of one of the most influential and venerated figures of antiquity, such as the physician-scientist Avicenna, cannot to be underestimated. The theory of the Avicennian *impetus* is the so-called *mayl theory*. Now, according to some critics [20], in his concept of force and motion Leonardo would be indebted to Nicholas of Cusa.

It is accepted that in the questions of dynamics Leonardo follows Aristotle and his commentators [21] and that in these commentators up to Leonardo there is "a vaguely anthropomorphic conception" [22] in the description of the motion.

Pierre Duhem [23] emphasizes the fact that Leonardo knows – and expressly mentions – the work of Albert of Saxony, who commented the Aristotle's theory of motion in his *questiones* [24].

Based on the above reported phrases, a third conceptual map can be formulated (Fig. 3).

According to the theory of the linear response [25–27], in the domain of time, one can write:

$$E(t) = \int_{-\infty}^{t} R(t = t\prime)C(t\prime)dt\prime \tag{1}$$

This relation has an immediate physical interpretation: the effect, at a certain moment t is given by the overlap (integral) of the causes that have taken place at any other past moment t', weighted by the response function $R(t-t')$ that is evaluated according to the time distance, $(t-t')$, separating the present instant t from the instant t' in which the cause value is considered. Let us observe that, in the above relation, the integration extremes range from $-\infty$ to t, which means that the integration variable t' must be smaller than t; in fact a value of t' greater than t would mean that causes that occurred after the effect are taken into account, which is not admissible according to the principle of causality (first the cause, later the effect).

Many processes can be mathematically represented by the same function type:

$$R(t) = R_0 \cdot exp(zt) \tag{2}$$

where R_0, the response function magnitude or its initial value $(t = 0)$, is real while $z = a + ib$ can be a complex number; a and b are constants whose value depends on the physical and geometric features of the system. By changing these values it is possible to pass continuously from an oscillating trend, apparently not dampened if a is much smaller than b, to a monotonously decreasing trend if a is much greater than b.
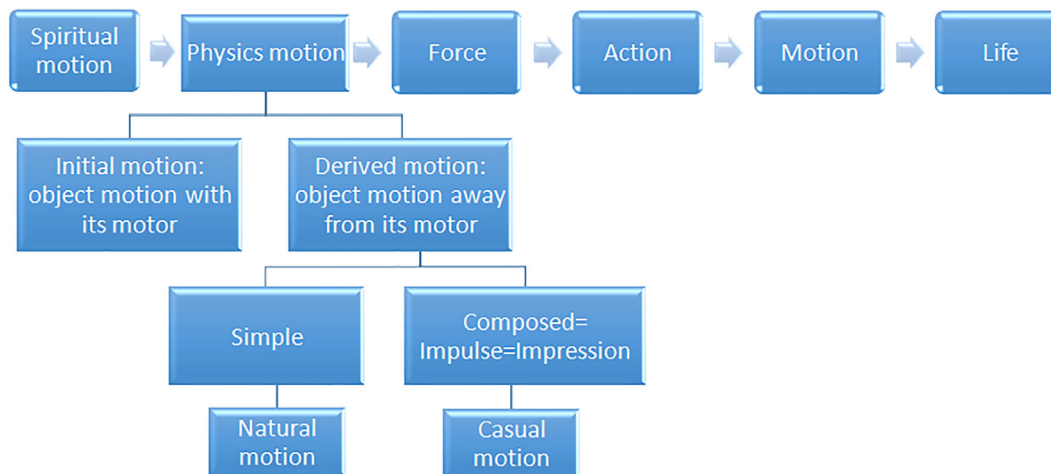

