



# The use of Protodiakonov and Hardgrove methods to determine the effect of coal quality on its grinding ability

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## ABSTRACT

The factors affecting the mechanical strength of coal and coal charges, deviations of the crushing capacity coefficients' actual values of coal charges from their calculated values, and the possibility of categorizing coal in relation to the degree of its resistance to crushing have been explored. The dependence permitting satisfactory precognition of the HGI value of the coal charge placed on the data of the crushing capacity coefficients of its ingredients was settled. The effect of the grinding ability of coal on its quality indicators, namely the degree of metamorphism, petrographic and elemental composition has been examined in this article. Determination of the coal grind ability was made by means of widely known Protodyakonov and Hardgrove methods, which allow estimating the resistibility of the material to the grind forces. For research 14 samples of coals (Ukraine, USA, and Kazakhstan), and 7 testers of coal charges at the basic coke chemical companies of Ukraine, were applied. The graphical and mathematical dependences are stated to have been developed to enable the predicting of the grinding ability of coals using the Hardgrove (HGI) and Protodyakonov (f) methods, based on determining the indicators of their quality ( $C^{daf}$ ,  $R_0$ ,  $O_d^{daf}$ ,  $v^{daf}$ ). It is shown that the graphical and mathematical dependencies are polynomials of the second degree, and the values of the coefficients of determination ( $R^2$ ) exceed 0.550–0.930. For the initial time, the coherency between the strength coefficient according to the Protodyakonov method and coal pulverization to the Hardgrove method was established. It was found the f and HGI values of measure are inversely in proportion, and a mathematical and diagram dependence based on one of them for its prognosticate has been established.

## 1. Introduction

It is known that the mining, burning, cooking, pulverized coal producing, or processes in a fluidized bed is an important part of the technological chain of the enterprise, and the method of crushing coal has a greater impact on the quality and quantity of products [1–6]. Therefore, at this time, coal remains a raw material that significantly affects the mechanical strength indicators of coke and the energy consumption of coal preparation plants.

The mechanical strength, brittleness, and grinding ability of coal and associated rocks are related to the particle size composition, sludge formation, and the effect of abrasives on the working surface. The mechanical strength of coal and related rocks is evaluated by

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their thermal stability, which shows their grinding ability, hardness, brittleness, temporary resistance to compression, and resistance to external forces at high temperatures. Previous studies have shown that with increasing metamorphism, the hardness of coal material increases, and the speed of crushing first increases and then decreases, which is associated with a decrease in the yield of volatile substances. Thus, low-metamorphosed coal is moderately firm but more viscous; moderately metamorphosed coal (due to well-developed fracturing) is the weakest; and highly-metamorphosed anthracite is the hardest and strongest [7].

When preparing coal for coking, it is necessary to achieve an optimal ratio of size classes in the charge to ensure a higher loading density; avoid the formation of a large number of fine classes, which reduce the sintering ability of coal. Large classes of charge should be similar to other classes in terms of cohesiveness, ash content and petrographic composition. With the same degree of grinding, characterized by the content of classes <3 mm, different methods of preparation of the charge can give different dust content in the final product (size class <0.5 mm). A heightening in the degree of coal crushing causes to a rise in the viscosity of the malleable mass. Nevertheless, this effect cannot be ascribed only to an increase in the specific surface of coal pieces in contact. When using any of the crushing schemes, the poorly sintering components (durene) should have a greater degree of crushing and well-sintering components - less.

The position of the grinding optimum varies depending on the depth of coal enrichment, its petrographic composition, and sintering. The more coal is mineralized and petrographically heterogeneous, the higher the degree of coal grinding should be to obtain stronger coke.

The works [5,8] have presented the research results of 26 samples of hard coal (pure or original) from the coal basin Zonguldak (Zonguldak - 17; Amasra - 9) and 17 low-quality coal from different places in Turkey. Regarding the newly proposed method for determining coal grinding ability, samples were done in the 1.7 + 1.18 mm size and crush in a ring roller mill. The ring mill was chosen here due to its common use for sample preparation and availability in each laboratory. This procedure involves placing the above samples in a ring mill (see Section 2, Subsection 2.2 - ISO 5074:2015 Hard Coal – Determination of Hardgrove Grindability Index) and grinding the samples for a predetermined time. After grinding by a ring mill, the samples have different size distributions at the end, depending on their ability to grind. More clearly, size distribution analysis of the ground samples (1.7 + 1.18 size group for 20 s for 50 g samples) using the Malvern Mastersizer showed their various D 10, D 50, D 90, D 32, and D 43 at the end. The present size division results correlated to the previously defined HGI values demonstrate that the coal grindability can be lightly determined by this method, as the calculated HGI values using a recommended method is dissimilar on  $\pm 0.05\%$  from the defined HGI values.

A set of coal Hardgrove Grindability Index (HGI) tests [9] were made to establish each content of iteration of the monomacrite microlithotypes vitrite (Vt) and inertite (In), the bima cerite microlithotypes clarite (Cl) and durite (Du), and the trimacrite microlithotypes duroclarite (Dc) and clarodurite (Cd). The breaking of larger particles during the HGI test results in particles that may consist of microlithotypes different from the original particle. Therefore, it is difficult to predict the general composition of upcoming iterations. Time series of composition values were built to the sequence of all iterations' content in another mesh sizes in agreement with the iterations. A specific way of analyzing a sequence of data points collected over an interval of time known as Seasonal Autoregressive Integrated Moving Average (SARIMA) was applied to foretell the ultimate iteration content of Vt, In, Cl, Du, Dc, and Cd. The suggested methodology allow it to predict the content of the last iteration, which was comparable to the present observation.

The research [10] gives the characteristics of experimental data for damp and dry crushing of coal samples during wet and dry grinding according to generally accepted distribution functions. The advantage of the R-R and Swrebec functions over the other investigated functions is their better-fitting performance for cumulative particle size curves. According to this, a time-dependent expression is made to describe the cumulative particle size distribution. A much better match of the relative mass distribution of ground products compared to others in a short grinding time is provided by the R-R function. The fit for all distribution functions was slightly worse after about 3 min, which may be due to a change in dominant breakage mechanisms from impact to abrasion. The G-G-S function is optimal for characterizing the particle size probability mass distributions of wet grinding with increasing grinding time. Unlike the previous one, the G-M function is the best for applying to dry-grinding experimental data. In addition, the probability density functions of the optimal particle size are related to the difference in breakage mechanisms between wet and dry grinding. In Ref. [11] an investigation was conducted to assess slope stability over old underground coal mines using wireless sensor networks (WSNs) and the Internet of Things (IoT). This paper discusses slope stability monitoring using a low-cost wireless sensor network. An integrated wireless data acquisition system for slope stability assessment is developed and validated using conventional instruments and numerical simulations. Scientists have repeatedly studied the connection between the Hardgrove index and the structural properties of coal.

