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Review article

Complex Fermatean fuzzy extended TOPSIS method and its applications in decision making

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ABSTRACT

The fuzzy set has its own limitations due to the membership function only. The fuzzy set does not describe the negative aspects of an object. The Fermatean fuzzy set covers the negative aspects of an object. The complex Fermatean fuzzy set is the most effective tool for handling ambiguous and uncertain information. The aim of this research work is to develop new techniques for complex decision-making based on complex Fermatean fuzzy numbers. First, we construct different aggregation operators for complex Fermatean fuzzy numbers, using Einstein t-norms. We define a series of aggregation operators named complex Fermatean fuzzy Einstein weighted average aggregation (CFFEWAA), complex Fermatean fuzzy Einstein ordered weighted average aggregation (CFFEOWAA), and complex Fermatean fuzzy Einstein hybrid average aggregation (CFFEHAA). The fundamental properties of the proposed aggregation operators are discussed here. The proposed aggregation operators are applied to the decision-making technique with the help of the score functions. We also construct different algorithms based on different aggregation operators. The extended TOPSIS method is described for the decision-making problem. We apply the proposed extended TOPSIS method to MAGDM problem "selection of an English language instructor". We also compare the proposed models with the existing models.

1. Introduction

Aggregation operators (AOs) are dominant because they collect diverse information in a single format. Aggregation operators are good tools in the multicriteria group decision-making (MCGDM) problem for selecting the most capable choice regarding the pivotal elements. When humans are unsure which option is the most valuable in the real-world MCGDM model, we use fuzzy set theory to solve the problem. In classical set theory, various AOs such as quasiarithmetic mean, harmonic mean, maximum, minimum, geometric mean, minimum weighted operator, median, maximum weighted operator, and arithmetic mean are used to collect crisp information. Many researchers presented MCGDM models that made use of Yager AOs. Symmetry, continuity, monotonicity, idempotency, associativity, bisymmetry, and invariance are the properties of AOs.

Zadeh [1] in 1965 developed the concept of the classical set into the fuzzy set and defined a few fuzzy set operators. Song et al. [2] discussed both the fuzzy set techniques and the attributes. The notion of intuitionistic fuzzy set IFS, put out by Atanassov [3], is

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one of the successful methods to manipulate unpredictability and uncertainty. A fuzzy set that incorporates degrees of satisfaction and dissatisfaction is known as an intuitionistic fuzzy set. For IFS, Yager [4] created the idea of ordered weighted average operators. In the intuitionistic fuzzy set (IFS) theory, weighted averaging operators were invented for the first time by Xu [5]. Wei [6] created the idea of "induced Geometric AOs for IF Data" and applied it to the multi-attribute decision-making (MADM) problem. The concept of generalized aggregating operations for IF information was first put forth by Zhao et al. [7]. The induced IF-correlated geometric operators and average aggregation operators were developed by Wei and Zhao and used in MADM. The Hamacher operators of the IFS theory were first introduced by Huang [8]. The idea of dynamic intuitionistic normal fuzzy aggregation operators was first forth by Yang et al. [9].

However, there are situations when the exports assign belongingness and non-belongingness values that are so high that the sum is greater than 1. Yager [10,11] presented another idea called the Pythagorean fuzzy set to solve such issues (PFS). The researchers created a wide range of aggregation operations using Pythagorean fuzzy data [12–21]. The study discovered that some values can be used to give belongingness and non-belongingness degrees and that their aggregate may be greater than 1. To get over this problem, Senapati and Yager [22] created a novel idea known as the Fermatean fuzzy set FFS. Finding higher accuracy than IFS and PFS is the area of expertise of FFS. The researchers used Fermatean fuzzy information to present [23–26] several AOs.

Later, Ramot [27] developed a brand-new idea known as the complex fuzzy set, which is better at regulating the restriction of fuzzy sets, intuitionistic fuzzy sets, and Pythagorean fuzzy sets. To make the most of fuzzy sets, experts can use Ramot's [28] illustration of complicated fuzzy logic to deal with such constraints. Geometric and arithmetic AOs are found by Bi et al. [29,30] using CFNs. In order to extend CFS, Alkouri et al. [31] developed a complex intuitionistic fuzzy set (CIFS) and described its operations. Akram et al. [32] put forward an MCDM issue to assess their suggested CIF Hamacher aggregation operators. Garg et al. [33] proposed the CIF-weighted geometric operator and the CIF-weighted average operator. Rani et al. [34] introduced power AOs for CIFNs. In order to handle more complex data, Akram et al. [35] developed a complex Pythagorean fuzzy set (CPFS) as an addition to CIFS. An MCDM based on CPFNs and improved Pythagorean fuzzy Einstein AOs was introduced by Janani et al. [36]. Rahman [37] also provided an MCDM model that allows CPFNs to make use of different Einstein Geometric AOs. Akram et al. [38] developed Dombi AOs for CPFNs and offered an MCDM challenge. Mahmood et al. [39] applied CPFAOs to an MCDM based on the confidence level for CPFNs. The concept of CPFHAOs was defined by Akram et al. [40]. Akram et al. created a hybrid technique for CPFNs in the same article. Akram et al. [41] defined Yager AOs for CPFNs as a concept.

In addition to CIFS and CPFS, Chinnadurai et al. [42] presented a complex Fermatean fuzzy set (CFFS). In the same article, Chinnadurai et al. introduced several AOs named complex Fermatean fuzzy weighted geometric operator (CFFWGO), complex Fermatean fuzzy weighted average operator (CFFWAO), complex Fermatean fuzzy weighted power geometric operator (CFFWPGO), and complex Fermatean fuzzy weighted power average operator to handle more uncertain and imprecise information in MADM problems (CFFWPAO).