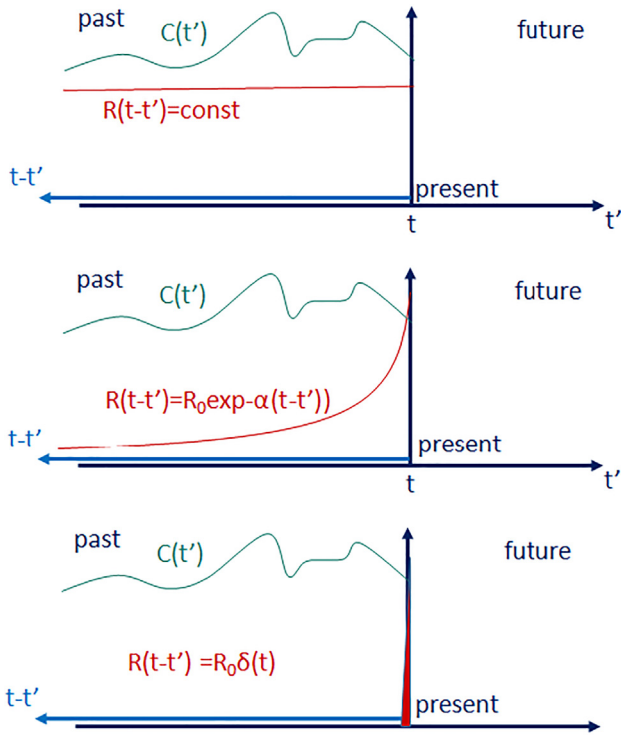
Fig. 3. Third conceptual map including impulse.

Fig. 4. Theory of the linear response and limit cases.

In the following we will consider for simplicity real response functions. In a process with real response functions, the effect is obtained starting from the general equation:

$$E(t) = \int_{-\infty}^{t} R_0 \exp[-\alpha(t - t\prime)] C(t\prime) dt\prime \tag{3}$$

which has an easily appreciable intuitive meaning in a graphical representation, as reported in Fig. 4.

Again, the effect at time t is obtained as an overlapping of all the past causes, weighted by the response function, whose value decreases going further back in the past: nearer causes have a greater relevance than the farther ones. So the larger the parameter α is, the smaller the response function becomes by going back in time. We can define $\tau = \alpha^{-1}$ as the characteristic time.

We observe that generally is: $\alpha > 0$; contrarily, in fact, a system carried out of equilibrium and abandoned to itself would not relax towards the equilibrium, but the initial effect would grow without limits, which is contrary to most experimental observations.

We have two cases: (i) $t < 0$ and $R(t) = 0$; (ii) $t > 0$ and (ii) $R(t) = R_0 \cdot \exp(-\alpha t)$ and we can distinguish two limit cases: i) the case of an infinitely long memory $\alpha^{-1} \to \infty$, i.e. $R(t) = const = R_0$ and (ii) the case of an infinitely short memory $\alpha^{-1} \to 0$, i.e. $R(t) = R_0 \delta(t)$.

First limit case

The first case corresponds to put $\alpha = 0$ in our response function, which means $R(t) = const = R_0$. In this case the effect will be given by:

$$E(t) = \int_{-\infty}^{t} R_0 C(t\prime) dt\prime = R_0 \int_{-\infty}^{t} C(t\prime) dt\prime \tag{4}$$

By deriving with respect to the time one gets

$$\frac{dE(t)}{dt} = R_0 C(t) \tag{5}$$

which is a relationship of "synchronous" proportionality between the cause and the *variation rapidity* of the effect. For instance, when one takes into account the velocity of the body as an effect and the force acting on it as a cause, this equation corresponds to the second law of dynamics $ma = F$, where $R_0 = m^{-1}$. This interpretation key leads to a different understanding of the first principle and of the concept of inertia: a body, which is not subject to forces, moves in a uniform straight motion because it continues to "remember" endlessly any cause that has put it in motion in the past.

Second limit case

Here we assume that the response function is reduced to a Dirac function $R(t) = R_0 \delta(t)$, which can be obtained by considering the limit for $\alpha \to \infty$. In this case it is:

$$E(t) = \int_{-\infty}^{t} R_0 \delta(t\prime) C(t\prime) dt\prime = R_0 C(t) \tag{6}$$

that is a relation of synchronous proportionality between cause and effect. Such a law corresponds, for example, to the laws of Ohm, Fourier, Stokes, Poiseuille, Fick, etc.; in other words to any law which governs the (constant) flow of something. If the system has no memory at all, it adapts itself, instant by instant, to the present value of the cause.

Conclusions

In this paper, some Leonardo da Vinci's texts are analyzed in order to show how, starting from the meanings of the used physical concepts and mechanical quantities, assuming the validity of the principle of causality, hypothesizing a linear cause-effect dependence and introducing an impression of mechanical systems, he made assumptions capable to conciliate the Aristotle's and the Newton's positions. The Aristotle's dynamics corresponds to a synchronous proportionality between the cause and the effect; for instance, when one considers the velocity of a body as an effect, E, and the force acting upon it as a cause, C, one recognizes the friction law $F = -bv$ (i.e $v = -b^{-1}F$) with $R_0 = -b^{-1}$. The Newton's dynamics corresponds to a synchronous proportionality relationship between the cause and the rapidity of variation of the effect, as, for example, when one considers the velocity of a body as an effect, E, and the force acting upon it as a cause, C; here, the second law of dynamics $ma = F$ with $R_0 = m^{-1}$ can be recognized.