The article [12] talk over the capability of gas adsorption on original pieces of TMT coal instead of grind coal particles and examine the influence of mechanical crushing during the preparation of coal structure characteristics cf. Foremost, N<sub>2</sub> and CO<sub>2</sub> adsorption experiments were performed on lumps of coal and pulverized coal (60 mesh particles) of primary coal structure and plastic TMT (Ro, max. = 0.93 %–1.15 %). Then, the effect of coal grinding on 60 mesh particle size on the micromesoporous structure of coal was investigated by contrast analysis. N<sub>2</sub> adsorption tests showed that the pore volume of mesopores >10 (especially mesopores >40 nm) increased significantly in pulverized coal compared to lump coal, and the noticed increase in pore volume gradually dropped down with increasing plastic deformation.

The paper [13] talk over the possibility of gas adsorption on original pieces of TDC coal instead of crushing coal particles, investigated the impact of mechanical crushing during the preparedness of coal structure characteristics cf. First at all, CO<sub>2</sub> and N<sub>2</sub> adsorption experiments were performed on lumps of coal and pulverized coal (60 mesh size fractions) of earliest coal form and plastic TDC (Ro, max. = 0.93 %–1.15 %). Then, the effect of coal grinding on 60-mesh particles on the micromesoporous structure of coal was investigated by contrast analysis. N<sub>2</sub> adsorption tests showed that the pore volume of mesopores >10 (especially mesopores >40 nm) increased significantly in pulverized coal compared to lump coal, and the observed increase in pore volume a bit at a time

dropped with growing plastic deformation.

In the study [14], a connection was established between the petrographic composition of Kentucky coal and the HGI index. In this article, a neural network access was proposed to work with the grindability behaviour of coal. The article mentioned above studied non-enriched coal samples. This is indicated by the extremely high ash content in the samples (from 28.00 to 67.00%). Despite the fact that the unnatural neural network-settled on the prognosis procedure introduced in this article can be additionally applied to predict the Hardgrove grindability index, this method is quite expensive and complex and requires significant investment in laboratory equipment.

The purpose of the research was to establish similar dependencies for enriched coal samples. We used simpler methods of analyzing the coal grinding abilities and their relationship with coal characteristics, which determine its degree of metamorphism.

There are also potential and alternative ways of establishing the mechanical strength of coal [15,16]. So, for example, in the study [16], there is a method of testing coal grinding, which adds a ring roller mill with dative conditions and Malvern Mastersizer. Comparing these size distribution results with the previously determined HGI values, it can be argued that coal grindability can be easily determined by this method, as the estimated HGI values using the proposed method differ by  $\pm 0.05\%$  from the determined HGI values.

We, in turn, proposed to use the Protodiakonov method described in this article with reliable results to determine the mechanical strength of coal.

Various factors influence the choice of the hardness measurement method: measurement volume, hardness of the measured samples, their geometry, linear dimensions, etc. This led to the appearance of different methods of measuring hardness. Papers [2,17] performance the advantages and also disadvantages a lot of existing methods for discovering the mechanical strength of coal. The Hardgrove method remains the head internationally admitted method since the HGI indicator is cognated to a great number of indicators of coal quality (wetness, ash content, degree of metamorphism, elemental, petrographic and mineral contents, oxidizability), as well as the fact that this method is standardized, relatively simple, correlated with other indicators of mechanical strength and power. It is also possible to use other methods that correlate well with the Hardgrove method [17].

Our previous studies [17,18] showed that, based on the indicators with which the HGI value is correlated, it could be concluded that the latter is a parameter that depends on the carbon content and the degree of irregularity of the structure of the organic mass of coal. An increase in its absolute value is associated with an increase in the content of total ( $C^{daf}$ ) and aromatic ( $C_{ar}$ ) carbon in coal, as well as the degree of unsaturation of its structure ( $\delta$ ).

The HGI increases with increasing vitrinite reflectance and ignition temperature and decreasing coal volatiles. It should be noted that these indicators also indirectly reflect the structural features of the Organic Mass of Coal (OMC). Thus, the vitrinite display index is related to the presence of one or another part of cyclically polymerized carbon in the OMC. The yield of volatile substances and the ignition temperature of coal reflect the thermal stability of OMC, which depends on the proportion of aliphatic and aromatic components of OMC macromolecules.

This article analyzes the adequacy and, in accordance, the possibility of the practical application of previously obtained dependencies. In the last case, coal is usually used in the form of mixtures (charges). Therefore, it is also essential to establish the possibility of forecasting the ability to grind raw materials, which consist of different grades of coal, which is also described in this article.

According to this work's research aim, a study of coal grinding ability determined by the Hardgrove and Protodyakonov methods from its quality indicators (degree of metamorphism, petrographic and elemental composition) was presented. Based on the obtained results of the coal quality determination ( $R^0$ ,  $V^{daf}$ ,  $C^{daf}$ ,  $O^{daf}$ ), experimental and statistical mathematical dependence models were developed for predicting the grinding ability of coal using the Protodiakonov ( $f$ ) and Hardgrove (HGI) methods, and their adequacy was assessed using the average relative error of approximation, determination coefficient, Fisher's criteria and statistics criteria. A prediction for calculating the grinding coefficient of coal charges based on the grinding coefficients of its components has been developed.

## 2. Methods

### 2.1. Raw materials

14 samples of coals (Ukraine, USA, and Kazakhstan), as well as 7 samples of coal charges at the basical coke chemical companies of Ukraine at the moment of the experiment (in January 2021), are studied. These coals are indicated by different degrees of metamorphism and petrographic sameness.