Garg [43,44], and [45] created various group decision-making based on Einstein geometric aggregation operators and Einstein averaging aggregation operators. Wang and Liu [46] introduced the Einstein averaging aggregation procedures for IFS. Zhao and Wei [47] created hybrid geometric and aggregation operators using Einstein's operations. Garg [48] introduced the Pythagorean fuzzy Einstein hybrid averaging aggregation operators and also posed a problem that needed to be resolved. Janani et al. [49] developed the concept of Einstein averaging aggregation operators for the complex Pythagorean fuzzy set. Rani et al. presented a MULTIMOORA method with Fermatean fuzzy Einstein aggregation operators [50].

Jushi and Kumar [51] developed the TOPSIS method, a flexible way of identifying the best choice, with the presentation of a MADM problem for IFS. For PFS, Zang and Xu [52] created the TOPSIS approach. Gul et al. [53] presented a TOPSIS approach for FFS. In their MADM, Kumar and Chen [54] used the Intuitionistic fuzzy Einstein weighted averaging (IFEWA) operator. Akram et al. [58] defined the N-Soft sets for CFFNs. Akram et al. [59] developed the VIKOR method and applied to a group decision-making problem. Chinnadurai et al. [60] defined the distance measures under CFFNs. Broumi et al. [61] the Complex Fermatean Neutrosophic fuzzy set under complex Fermatean fuzzy numbers.

This article has the following motivations:

1. Due to the existence of phase terms, which the FS theory lacks, the CFFEA AOs are more adaptable solutions for handling data of a periodic nature.
2. Because they have a cube on both terms, the CFFEA AOs are cleverer at communicating two-dimensional ambiguous data. When it comes to periodic information, CFFEA AOs provide good communication tools if the outcomes of CIFEA AOs and CPFEA AOs are questionable.
3. If the CFS theory and CIFS theory fall short in terms of performance, the CFFS theory is more adaptable in dealing with periodic information in terms of Einstein t-norm and Einstein t-conorm. The experts frequently come up with ambiguous conclusions when choosing the best option while applying the CIFEA AOs and CPFEA AOs constraints. The CFFEA AOs are readily applicable and can be used to bypass these limitations.
4. When applied to an MAGDM challenge, Einstein AOs are useful tools for discovering more creative and adaptable results. The purpose of this paper is to outline the MAGDM issues for CFF information while using Einstein techniques.
5. In terms of Einstein operations, the CFFE-TOPSIS method's objective is to select the best option from a group of decision alternatives based on a variety of factors. It is a multi-criteria decision-making method that chooses the optimal option by comparing alternatives relative to the ideal answer. The best option is the one that comes closest to the ideal resolution.

Contribution

In this research study, we present;

1). The concept of the complex Fermatean fuzzy Einstein averaging aggregation operators.
2). We present a MAGDM problem for selecting the best English language instructor.
3). We also propose the TOPSIS method for the complex Fermatean fuzzy Einstein averaging aggregation (CFFEAA) operators.
4). We compare our proposed AOs with existing AOs.

The organization of this study is as follows:

We provide some fundamental definitions in section 2. We provide Einstein's operations based on CFNs in the same section, which inspired us to create the concept of CFFEAAOs and their characteristics in section 3. Section 4 of this paper details the procedure for utilizing the CFFEA operator and the CFFE-TOPSIS technique to resolve the MAGDM issue using CFF data. We also provided a flowchart for the CFFE-TOPSIS approach in the same section. We create a MAGDM model in section 5 and use the suggested operators to solve it. In section 6, a comparison of the proposed operators with existing operators is developed. Section 7 of this research study contain the conclusion, future directions, limitation and discussion, author contribution, data availability and declaration of interest.

2. Preliminaries

Definition 1. [55]. If we consider S as a universal set, a complex Fermatean fuzzy set M over the universal set S is defined as:

$$M = \{\langle s, \Psi_M(s), \varpi_M(s)\rangle \mid s \in S\}, \tag{1}$$

where Ψ_M is called the degree of membership and ϖ_M is called the degree of nonmembership that has a mapping $\Psi_M, \varpi_M : S \longrightarrow \{l \mid l \in L : |l| \le 1\}$. For every $s \in S$, the degrees of membership and nonmembership is $\Psi_M(s) = A_M(s)e^{ia_M(s)}$ and $\varpi_M(s) = B_M(s)e^{ib_M(s)}$ respectively. Where $A_M, B_M \in [0,1], a_M, b_M \in [0, 2\pi], 0 \le A^3_M(s) + B^3_M(s) \le 1, i = \sqrt{-1}$ and $0 \le (\frac{a_M(s)}{2\pi^3}) + (\frac{b_M(s)}{2\pi^3}) \le 1$. The pair (Ψ_M, ϖ_M) represents complex Fermatean fuzzy numbers. Where Ψ_M is a membership function and ϖ_M is a nonmembership function. Eq. (1) shows the CFFS.

Definition 2. [55]. The score function of the complex Fermatean fuzzy numbers (CFFNs) $C = (\Psi_C, \varpi_C)$ has the following formula:

$$s(C) = (A^3 - B^3) + \frac{1}{8\pi^3}\left(a^3 - b^3\right); \tag{2}$$

where $s(C) \in [-2, 2]$. Eq. (2) shows the score function.