From the analysis of the Leonardo sentences, we show how the motion of a body that is characterized by a constant impression, or a memory whose decay time is infinite, will persist indefinitely in its state; on the other hand, in the motion of a body with finite memory, that is with a finite decay time, the impression of the power of the motion is lost over time. On this ground, the dynamics of Leonardo's, which encompasses both the Aristotle's and Newton's dynamics, can be framed within the modern linear response theory.

Conflict of interest

The authors have declared no conflict of interest.

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

References

[1] Frosini F. Il concetto di forza in Leonardo da Vinci. In Bernardoni A, Fornari G. Il Codice Arundel di Leonardo: ricerche e prospettive, CB Edizioni, Poggio a Caiano; 2011. p. 113–28.

[2] Lee HDP. Aristotle. Cambridge (MA): Meteorologica. Harvard University Press; 1952.

[3] Ficino M. Platonic theology. (MA): Allen MJB, Harvard University Press; 2001.

[4] Galloni P. Il sacro artefice. Bari: Mitologie degli artigiani medievali. Laterza; 1998.

[5] Nova A. Valore e limiti del metodo analogico nell'opera di Leonardo da Vinci. In Marani PC, Maffeis R. Metodi e tecniche per la costruzione della conoscenza, Nomos Edizioni, Busto Arsizio; 2016. p. 25–36.

[6] Kemp M. Leonardo da Vinci: Science and the poetic impulse. J Royal Soc Arts 1985;133:196–214.

[7] Weill-Parot N. Astrology, astral influences, and occult properties in the thirteenth and fourteenth centuries. Traditio 2010;65:201–30.

[8] Keele KD. Leonardo da Vinci's Elements of the Science of man. Academic Press; 1983.

[9] Magnus A. Physica III, I, 8. In Jammy P. Opera omnia, Lyon; 1651.

[10] Magnus A. De Causi et Processu Universitatis, I, IV, 3. In Jammy P. Opera omnia, Lyon; 1651.

[11] Lizzini O. Fluxus (Fayḍ) – Survey on the fundamentals of Physics and Metaphysics of Avicenna. Bari; 2011.

[12] Kemp M. Leonardo. Oxford University Press; 2004.

[13] Plato. Teeteto. 191D. Laterza, Bari; 2006.

[14] Aristotle. Opere IV, Della generazione della corruzione, dell'anima, piccoli trattati di storia naturale. Laterza, Bari; 2007.

[15] Aristotle. Physica IV, c. 8. Mimesis, Milano; 2007.

[16] Aristotle, De Coelo III, c. 2. Bompiani, Milano; 2002.

[17] Aristotle. Physica VII, c. 10. In Ferrini MF. Meccanica. Bompiani, Milano; 2010.

[18] Clagett M. The science of mechanics in the middle ages. University of Wisconsin Press (Madison); 1959.

[19] Buridan J. Quaestiones super octo physicorum libris Aristotelis. Leiden: Brill; 2015.

[20] Pannaria F. Memorie Scelte. Edizioni Alla Via Jacobea 2016.

[21] Dijksterhuis EJ. The mechanization of the world picture. Oxford University Press; 1961.

[22] De Paoli M. Theoria Motus. Principio di relatività e orbite dei pianeti. Edizioni Franco Angeli, Milano; 2013.

[23] Duhem P. The origin of statics: The sources of physical theory. Dordrecht: Springer, Netherlands; 1991.

[24] De Saxonia A. Quaestiones in Aristotelis De Caelo. Editions Critiques (Philosophes Medievaux). EEters Pub, Verlag; 2008.

[25] Kubo R. Statistical mechanical theory of irreversible processes I. J Phys Soc Jpn 1957;12:570–86.

[26] Wanderlingh F. Discorso sulla Fisica, unpublished work.

[27] Wanderlingh F. Orientamenti e prerequisiti - Una sintetica esposizione metodologica per l'insegnamento della Fisica, unpublished notes.