Tables 1–6 show the maximum, minimum, and average indicators of technological properties, petrographic characteristics, and

**Table 1**  
Technological properties of coal.

Value	Proximate analysis, %			Hardgrove grindability index, un.	Strength coefficient according to Protodyakonov, un.
	$A^d$	$S^d$	$V^{daf}$	HGI	$f$
Max	10.8	1.32	38.6	86	1.37
Min	7.2	0.27	18.3	38	0.65
Mean	8.5	0.64	28.2	68.2	0.82

elemental composition of the studied coal samples and their mixtures.

## 2.2. Methods

GOST 21153.1–75 Rocks. The strength coefficient, which determining up to Protodyakonov, uses to hard rocks (including coals) and sets a method for determining the coefficient of its strength for categorizing it in technical documentation, in the calculations and project of modern mining operations, mining tools and equipment, besides, in research effort [19].

The method is to determine the coefficient of hardness, which is proportional to the ratio of the work spent on crushing the rock to the newly formed surface during crushing, estimated by the particles total volume less than 0.5 mm. The destruction of coal grains by the impact of a falling weight imitates the conditions of coal grinding in hammer crushers, and the sieve composition of the crushed sample approaches industrial values.

The device consists of the glass No 1, a tubular impact tester 2 placed into it, inward which a weight 3 weighing  $2.4 \pm 0.01$  kg is freely placed with a holder 4 knotted to a weight with rope. Driving tubular steel piles has gaps in the higher part, into which pins 5 are placed, limiting the raising of the weight. The device complex contains a volume meter concluding of a glassware 6 and the plunger 7 with a measuring dial with a reading series from 0 to 150 mm along its lengthwise axis.

A tubular pile driver 2 is inserted into glass 1 with a freely placed load 3 weighing  $2.4 \pm 0.01$  kg, and handle 4 is tied to the load with a cord. The tubular pile driver is equipped with a load-lifting limiter 5. The device also includes a glass 6 for measuring the volume of the crushed sample the plunger 7 with a measuring dial with a reading series from 0 to 150 mm along its lengthwise axis.

The device for determining the crushability of coal is shown in Fig. 1.

A sample of coal on a solid base is split with a hammer to obtain pieces 20–40 mm in size. 20 samples 20 tests weighing 40–60 g each are taken from the smashed material. Each sample is evenly distributed on the bottom of glass 1, copra is collected, and weight is dropped onto it from a height of 60 cm. When crushing the first five samples, the number of weight drops for each sample is set. To do this, the first five crushed samples are scattered on a 0.5 mm sieve, the undersized product is transferred to volume meter 6, and the altitude of the charge is marked off with plunger 7. When a column with a height of 20–100 mm is obtained, the sample of drops is stored for the remaining 15 samples. With a greater or lesser height, the number of drops is adjusted in one direction or another. The leavings fifteen examples are pressed in the device progressively in the determined test mode: with a standing number of load drops and a load raising height to 60 cm. According to Ref. [19] rock hardness coefficient ( $f$ ) is counted by formula (1):

$$f = \frac{20 \cdot n}{h}, \quad (1)$$

where 20 is an experiential numerical coefficient that ensures on the whole taken values of the strength coefficient and leads on into account the job expend on crushing.

$n$  is the number of weight drops when testing one sample;

$h$  is the height of the fine fraction column in the volumetric meter after testing five samples, mm.

The arithmetic mean is accepted as the last test result.

ISO 5074:2015 *Hardgrove* – Definition of Hard coal grindability index specifies the method for determining the grindability index of hard coal using the Hardgrove Grindability machine Except it determines the procedure for regulating the test machine and arranging the standard referential coal examples [20]. An air-dry sample of solid fuel prepared for testing with a given particle size is ground in a graduated Hardgrove instrument under standard positions. After this, a sifter research of the reached material is performed.

Grinding research resulting in the determination of the HGI index enables consumers of different types of coal to evaluate the properties of raw materials for crushing plants, as well as establish grinding power requirements and throughput.

The HGI coefficient is determined in a multi-step procedure. Fig. 2 shows a photo and diagram of the Hardgrove apparatus.

1. One-dimensional sample of enriched coal weighing 50 g was placed in the crushing unit.
2. The unit performs the standard number of revolutions at the specified pressure.
3. Steel balls inside the machine crush the sample.
4. The crushed coal is sorted, after which it is fixed the amount of coal, the size of which is less than the specified value. The resulting figure is converted into a coefficient value grindability, according to Hardgrove (HGI).

**Table 2**  
Technological properties of coal mixtures.

Value	Proximate analysis, %			Hardgrove grindability index, un.	Strength coefficient according to Protodyakonov, un.
	A <sup>d</sup>	S <sup>d</sup>	V <sup>daf</sup>	HGI	f
Max	7.6	0.77	32.2	62	0.94
Min	7.5	0.71	31.5	58	0.90
Mean	7.57	0.74	31.97	59.1	0.93

**Table 3**  
Petrographic characteristics of coal.

Value	Petrographic composition, %					Mean random vitrinite reflectance, %
	Vt	Sv	I	L	FC	R <sub>0</sub>
Max	95	2	75	11	76	1.54
Min	24	0	5	0	9	0.72
Mean	70.7	0.5	27.7	1.7	28.9	1.10

**Table 4**  
Petrographic characteristics of coal mixtures.

Value	Petrographic composition, %					Mean random vitrinite reflectance, %
	Vt	Sv	I	L	FC	R <sub>0</sub>
Max	84	0	18	1	18	1.03
Min	81	0	15	1	15	0.99
Mean	83	0	16	1	16	1.00

**Table 5**  
Ultimate analysis of coal.

Value	Ultimate analysis, %				
	C <sup>daf</sup>	H <sup>daf</sup>	N <sup>daf</sup>	S <sub>t</sub> <sup>d</sup>	O <sub>d</sub> <sup>daf</sup>
Max	90.51	5.84	2.97	1.32	8.55
Min	83.39	4.52	1.32	0.27	2.62
Mean	87.18	5.18	1.99	0.65	5.00

**Table 6**  
Ultimate analysis of coal mixtures.

Value	Ultimate analysis, %				
	C <sup>daf</sup>	H <sup>daf</sup>	N <sup>daf</sup>	S <sub>t</sub> <sup>d</sup>	O <sub>d</sub> <sup>daf</sup>
Max	86.54	5.32	2.07	0.74	5.83
Min	86.15	5.31	1.98	0.69	5.45
Mean	86.27	5.31	2.03	0.71	5.68

The typical value of the HGI coefficient is in the range of 30 (increased resistance to pulverization) to 100 (light pulverization). Hardgrove's coefficient tester grindability is shown in Fig. 2. Hardgrove's coefficient tester HGI consists of a fixed grinding bowl made of hardened steel with a horizontal track on which eight steel balls with a diameter of 25.4 mm. The balls are set in a motion upper-pressure ring that rotates at a speed of 19–21 min<sup>-1</sup>. The top pressure ring is connected to the shaft and actuated electric motor through a gearbox. A load is mounted on the shaft. The general vertical force on the balls, created by the upper-pressure ring, gearing, shaft, and load, should be (284 ± 2) N or have a total mass of (29 ± 0.2) kg.