Definition 3. [55]. The formula for the accuracy function f of the CFFNs $C = (\Psi_C, \varpi_C)$ is given below:

$$f(C) = (A^3 + B^3) + \frac{1}{8\pi^3}\left(a^3 + b^3\right); \tag{3}$$

where $f(C) = [0, 2]$. Eq. (3) shows the accuracy function.

Definition 4. [55]. In order to compare two CFFNs $C_1 = (A^3_1 e^{ia_1}, B_1 e^{ib_1})$ and $C_2 = (A_2 e^{ia_2}, B_2 e^{ib_2})$ then

(1) $C_1 \succ C_2$ (C_1 is superior to C_2) if $s(C_1) \succ s(C_2)$;
(2) If $s(C_1) = s(C_2)$ then
   (a) If $f(C_1) \succ f(C_2)$, then $C_1 \succ C_2$ (C_1 is superior to C_2);
   (b) If $f(C_1) = f(C_2)$, then $C_1 \sim C_2$ (C_1 is equivalent to C_2).

2.1. Einstein t-norm and Einstein t-conorm

In fuzzy set theory, t-norm and t-conorm are used to define operations. Einstein developed t-norm and t-conorm into a specific form, Einstein sum, and Einstein product respectively. The definition of Einstein's t-norm and Einstein's t-conorm are given below:

$$(T)^E(q,r) = q \otimes r = \frac{qr}{1 + (1-q)(1-r)} \tag{4}$$

$$\left(T^*\right)^E(q,r) = q \oplus r = \frac{q+r}{1+qr} \tag{5}$$

for all $q, r \in [0,1]$. Eq. (4) and (5) represent Einstein's norms.

2.2. Einstein operations for complex Fermatean fuzzy numbers

If we suppose three CFFNs $C = (A_C e^{ia_C}, B_C e^{ib_C})$, $C_1 = (A_{C_1} e^{ia_{C_1}}, B_{C_1} e^{ib_{C_1}})$, and $C_2 = (A_{C_2} e^{ia_{C_2}}, B_{C_2} e^{ib_{C_2}})$. Then we have;

1.

$$C_1 \otimes C_2 = \frac{A_1^3 A_2^3}{\sqrt[3]{1+(1-A_1^3)(1-A_2^3)}} e^{i2\pi\left(\frac{(a_1/2\pi)(a_2/2\pi)}{\sqrt[3]{1+\left((1-(a_1/2\pi)^3(1-(a_2/2\pi)^3\right)}}\right)},$$

$$\sqrt[3]{\frac{B_1^3+B_2^3-B_1^3B_2^3}{1+B_1^3B_2^3}} e^{i2\pi\left(\sqrt[3]{\frac{(b_1/2\pi)^3+(b_2/2\pi)^3-(b_1/2\pi)^3(b_2/2\pi)^3}{1+(b_1/2\pi)^3(b_2/2\pi)^3)}}\right)}$$

2.

$$C_1 \oplus C_2 = \sqrt[3]{\frac{A_1^3+A_2^3-B_1^3B_2^3}{1+A_1^3A_2^3}} e^{i2\pi\left(\sqrt[3]{\frac{(a_1/2\pi)^3+(a_2/2\pi)^3-(a_1/2\pi)^3(a_2/2\pi)^3}{1+(a_1/2\pi)^3(a_2/2\pi)^3)}}\right)},$$

$$\frac{B_1B_2}{\sqrt[3]{1+(1-B_1^3)(1-B_2^3)}} e^{i2\pi\left(\frac{(b_1/2\pi)(b_2/2\pi)}{\sqrt[3]{1+(1-(b_1/2\pi)^3)(1-(b_2/2\pi)^3)}}\right)}$$

3.

$$\alpha C = \left(\begin{array}{l} \sqrt[3]{\frac{\left(1+A_1^3\right)^\alpha-(1-A_1^3)^\alpha}{\left(1+A_1^3\right)^\alpha+(1-A_1^3)^\alpha}} e^{i2\pi\left(\sqrt[3]{\frac{\left(1+(a/2\pi)^3\right)^\alpha-\left(1-(a/2\pi)^3\right)^\alpha}{\left(1+(a/2\pi)^3\right)^\alpha+\left(1-(a/2\pi)^3\right)^\alpha}}\right)}, \\ \frac{\sqrt[3]{2}(B)^\alpha}{\sqrt[3]{\left(1+(1-B^3)\right)^\alpha+(B^3)^\alpha}} e^{i2\pi\left(\frac{\sqrt[3]{2}(b/2\pi)^\alpha}{\sqrt[3]{\left(1+\left(1-(b/2\pi)^3\right)\right)^\alpha+(b/2\pi)^{3\alpha}}}\right)} \end{array}\right)$$

4.

$$C^\alpha = \frac{\sqrt[3]{2}(A)^\alpha}{\sqrt[3]{\left(1+(1-A_1^3)\right)^\alpha+(A_1^3)^\alpha}} e^{i2\pi\left(\frac{\sqrt[3]{2}(a/2\pi)^\alpha}{\sqrt[3]{\left(1+\left(1-(a/2\pi)^3\right)\right)^\alpha+(a/2\pi)^{3\alpha}}}\right)},$$

$$\sqrt[3]{\frac{\left(1+B^3\right)^\alpha-(1-B^3)^\alpha}{\left(1+B^3\right)^\alpha+(1-B^3)^\alpha}} e^{i2\pi\left(\sqrt[3]{\frac{\left(1+(b/2\pi)^3\right)^\alpha-\left(1-(b/2\pi)^3\right)^\alpha}{\left(1+(b/2\pi)^3\right)^\alpha+\left(1-(b/2\pi)^3\right)^\alpha}}\right)}$$

3. Complex Fermatean fuzzy Einstein averaging aggregation operators

In the above section we have developed Einstein operational laws for CFFNs. Now using these developed Einstein operational laws, a series of aggregation operators is discussed. These series of aggregation operators is named as complex Fermatean fuzzy Einstein weighted average aggregation operators, complex Fermatean fuzzy Einstein ordered weighted average aggregation operators and complex Fermatean fuzzy Einstein hybrid weighted average aggregation operators. Also we have discussed the basic properties for the developed AOs under CFFS information.