The device is equipped with a rev counter and an automatic device that turns off the device after (60 ± 0.2) revolutions. Before starting the determination of the grindability factor according to the Hardgrove method, the device must be tared. For calibration, use four standard samples with HGI around 40, 60, 80, and 110. The calibration graph is built in the coordinates HGI = f(m), where m is the mass of coal (g) passed through a 75 mm sieve.

The preparation of samples for testing is carried out as follows. The initial sample of coal with a mass of 1 kg and a particle size of 0–4.75 mm is adjusted to air dry condition. Dried samples in portions of approximately 200 g were sieved for 2 min in a sieving machine on a set of sieves consisting of sieves with a hole size of 1.18 mm and 600 μm. Remaining on the sieve with a hole size of 1.18 mm, the material is crushed in a crusher and adjusted so that only the largest particles are ground. Then the crushed material is sieved for 3 min on the same sieves. Crushing and screening are continued until all the material passes through a 1.18 mm sieve. Coal passed through a sieve with a hole size of 600 μm is discarded, and the remaining on it weighed with an accuracy of 1 g. Weigh an experimental portion of the material weighing (50 ± 0.01) g, carefully pour it into the grinding bowl, and turn on the device. It turns off after the shaft has made (60 ± 0.2) revolutions. The crushed coal is placed in a sieving machine on a sieve with holes 75 mm in size, and sieving is performed for 10 min. After this sieve is removed from the machine, carefully clean the shell, mesh and cover so that all the coal from the sieve gets into the collection tank (collector). Into the purified sieve with a bottom, pour the coal from the collector, close the lid, again place in a sieving machine, and sieve for another 5 min. Coal cleaned from a 75 mm sieve and coal passed through this sieve are weighed separately to the nearest 0.01 g. If the sum of these masses differs from the initial mass of the experimental portion by more than 0.5 g, then the analysis is repeated. According to Ref. [20] the mass of coal (m, grams) passed through a sieve with the size of

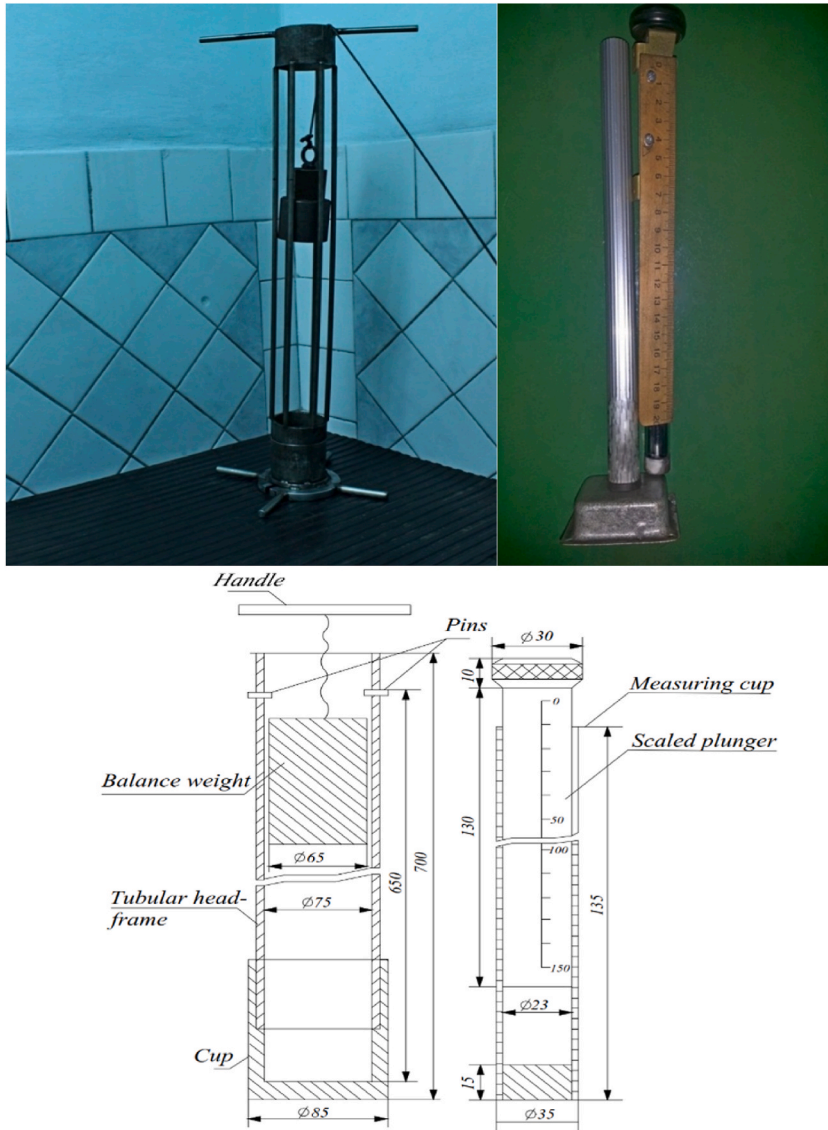


Fig. 1. Photo and diagram of Protodiakonov's apparatus.

the holes 75 mm, calculated by formula (2):

$$m = m_1 - m_2, \tag{2}$$

where  $m_1$  is the mass of the sample selected for testing;  $m_2$  is the mass of coal remaining on a sieve with holes of 0.075 mm.

The grindability factor HGI is defined by the calibration graphics pursuant to ISO 5074:2015 Hard Coal – definition of Hardgrove Grindability Index [20].