3.1. Complex Fermatean fuzzy Einstein weighted average operator

Definition 5. If we have a collective set of CFFNs denoted by $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ with $(s = 1, 2, ..., n)$. If the weight vector exists and represented by $\eta = (\eta_1, \eta_2, ..., \eta_n)^T$, then we can define a (CFFEWAO) as:

$$CFFEWA_\eta(K_1, K_2, ..., K_n) = \bigoplus_{p=1}^{n} \eta_p K_p \tag{6}$$

where the weight vector of K_p can be represented by η_p with the condition $\Sigma_{p=1}^n \eta_p = 1$, where η_p lies in $[0, 1]$. Eq. (6) represents the CFFEWA operator.

Theorem 6. *If there exists a collective set of $CFFNs$ represented by $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_1}, B_p e^{ib_1})$ with $(p = 1, 2, ..., n)$. If $\eta = (\eta_1, \eta_2, ..., \eta_n)^T$ represents the weight vector, then the assembled values of CFFNs can be acquired by using the CFFEWA operator that is given as:*

$$CFFEWA_\eta(K_1, K_2, ..., K_n) = \bigoplus_{p=1}^{n} \eta_p K_p$$

$$\left(\begin{array}{c} \sqrt[3]{\dfrac{\prod_{p=1}^{n}\left(1+A_1^3\right)^{\eta_p} - \prod_{p=1}^{n}(1-A_1^3)^{\eta_p}}{\prod_{p=1}^{n}\left(1+A_1^3\right)^{\eta_p} + \prod_{p=1}^{n}(1-A_1^3)^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\dfrac{\prod_{p=1}^{n}\left(1+(a/2\pi)^3\right)^{\eta_p} - \prod_{p=1}^{n}\left(1-(a/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{n}\left(1+(a/2\pi)^3\right)^{\eta_p} + \prod_{p=1}^{n}\left(1-(a/2\pi)^3\right)^{\eta_p}}}\right)}, \\ \dfrac{\prod_{p=1}^{n}\sqrt[3]{2}(B)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+(1-B^3)\right)^{\eta_p} + \prod_{p=1}^{n}(B^3)^{\eta_p}}}\, e^{i2\pi\left(\dfrac{\prod_{p=1}^{n}\sqrt[3]{\delta}(b/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+\left(1-(b/2\pi)^3\right)\right)^{\eta_p} + \prod_{p=1}^{n}(b/2\pi)^{3\eta_p}}}\right)} \end{array} \right) \tag{7}$$

where the weight of K_p is represented by η_p that must lie in the interval $[0, 1]$, *where the condition that* $\Sigma_{p=1}^{n}\eta_p = 1$ *must be satisfied. Mathematical induction can be useful to proving the theorem.*

Proof. Case 1. Let us check it by the condition that if $n = 1$, so we get □

$$CFFEWA_\eta(K_1, K_2, ..., K_n) = \Sigma_1\eta_1 = \eta_1(\text{ as we know that } \Sigma_1 = 1)$$

$$\left(\begin{array}{c} \sqrt[3]{\dfrac{\left(1+A_1^3\right)^{\alpha} - (1-A_1^3)}{\left(1+A_1^3\right)+(1-A_1^3)}}\, e^{i2\pi\left(\sqrt[3]{\dfrac{\left(1+(a_1^3/2\pi)^3\right)-\left(1-(a_1^3/2\pi)^3\right)}{\left(1+(a_1^3/2\pi)^3\right)+\left(1-(a_1^3/2\pi)^3\right)}}\right)}, \\ \dfrac{\sqrt[3]{2}(B_1)}{\sqrt[3]{\left(1+(1-B_1^3)\right)+(B_1^3)}}\, e^{i2\pi\left(\dfrac{\sqrt[3]{2}(b_1/2\pi)}{\sqrt[3]{\left(1+\left(1-(b_1/2\pi)^3\right)\right)+(b_1/2\pi)^3}}\right)} \end{array} \right) = (A_1^3 e^{ia_1^3}, B_1 e^{ib_1}) \tag{8}$$

Here, the result is satisfactory for $n = 1$. Eq. (8) shows the special case of our proposed study.

Case 2. Now let us check the equation for the condition that $(n = 1)$ and $n = r$ denotes a natural number, so

$$CFFEWA_\eta(K_1, K_2, ..., K_r) = \bigoplus_{p=1}^{n} \eta_p K_p =$$

$$\left(\begin{array}{c} \sqrt[3]{\dfrac{\prod_{p=1}^{r}\left(1+A_p^3\right)^{\eta_p} - \prod_{p=1}^{r}(1-A_p^3)^{\eta_p}}{\prod_{p=1}^{r}\left(1+A_p^3\right)^{\eta_p} + \prod_{p=1}^{r}(1-A_p^3)^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\dfrac{\prod_{p=1}^{r}\left(1+(a_p/2\pi)^3\right)^{\eta_p} - \prod_{p=1}^{r}\left(1-(a_p/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{r}\left(1+(a_p/2\pi)^3\right)^{\eta_p} + \prod_{p=1}^{r}\left(1-(a_p/2\pi)^3\right)^{\eta_p}}}\right)}, \\ \dfrac{\prod_{p=1}^{r}\sqrt[3]{2}(B_p)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{r}\left(1+(1-B_p^3)\right)^{\eta_p} + \prod_{p=1}^{r}(B_p^3)^{\eta_p}}}\, e^{i2\pi\left(\dfrac{\prod_{p=1}^{r}\sqrt[3]{2}(b_p/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{r}\left(1+\left(1-(b_p/2\pi)^3\right)\right)^{\eta_p} + \prod_{p=1}^{r}(b_p/2\pi)^{3\eta_p}}}\right)} \end{array} \right) \tag{9}$$

suppose that $n = r + 1$

$$CFFEWA_\eta(K_1, K_2, ..., K_{r+1}) = \bigoplus_{p=1}^{n} \eta_p K_p \bigoplus \eta_{r+1} K_{r+1} =$$

$$
\left(
\begin{array}{l}
\sqrt[3]{\dfrac{\prod_{p=1}^{r}\left(1+A_p^3\right)^{\eta_p}-\prod_{p=1}^{r}(1-A_p^3)^{\eta_p}}{\prod_{p=1}^{r}\left(1+A_p^3\right)^{\eta_p}+\prod_{p=1}^{r}(1-A_p^3)^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\prod_{p=1}^{r}\left(1+(a_p/2\pi)^3\right)^{\eta_p}-\prod_{p=1}^{r}\left(1-(a_p/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{r}\left(1+(a_p/2\pi)^3\right)^{\eta_p}+\prod_{p=1}^{r}\left(1-(a_p/2\pi)^3\right)^{\eta_p}}}\right)}, \\
\dfrac{\prod_{p=1}^{r}\sqrt[3]{2}(B_p)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{r}\left(1+(1-B_p^3)\right)^{\eta_p}+\prod_{p=1}^{r}(B_p^3)^{\eta_p}}}\, e^{i2\pi\left(\frac{\prod_{p=1}^{r}\sqrt[3]{2}(b_p/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{r}\left(1+\left(1-(b_p/2\pi)^3\right)\right)^{\eta_p}+\prod_{p=1}^{r}(b_p/2\pi)^{3\eta_p}}}\right)}
\end{array}
\right)
$$

$$
\oplus\left(
\begin{array}{l}
\sqrt[3]{\dfrac{\left(1+A_{r+1}^3\right)^{\eta_{r+1}}-(1-A_{r+1}^3)^{\eta_{r+1}}}{\left(1+A_{r+1}^3\right)^{r+1}+(1-A_{r+1}^3)^{r+1}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\left(1+(a_{r+1}/2\pi)^3\right)^{\eta_{r+1}}-\left(1-(a_{r+1}/2\pi)^3\right)^{\eta_{r+1}}}{\left(1+(a_{r+1}/2\pi)^3\right)^{\eta_{r+1}}+\left(1-(a_{r+1}/2\pi)^3\right)^{r+1}}}\right)}, \\
\dfrac{\sqrt[3]{2}(B_p)^{r+1}}{\sqrt[3]{\left(1+(2-1)(1-B_{r+1}^3)\right)^{\eta_{r+1}}+(2-1)B_{r+1}^{3\xi_{r+1}}}}\, e^{2\pi\left(\frac{\sqrt[3]{2}(B_{r+1}/2\pi)^{\eta_{r+1}}}{\sqrt[3]{\left(1+\left(1-(b_{r+1}/2\pi)^3\right)\right)^{\eta_{r+1}}+(b_{r+1}/2\pi)^{3\eta_{r+1}}}}\right)}
\end{array}
\right)
$$

$$
=\left(
\begin{array}{l}
\sqrt[3]{\dfrac{\prod_{p=1}^{r+1}\left(1+A_p^3\right)^{\eta_p}-\prod_{p=1}^{r+1}(1-A_p^3)^{\eta_p}}{\prod_{p=1}^{r+1}\left(1+A_p^3\right)^{\eta_p}+\prod_{p=1}^{r+1}(1-A_p^3)^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\prod_{p=1}^{r+1}\left(1+(a_p^3/2\pi)^3\right)^{\eta_p}-\prod_{p=1}^{r+1}\left(1-(a_p^3/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{r+1}\left(1+(a_p^3/2\pi)^3\right)^{\eta_p}+\prod_{p=1}^{r+1}\left(1-(a_p^3/2\pi)^3\right)^{\eta_p}}}\right)}, \\
\dfrac{\sqrt[3]{2}\prod_{p=1}^{r+1}(B_p)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{r+1}\left(1+(1-B_p^3)\right)^{\eta_p}+\prod_{p=1}^{r+1}B_p^{3\eta_p}}}\, e^{i2\pi\left(\frac{\sqrt[3]{2}\prod_{p=1}^{r+1}(b_p/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{r+1}\left(1+\left(1-(b_p/2\pi)^3\right)\right)^{\eta_p}+\prod_{p=1}^{r+1}(b_p/2\pi)^{3\eta_p}}}\right)}
\end{array}
\right) \tag{10}
$$

So we found a satisfactory output for all the values of n because for $n = r + 1$, the equation is satisfied. Where n belongs to natural numbers. Eq. (10) is the special case of our proposed work.