The absolute difference between the resumes of two dividational tests performed in consequence in the that one laboratory by the same manipulator applying the similarly equipment should not outweigh the value of the repeatability limit given in Table 7. The absolute difference between the decisive results of tests exercised in another laboratories obtained using a divider, duplicates of the same sample with a maximum piece size of 4.75 mm shall not exceed the value of the reproducibility limit given in Table 7.

### 2.3. Mathematical processing of results

Experimental-statistical mathematical models (equivalent regression) were composed based on the obtained experimental results. The STATISTICA application package was selected to know the performance of many regressions, such as the description of deposits and functions in response to the process using functionals. The assessment of the adequacy of the statistical significance of the regressions was successful [21,22].

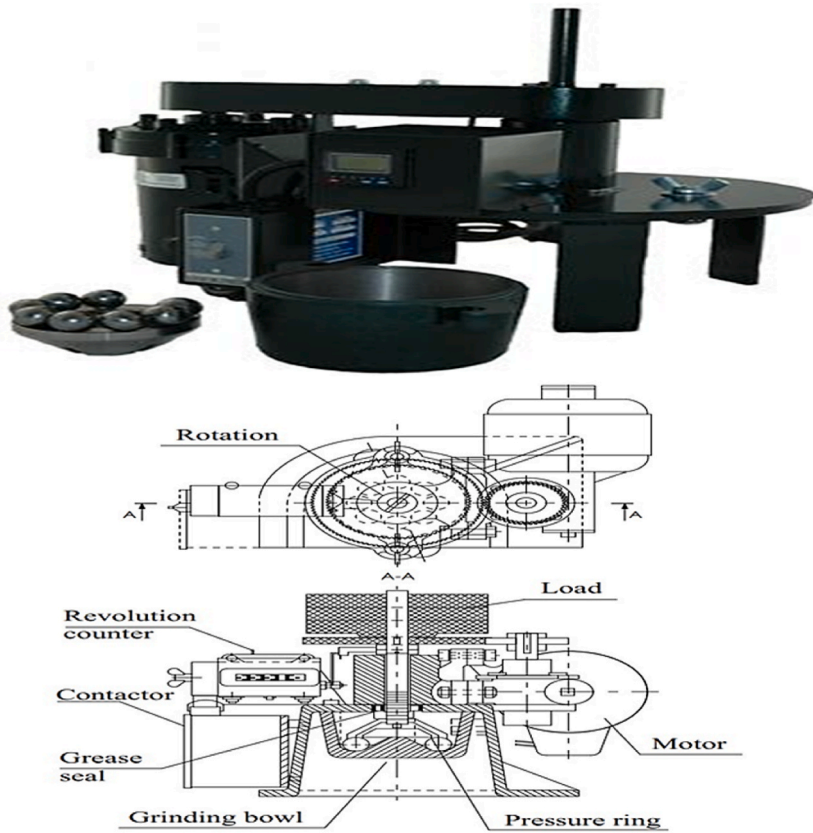


Fig. 2. Photo and diagram of Hardgrove apparatus.

Table 7  
Repeatability and reproducibility.

Repeatability, un.	Reproducibility, un.
3	7

The estimation of model adequacy was carried out according to the following appearances.

- mean relative error of approximation ( $\epsilon_i$ );
- coefficient of determination ( $R^2$ );
- Fisher's criterion (Fi), statistics criterion (Fri).

The adequacy of the obtained regression equations was evaluated by substituting the given experimental parameters ( $X_{ij}$ ) and finding the expected (regressive) values of the response functions ( $Y_{ijreg}$ ).

The coefficient of determination ( $R^2$ ), which shows the significance of the functions accumulation in the response (Y) in the process (X) and the amount of swelling from 0 to 1, was determined according to standard methods [21].

The mean of evaluating relative blunder of approximation was counted by formula (3):

$$\epsilon_i = \frac{1}{n} \sum_{j=1}^n \left| \frac{Y_{ij} - Y_{ij}^{reg}}{Y_{ij}} \right| \tag{3}$$

which  $n$  is the amount of representatives (number of experiments),  $Y_{ij}$  – values noticed during the experiments,  $Y_{ij}^{reg}$  – values of response functions counted with helping the regression equations,  $i$  is response function figure, and  $j$  is experiment figure.

Fisher's criterion was calculated by formula (4):

$$F = \frac{S_{reg}^2}{S_{res}^2}, \tag{4}$$

where  $S_{reg_i}^2$  is the dispersion of a scientific test's response functions relative to its indicate values and  $S_{(res_i)}^2$  is the remnant dispersion of rejoinder functions.

The dispersion of experimental response functions  $S_{(reg_i)}^2$  was calculated according to formula (5) and the residual dispersion of response functions  $S_{(res_i)}^2$  was calculated according to formula (6):

$$S_{reg_i}^2 = \frac{1}{n - 1} \sum_{j=1}^n (Y_{ij} - \underline{Y}_i)^2 \tag{5}$$

where  $\underline{Y}_i$  is the average experimental value of the response function.

$$S_{res_i}^2 = \frac{1}{n - m_i} \sum_{j=1}^n (Y_{ij}^{reg} - Y_{ij})^2 \tag{6}$$

where  $m_i$  is the coefficients numbers in the equation of regression. Calculation according to formulas (3) and (4) is specified in Applied regression analysis [22], and (5) and (6) formulas in Statistical modeling and forecasting [22].

After the countings, Fisher's criterion have to be bigger than the value into table at the significance standard and figures of freeness  $(n-1)$  and  $(n - m_i)$ . This refers to the dispersion quantitative change of results scattering relative to the line of obtained regression equation compared to the scattering relative to the mean value [22].

### 3. Results and discussion

#### 3.1. Study of the coal grinding ability

For the studied samples, the coefficients of pair correlation between the properties of coals and the values of f and HGI were calculated (Table 8).

In the numerator for f; in the denominator - for HGI.

As can be seen from Table 8 data between indicators f and HGI, as well as indicators  $R_0$ ,  $V^{daf}$ ,  $C^{daf}$ ,  $O_d^{daf}$ , characterizing the composition and structure of coals, the values of correlation coefficients exceeding 0.5.