Example 7. Suppose that there are three $CFFNs$, $K_1 = \left(0.68e^{i2\pi(0.88)}, 0.82e^{i2\pi(0.60)}\right)$, $K_2 = \left(0.85e^{i2\pi(0.64)}, 0.63e^{i2\pi(0.80)}\right)$, and $K_3 = \left(0.83e^{i2\pi(0.55)}, 0.73e^{i2\pi(0.90)}\right)$. If the weight vector is $\eta = (0.37, 0.43, 0.2)^T$. So putting the values in the given equation

$$
CFFEWA_\eta(K_1, K_2, K_3) = \bigoplus_{p=1}^{3}\eta_p K_p =
$$

$$
\left(
\begin{array}{l}
\sqrt[3]{\dfrac{\prod_{p=1}^{3}\left(1+3A_p^3\right)^{\eta_p}-\prod_{p=1}^{3}(1-A_p^3)^{\eta_p}}{\prod_{p=1}^{3}\left(1+3A_p^3\right)^{\eta_p}+3\prod_{p=1}^{3}(1-A_p^3)^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\prod_{p=1}^{3}\left(1+3(a_p^3/2\pi)^3\right)^{\eta_p}-\prod_{p=1}^{3}\left(1-(a_p^3/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{3}\left(1+3(a_p^3/2\pi)^3\right)^{\eta_p}+3\prod_{p=1}^{3}\left(1-(a_p^3/2\pi)^3\right)^{\eta_p}}}\right)}, \\
\dfrac{\prod_{p=1}^{3}\sqrt[3]{2}(B_p)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{3}\left(1+3(1-B_p^3)\right)^{\eta_p}+3\prod_{p=1}^{3}(B_p^3)^{\eta_p}}}\, e^{i2\pi\left(\frac{\prod_{p=1}^{3}\sqrt[3]{2}(b_p/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{3}\left(1+3\left(1-(b_p/2\pi)^3\right)\right)^{\eta_p}+3\prod_{p=1}^{3}(b_p/2\pi)^{3\eta_p}}}\right)}
\end{array}
\right)
$$

$$= 0.8023349885e^{i2\pi(0.1188531875)}, 0.7563003774e^{i2\pi(0.1018040977)}$$

Theorem 8 *(Idempotent property). Let us denote the family of $CFFNs$ by $K_p = (\Psi_p, \varpi_p) = (A_p^3 e^{ia_p}, B_p e^{ib_p})$ with $(p = 1, 2, ..., n)$. Suppose that $K_p = K\ \forall p$, then:*

$$CFFEWA_\eta(K_1, K_2, ..., K_n) = K. \tag{11}$$

Proof. As it is known that $K_p = K = (A_p e^{ia}, B_p e^{ib})\forall p$, so equation (7) implies that: □

$$CFFEWA_\eta(K_1, K_2, ..., K_n) = \bigoplus_{p=1}^{n} \eta_p K_p =$$

$$\left(\begin{array}{c} \sqrt[3]{\dfrac{\prod_{p=1}^{n}\left(1+A_p^3\right)^{\eta_p} - \prod_{p=1}^{n}(1-A_p^3)^{\eta_p}}{\prod_{p=1}^{n}\left(1+A_p^3\right)^{\eta_p} + \prod_{p=1}^{n}(1-A_p^3)^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\dfrac{\prod_{p=1}^{n}\left(1+(a_p^3/2\pi)^3\right)^{\eta_p} - \prod_{p=1}^{n}\left(1-(a_p^3/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{n}\left(1+(a_p^3/2\pi)^3\right)^{\eta_p} + \prod_{p=1}^{n}\left(1-(a_p^3/2\pi)^3\right)^{\eta_p}}}\right)}, \\ \dfrac{\prod_{p=1}^{n}\sqrt[3]{2}(B_p)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+(1-B_p^3)\right)^{\eta_p} + \prod_{p=1}^{n}(B_p^3)^{\eta_p}}}\, e^{i2\pi\left(\dfrac{\prod_{p=1}^{n}\sqrt[3]{2}(b_p/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+\left(1-(b_p/2\pi)^3\right)\right)^{\eta_p} + \prod_{p=1}^{n}(b_p/2\pi)^{3\eta_p}}}\right)} \end{array} \right)$$

$$= (Ae^{ia}, Be^{ib}) = K = \left(\begin{array}{c} \sqrt[3]{\dfrac{\left(1+A^3\right)-(1-A^3)}{\left(1+A^3\right)+(1-A^3)}}\, e^{i2\pi\left(\sqrt[3]{\dfrac{\left(1+(a/2\pi)^3\right)-\left(1-(a/2\pi)^3\right)}{\left(1+(a/2\pi)^3\right)+\left(1-(a/2\pi)^3\right)}}\right)}, \\ \dfrac{\sqrt[3]{2}(B)}{\sqrt[3]{\left(1+(1-B^3)\right)+(B^3)}}\, e^{i2\pi\left(\dfrac{\sqrt[3]{2}(b/2\pi)}{\sqrt[3]{\left(1+\left(1-(b/2\pi)^3\right)\right)+(b/2\pi)^3}}\right)} \end{array} \right) \tag{12}$$

So the result is

$$CFFEWA_\eta(K_1, K_2, ..., K_n) = K. \tag{13}$$

Eq. (13) shows that the idempotent property is satisfied.

Theorem 9 *(Boundedness property). Let us denote the family of $CFFNs$ by $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ with $(p = 1, 2, ..., n)$. Suppose that $K_p = K\ \forall p$, then*

$$K^+ = \left(\max_p A_p e^{i \max_p a_p}, \min_p B_p e^{i \min_p b_p} \right) \tag{14}$$

and

$$K^- = \left(\min_p A_p e^{i \min_p a_p}, \max_p B_p e^{i \max_p b_p} \right) \tag{15}$$

then

$$K^- \le CFFEWA_\eta(K_1, K_2, ..., K_n) \le K^+. \tag{16}$$

Eq. (14) and (15) show the maximum and minimum parts of the family of the complex Fermatean fuzzy numbers respectively. While Eq. (16) shows the value of CFFEWA operator.

Theorem 10 *(Monotonicity). Let us consider families of CFFNs by $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ and $K_p^* = (\Psi_p^*, \varpi_p^*) = (A_p^* e^{ia_p*}, B_p^* e^{ib_p*})$ with $(p = 1, 2, ..., n)$ so if $A_p \le A_p^*$, $a_p \le a_p^*$, $B_p \ge B_p^*$ and $b_p \ge b_p^*$ then*

$$CFFEWA_\eta(K_1, K_2, ..., K_n) \le CFFEWA_\eta(K_1^*, K_2^*, ..., K_n^*). \tag{17}$$

Eq. (17) shows the monotonicity of the CFFEWA operator.

3.2. Complex Fermatean fuzzy Einstein ordered weighted average operator

Definition 11. Suppose that a collective set of the CFFNs is denoted by K_p that is $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ with $(p = 1, 2, ..., n)$, where the weight vector is given as $\eta = (\eta_1, \eta_2, ..., \eta_n)^T$, then we can define a complex Fermatean fuzzy Einstein ordered weighted average (CFFEOWA) operator as:

$$CFFEOWA_\eta(K_1, K_2, ..., K_n) = \bigoplus_{p=1}^{n} \eta_p K_{f(p)}, \tag{18}$$

here the permutation of $(1, 2, 3, ..., n)$, is denoted by $f(1), f(2), ..., f(n)$ and $\forall s = 1, 2, 3, ..., n-1$, $Z_{f(p)} \geq Z_{f(p+1)}$, where the η_p denotes the weight vector of K_p that lies in the interval $[0, 1]$ with a satisfactory condition of $\prod_{p=1}^{n} \eta_p = 1$. Eq. (18) shows the CFFEOWA operator.

Theorem 12. *Let us consider a collective set of CFFNs denoted by K_p and defied as $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ with $(p = 1, 2, ..., n)$, a weight vector $\eta = (\eta_1, \eta_2, ..., \eta_n)^T$, the assembled value can be got by utilizing CFFEOWA operator, where the assembled value should also be a CFFN is the following:*

$$CFFEOWA_\eta(K_1, K_2, ..., K_n) = \bigoplus_{p=1}^{n} \eta_p K_{f(p)} =$$

$$\left(\begin{array}{c} \sqrt[3]{\dfrac{\prod_{p=1}^{n}\left(1+A_{f(p)}^3\right)^{\eta_p} - \prod_{p=1}^{n}(1-A_{f(p)}^3)^{\eta_p}}{\prod_{p=1}^{n}\left(1+A_{f(p)}^3\right)^{\eta_p} + \prod_{p=1}^{n}(1-A_{f(p)}^3)^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\dfrac{\prod_{p=1}^{n}\left(1+(a_{f(p)}/2\pi)^3\right)^{\eta_p} - \prod_{p=1}^{n}\left(1-(a_{f(p)}/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{n}\left(1+(A_{f(p)}/2\pi)^3\right)^{\eta_p} + \prod_{p=1}^{n}\left(1-(a_{f(p)}/2\pi)^3\right)^{\eta_p}}}\right)}, \\ \dfrac{\prod_{p=1}^{n}\sqrt[3]{2}(B_{f(p)})^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+(1-B_{f(p)}^3)\right)^{\eta_p} + \prod_{p=1}^{n}(B_{f(p)}^3)^{\eta_p}}}\, e^{i2\pi\left(\dfrac{\prod_{p=1}^{n}\sqrt[3]{2}(b_{f(p)}/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+\left(1-(b_{f(p)}/2\pi)^3\right)\right)^{\eta_p} + \prod_{p=1}^{n}(b_{f(p)}/2\pi)^{3\eta_p}}}\right)} \end{array} \right) \tag{19}$$

here the permutation of $(1, 2, 3, ..., n)$, is denoted by $f(1), f(2), ..., f(n)$ and $\forall s = 1, 2, 3, ..., n-1$, $Z_{f(p)} \geq Z_{f(p+1)}$, where the η_p denotes the weight vector of K_p that lies in the interval $[0, 1]$ with a satisfactory condition of $\prod_{p=1}^{n} \eta_p = 1$. Eq. (19) shows the CFFEOWA operator.