Table 9 presents the obtained values of f and HGI of the studied samples of coal concentrates with the corresponding indicators of their quality -  $R_0$ ,  $V^{daf}$ ,  $C^{daf}$ ,  $O_d^{daf}$ , and in Fig. 3 is a graph of the strength factor f versus the grindability factor HGI, from the analysis of which it can be concluded that these two indicators are interrelated, and the correlation coefficient is 0.960.

Tables 10 and 11 present results regarding the influence of process factors (Factors  $(X_{ij})$ ), those are the content of volatile substances  $V^{daf}$ , mean random vitrinite reflectance  $R_0$ , carbon content  $C^{daf}$  and as well oxygen  $O_d^{daf}$  in coal on the response function Y — coefficients f and HGI, respectively. It was settled that the rise of the Hardgrove grindability coefficient is made happen by a grow up in the content of total ( $C^{daf}$ ) and aromatic ( $C_{ar}$ ) carbon inside coal, also the unsaturation degree ( $\delta$ ) of the structure. Therefore, a reduction in the vitrinite index, made by an increase an aliphatic carbon in the content and a decrease of the organic mass structure of coal in the unsaturation degree, causes decreasing of the Hardgrove grindability coefficient. The influence of the mentioned parameters on the strength coefficient of Protodiakonov is inverse.

Placed on the procured experimental data (look at Tables 10 and 11), known as ESMM (experimental-statistical mathematical models) were performed, which showed the dependence of the Strength coefficient, according to Protodyakonov and the Hardgrove grindability index from coal quality indicators -  $V^{daf}$ ,  $R_0$ ,  $C^{daf}$  and  $O_d^{daf}$ . These equations are given in Table 12. Form on the received equations, substituting the values of the factors in them for everyone experiment, we established the values of the response functions ( $Y_{ij}^{reg}$ ) which we expected and the relative blunders of the ESMM ( $\epsilon$ ). Their datas are shown in Tables 10–12. Setted on the regression values of the response functions, the adequacy of the samples was checking and accounted by the Fisher criterion, the values of which are as well shown in Table 12.

The average relative blunders  $\epsilon$  demonstrate that the correspondence of the built models for the coefficients f and HGI with the experimental data can mostly be pondered high (in cause  $\epsilon = 0-10\%$  – the accuracy of the prediction is high). For equation 7 f from  $C^{daf}$  it is satisfactory ( $\epsilon = 20-50\%$ ) [22].

Table 12 shows mathematical equations (7)–(15) and a statistical assessment of the parameter dependencies of the studied parameters of coal properties, doing it possible to foresight the f and HGI indicators successfully. The obtained dependencies are described satisfactorily by polynomials of the second degree.

It is given that the dependence of high quality indicators of coals with the modulus of their strength (f) is much lower ( $R^2 = 0.716-0.550$ ) than with the coefficient of grindability by Hardgrove (HGI) ( $R^2 = 0.937-0.807$ ). The obtained dependences of HGI=f

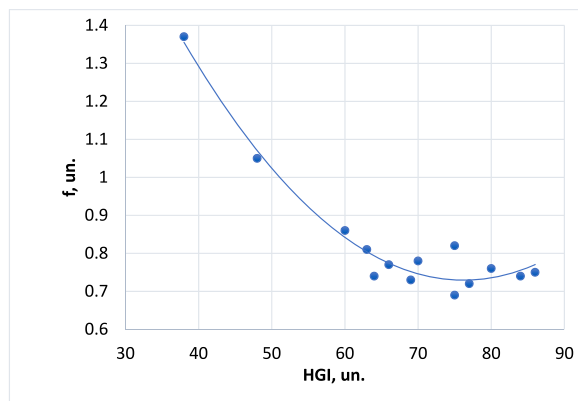
**Table 8**  
Pair correlation coefficients  $r$ .<sup>1</sup>

Value	$R_0$	$V^{daf}$	$C^{daf}$	$O_d^{daf}$
$ r $	<u>0.843</u>	<u>0.741</u>	<u>0.846</u>	<u>0.821</u>
	0.968	0.919	0.947	0.898



**Table 9**  
Quality indicators of the studied samples of coal concentrates.

Sample number	Index					
	$R_0, \%$	$V^{daf}, \%$	$C^{daf}, \%$	$O_d^{daf}, \%$	f, un.	HGI, un.
1	0.73	37.7	83.51	8.55	1.05	48
2	0.72	38.6	83.39	7.93	1.37	38
3	0.96	34.3	86.4	5.37	0.86	60
4	0.94	32.5	86.28	6.01	0.81	63
5	0.93	35.7	86.38	4.63	0.77	66
6	1.01	32.5	86.22	4.5	0.74	64
7	1.22	25.2	87.96	4.15	0.76	80
8	1.2	27.6	87.5	4.78	0.69	75
9	1.18	26.3	88.8	3.91	0.73	69
10	1.2	21.3	87.45	5.19	0.82	75
11	1.03	24.7	87.85	4.78	0.78	70
12	1.19	20.3	88.31	4.65	0.72	77
13	1.54	19.2	89.89	2.62	0.75	86
14	1.51	18.3	90.51	2.93	0.74	84



**Fig. 3.** The influence of HGI on the f.

**Table 10**  
Experimental and calculated (regression) values of the strength factor f.

№	Strength coefficient according to Protodyakonov, un. (experimental values, $Y_{ij}$ )	Factors ( $X_{ij}$ )				$f^{reg}$ , un. (calculated values, $Y_{ij}^{reg}$ )			
		$R_0, \%$	$V^d, \%$	$C^{daf}, \%$	$O_d^{daf}, \%$	$f_{1j}^{reg} = f(R_0)$	$f_{2j}^{reg} = f(V^d)$	$f_{3j}^{reg} = f(C^{daf})$	$f_{4j}^{reg} = f(O_d^{daf})$
1.	1.05	0.73	37.7	83.51	8.55	1.17	1.08	1.07	1.23
2.	1.37	0.72	38.6	83.39	7.93	1.18	1.13	1.09	1.13
3.	0.86	0.96	34.3	86.4	5.37	0.86	0.90	0.70	0.83
4.	0.81	0.94	32.5	86.28	6.01	0.88	0.82	0.72	0.89
5.	0.77	0.93	35.7	86.38	4.63	0.89	0.96	0.71	0.77
6.	0.74	1.01	32.5	86.22	4.50	0.81	0.82	0.72	0.76
7.	0.76	1.22	25.2	87.96	4.15	0.69	0.69	0.60	0.73
8.	0.69	1.20	27.6	87.50	4.78	0.70	0.70	0.62	0.78
9.	0.73	1.18	26.3	88.80	3.91	0.70	0.69	0.57	0.72
10.	0.82	1.20	21.3	87.45	5.19	0.70	0.72	0.63	0.81
11.	0.78	1.03	24.7	87.85	4.78	0.80	0.69	0.61	0.78
12.	0.72	1.19	20.3	88.31	4.65	0.70	0.74	0.59	0.77
13.	0.75	1.54	19.2	89.89	2.62	0.73	0.77	0.57	0.67
14.	0.74	1.51	18.3	90.51	2.93	0.72	0.79	0.57	0.68
Average value	$f(f_i) = 0.83$	-	-	-	-	-	-	-	-

**Table 11**  
Experimental and calculated (regression) values of the Hardgrove grindability index HGI.

№	Hardgrove grindability index, un. (experimental values, Y <sub>ij</sub> )	Factors (X <sub>ij</sub> )				HGI <sup>reg</sup> , un. (calculated values, Y <sub>ij</sub> <sup>reg</sup> )			
		R <sub>0</sub> , %	V <sup>d</sup> , %	C <sup>daf</sup> , %	O <sub>d</sub> <sup>daf</sup> , %	HGI <sub>ij</sub> <sup>reg</sup> = f (R <sub>0</sub> )	HGI <sup>reg</sup> <sub>2j</sub> = f (V <sup>d</sup> )	HGI <sup>reg</sup> <sub>3j</sub> = f (C <sup>daf</sup> )	HGI <sup>reg</sup> <sub>4j</sub> = f (O <sub>d</sub> <sup>daf</sup> )
1.	48	0.73	37.7	83.51	8.55	44	49	43	42
2.	38	0.72	38.6	83.39	7.93	43	46	42	47
3.	60	0.96	34.3	86.40	5.37	63	59	65	66
4.	63	0.94	32.5	86.28	6.01	61	63	64	61
5.	66	0.93	35.7	86.38	4.63	61	55	65	71
6.	64	1.01	32.5	86.22	4.50	66	63	64	72
7.	80	1.22	25.2	87.96	4.15	77	76	74	74
8.	75	1.20	27.6	87.50	4.78	76	73	72	70
9.	69	1.18	26.3	88.80	3.91	75	74	78	76
10.	75	1.20	21.3	87.45	5.19	76	80	72	67
11.	70	1.03	24.7	87.85	4.78	67	76	74	70
12.	77	1.19	20.3	88.31	4.65	76	80	76	71
13.	86	1.54	19.2	89.89	2.62	85	81	83	86
14.	84	1.51	18.3	90.51	2.93	85	81	85	83
Average value	$\frac{HGI}{Y_i} = 68$	-	-	-	-	-	-	-	-

**Table 12**  
Mathematical equations and their statistical evaluation.

No.	Type of equation	Assessment of statistical significance and adequacy		
		R <sup>2</sup>	ε, %	F
(7)	$f = 1.3695 \bullet R_0^2 - 3.6455 \bullet R_0 + 3.0978$	0.711	7.3	3.59
(8)	$HGI = - 51.754 \bullet R_0^2 + 167.42 \bullet R_0 - 50.297$	0.937	9.9	13.33
(9)	$f = 0.0024 \bullet (V^{daf})^2 - 0.1199 \bullet V^{daf} + 2.1846$	0.550	8.8	2.44
(10)	$HGI = - 0.0738 \bullet (V^{daf})^2 + 2.487 \bullet V^{daf} + 60.081$	0.845	6.4	5.45
(11)	$f = 0.0136 \bullet (C^{daf})^2 - 2.4376 \bullet C^{daf} + 109.79$	0.716	20.7	1.04
(12)	$HGI = - 0.396 \bullet (C^{daf})^2 + 74.843 \bullet C^{daf} - 3445.1$	0.897	5.2	8.22
(13)	$f = 0.0119 \bullet (O_d^{daf})^2 - 0.0385 \bullet O_d^{daf} + 0.6895$	0.674	7.3	2.92
(14)	$HGI = - 0.0141 \bullet (O_d^{daf})^2 - 7.164 \bullet O_d^{daf} + 104.42$	0.807	7.8	4.38
(15)	$f = 0.0004 \bullet HGI^2 - 0.0599 \bullet HGI + 3.0722$	0.960	9.4	2.82

(qualitative characteristics of coal) according to the R<sup>2</sup> indicator exceed or are equal to the data given in Ref. [8].

Therefore, to predict the grinding ability of coals, it is preferable to use the Hardgrove method rather than the Protodyakonov method.

In addition, Table 12 shows equation (15), which makes it possible to predict the grinding ability of coals in the presence of one value out of two.

Equations 9 and 11 describe the dependence of f on coal characteristics inadequately. The estimated values of the Fisher’s criterion are smaller than its critical value (F<sub>1cr</sub> = 2.78). The obtained data for equations 7, 8, 10, 12, 13, 14, and 15 testify to the adequacy of the developed linear ECMM. Therefore, they can be used to predict the dependence of the grinding ability of coal (f and HGI) on its indicators, characterizing the degree of metamorphism.

### 3.2. Study of the charge grinding ability

We examined 7 models of coal charges of the head coke chemical companies, which had different degrees of metamorphism and petrographic homogeneity.

Besides calculation of the HGI<sub>fact</sub> for examined samples, the calculation of the HGI<sub>calc.</sub> was also made with a few methods.

1. By the additivity method.
2. According to the well-known formula (16) for the dependence of HGI on V<sup>daf</sup>:

$$HGI = - 0,0238 \bullet V^{daf^2} - 0,361 \bullet V^{daf} + 99,321 \tag{16}$$

where V<sup>daf</sup> – is the content of volatile substances in the charge.

3. According to the HGI formula (17) determined by us for the mixture with the simultaneous preparation of samples:

$$HGI_{fact} = 1,0313 \bullet HGI_{calc.} - 3,6162, \quad (17)$$

where  $HGI_{calc.}$  – is the value of the charge's disintegrability coefficient, calculated by the method of additivity of its components.

Table 13 shows the results of determinations of the HGI disintegrability coefficient of the presented charges - actual and calculated, obtained by the additivity method and according to equations (16) and (17).

Analyzing the data given in Tables 13 and it can be stated that the discrepancies between the actual and calculated values of the youth coefficient are the largest when calculated according to equation (16)  $f(HGI)$  from  $V^{daf}$  - (2–5 units), and the smallest (0–2 units) when calculated according to equation (17). It is also worth noting that with the growth of the experimentally obtained grinding power coefficients from 58 to 60–62 units, their deviation from the calculated values decreases in all cases and is equal to zero according to equation (17). In our opinion, this is because the increase in the content of gas coal in the charges (up to 31 %) during the preparation of the sample mainly gets into the grinding bowl and affects the change in the charge HGI coefficient to the HGI value of solid coal.

#### 4. Conclusions

The quality indicators of 14 coals of Ukraine, USA, and Kazakhstan, characterized by various degrees of metamorphism, technical analysis, petrographic, plastometric, and elemental compositions, as well as grinding ability indicators, were determined.

Determination of the coal grind ability was made by widely known Protodyakonov and Hardgrove methods, which allows evaluating the resistibility of the substance to the friction forces.

Graphical and mathematical dependences have been appeared that allow predicting the grinding capability of coals applying the Hardgrove (HGI) and Protodyakonov ( $f$ ) techniques, founded on definitiving special indicators ( $R^0$ ,  $V^{daf}$ ,  $C^{daf}$ ,  $O_2^{daf}$ ) of their quality. It is shown that the graphical and mathematical dependencies are polynomials of the second degree, and the values of the coefficients of determination ( $R^2$ ) exceed 0.674.

It accepted that a growth of the Hardgrove grindability coefficient is resulted by an increase in the meaning of full ( $C^{daf}$ ) and aromatic ( $C_{ar}$ ) carbon in coal, also the unsaturation degree ( $\delta$ ) of the structure. Suitably, a decrease in the vitrinite index, the reason why it happens is an increase in the content of aliphatic carbon and a decrease in the desaturation of the organic mass structure of a coal. It leads to reduce the Hardgrove grindability coefficient. The influence of the reference parameters on the strength coefficient according to the M.M. scale of Protodyakonov is opposite.

First-ever the relationship between the strength coefficient in accord with the Protodyakonov method and the coal's Grindability Index in consonance with the Hardgrove method was established. This dependency is satisfactorily put into by a second-degree polynomial and a 0.960 correlation coefficient.

It accepted that the senses of the  $f$  and HGI indicators are inversely proportional; a mathematical and graphical dependence of their prediction which based on the values of one of them.

Received results can be used in the coal preparation process for its burning and coking at special facilities. Moreover, developed dependencies can be used to calculate the amount of electricity that is needed for fuel braking and burning.

Equations 9 and 11 describe the dependence on coal characteristics inadequately. Equations 7, 8, 10, 12, 13, and 14 can be used to predict the dependence of coal grinding ability ( $f$  and  $HGI$ ) on its indicators characterizing the degree of metamorphism. And equation (15) acts it able to predict the value of the grinding's ability coefficients of coal  $f$  or HGI in the attendance of one of the two values.

The modulus of HGI grind ability capacity of coal charges in the Ukraine's leading coke chemical companies, which included coal of different degrees of metamorphism in four primary grades, were definited.

Systematical rejections of the topical HGI values of coal charges during their associate preparation from their counted values in the direction of a decrease in the HGI coefficient (to harder coal) were acknowledged.

It was also admitted that with the growth of the experimentally obtained coefficients of crushing capacity, their deviations from the calculated values decreased.

**Table 13**  
Comparison of actual and estimated HGI values.

Charge	Hardgrove disintegrability coefficient						
	Experimentally obtained	Calculated by additivity	$ HGI_{fact} - HGI_{calc.} $	Calculated from Eq. (16) from $V^{daf}$	$ HGI_{fact} - HGI_{calc.} $	Calculated from Eq. (17) for the mixture	$ HGI_{fact} - HGI_{calc.} $
PJSC "YUZHOKOKS"	58	61	3	63	5	60	2
PJSC "DKHZ"	58	62	4	63	5	60	2
PJSC "AKHZ" KC-1	59	62	3	64	5	60	1
PJSC "AKHZ" KC-2	58	61	3	63	5	60	2
PJSC "AKHZ" KC-4	59	62	3	63	4	61	2
PJSC MK "Azovstal" ("KHP JSC")	62	63	1	64	2	62	0
PJSC "ZAPORIZHKOKS"	60	62	2	64	4	60	0
Maximum	62	63	4	64	5	62	2
Minimal	58	61	1	63	2	60	0
Average	59.1	61.9	2.7	63.4	4.3	60.4	1.3

A mathematical dependence makes known, which produces it possible to satisfactorily foresee the value of the HGI of the coal charge in the attendance the data about the coefficients of grind ability capacity of its peculiar components. It has been established that this dependence makes it possible to predict the grind ability coefficient of coal charges much more accurately than the previously used formulas for calculating the dependence of HGI on  $V^{\text{daf}}$  and by the additivity method.

Thus, it is visible that the Hardgrove and Protodiakonov strength coefficients satisfactorily categorize coal in keeping with the degree of its resistance to destructing forces. Moreover, both the well-known ways of determining the destructive capacity of Hardgrove and the alternative, more accessible crushing Protodyakonov method can be recommended for implementation at coke chemical enterprises.

We also recommended the proposed methodology for calculating the strength of coal charges based on the HGI coefficients of individual components.

Only samples of hard coal were used. The study's limitations are as follows: the obtained results cannot be applied to lignite and anthracite.

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## Data availability statement

Data included in article/supplementary material/referenced in article.

## Additional information

No additional information is available for this paper.

## CRediT authorship contribution statement

**Serhiy Pyshyev:** Analyzed and interpreted the data; Wrote the paper. **Denis Miroshnichenko:** Performed the experiments; Contributed reagents, materials, analysis tools or data. **Valentine Koval:** Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data. **Taras Chipko:** Contributed reagents, materials, analysis tools or data; Wrote the paper. **Mariia Shved:** Analyzed and interpreted the data; Wrote the paper.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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