Example 13. Let us consider three CFFNs $K_1 = \left(0.65e^{i2\pi(0.53)}, 0.25e^{i2\pi(0.48)}\right)$, $K_2 = \left(0.66e^{i2\pi(0.73)}, 0.21e^{i2\pi(0.49)}\right)$ and $K_3 = (0.38e^{i2\pi(0.19)}, 0.16e^{i2\pi(0.17)})$. Suppose that the weight vector is $\eta = (0.42, 0.3, 0.28)^T$, then by applying equation (2), the calculated score function is given as:

$$s(K_1) = (0.65)^3 - (0.25)^3 + (0.53)^3 - (0.48)^3 = 0.297285$$

$$s(K_2) = (0.66)^3 - (0.21)^3 + (0.73)^3 - (0.49)^3 = 0.549603$$

$$s(K_3) = (0.38)^3 - (0.16)^3 + (0.19)^3 - (0.17)^3 = 0.052722$$

so we have

$$s(K_2) \succ s(K_1) \succ s(K_3).$$

Now we can write

$$K_{f(2)} = K_2 = \left(0.66e^{i2\pi(0.73)}, 0.21e^{i2\pi(0.49)}\right)$$

$$K_{f(1)} = K_1 = \left(0.65e^{i2\pi(0.53)}, 0.25e^{i2\pi(0.48)}\right)$$

$$K_{f(3)} = K_3 = \left(0.38e^{i2\pi(0.19)}, 0.16e^{i2\pi(0.17)}\right).$$

$$CFFEOWA_\eta(K_1, K_2, K_3) = \bigoplus_{p=1}^{3} \eta_p K_p$$

$$= \left(\sqrt[3]{\frac{\prod_{p=1}^{3}\left(1+3A_p^3\right)^{\eta_p}-\prod_{p=1}^{3}(1-A_p^3)^{\eta_p}}{\prod_{p=1}^{3}\left(1+3A_p^3\right)^{\eta_p}+3\prod_{p=1}^{3}(1-A_p^3)^{\eta p}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\prod_{p=1}^{3}\left(1+3(a_p^3/2\pi)^3\right)^{\eta_p}-\prod_{p=1}^{3}\left(1-(a_p^3/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{3}\left(1+3(a_p^3/2\pi)^3\right)^{\eta_p}+3\prod_{p=1}^{3}\left(1-(a_p^3/2\pi)^3\right)^{\eta_p}}}\right)}, \frac{\prod_{p=1}^{3}\sqrt[3]{2}(B_p)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{3}\left(1+3(1-B_p^3)\right)^{\eta_p}+3\prod_{p=1}^{3}(B_p^3)^{\eta_p}}}\, e^{i2\pi\left(\frac{\prod_{b=1}^{3}\sqrt[3]{2}(b_p/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{3}\left(1+3\left(1-(b_p/2\pi)^3\right)\right)^{\eta_p}+3\prod_{p=1}^{3}(b_p/2\pi)^{3\eta_p}}}\right)} \right)$$

we get

$$= (0.5965357531e^{i2\pi(0.1589770329)}, 0.1969876180e^{i2\pi(0.05665173660)})$$

Theorem 14 *(Idempotent property). Let us consider a collective set of the CFFNs denoted by K_p and defied as: $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ with $(p = 1, 2, ..., n)$, and a weight vector $\eta = (\eta_1, \eta_2, ..., \eta_n)^T$, the assembled value can be got by utilizing the CFFEOWA operator, where the assembled value should also be a CFFN is the following.*

Proof. As it is known that $K_p = K = (A_p e^{ia_p}, B_p e^{ib_p}) \forall p$, then equation (11) becomes: □

$$CFFEOWA_\eta(K_1, K_2, ..., K_n) = \bigoplus_{p=1}^{n}\eta_p K_p =$$

$$\left(\sqrt[3]{\frac{\prod_{p=1}^{n}\left(1+A_{f(p)}^3\right)^{\eta_p}-\prod_{p=1}^{n}(1-A_{f(p)}^3)^{\eta_p}}{\prod_{p=1}^{n}\left(1+A_{f(p)}^3\right)^{\eta_p}+\prod_{p=1}^{n}(1-A_{f(p)}^3)^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\prod_{p=1}^{n}\left(1+(a_{f(p)}/2\pi)^3\right)^{\eta_p}-\prod_{p=1}^{n}\left(1-(a_{f(p)}/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{n}\left(1+(a_{f(p)}/2\pi)^3\right)^{\eta_p}+\prod_{p=1}^{n}\left(1-(a_{f(p)}/2\pi)^3\right)^{\eta_p}}}\right)}, \frac{\prod_{p=1}^{n}\sqrt[3]{2}(B_{f(p)}^3)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+(1-B_{f(p)}^3)\right)^{\eta_p}+\prod_{p=1}^{n}(B_{f(p)}^3)^{\eta_p}}}\, e^{i2\pi\left(\frac{\prod_{p=1}^{n}\sqrt[3]{2}(b_{f(p)}/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+\left(1-(b_{f(p)}/2\pi)^3\right)\right)^{\eta_p}+\prod_{p=1}^{n}(b_{f(p)}/2\pi)^{3\eta_p}}}\right)} \right)$$

$$= \left(\sqrt[3]{\frac{\left(1+A_1^3\right)-(1-A_1^3)}{\left(1+A_1^3\right)+(1-A_1^3)}}\, e^{i2\pi\left(\sqrt[3]{\frac{\left(1+(a/2\pi)^3\right)-\left(1-(a/2\pi)^3\right)}{\left(1+(a/2\pi)^3\right)+\left(1-(a/2\pi)^3\right)}}\right)}, \frac{\sqrt[3]{2}(B)}{\sqrt[3]{(1+(1-B^3))+(B^3)}}\, e^{i2\pi\left(\frac{\sqrt[3]{2}(b/2\pi)}{\sqrt[3]{\left(1+\left(1-(b/2\pi)^3\right)\right)+(b/2\pi)^3}}\right)} \right)$$

$$= (Ae^{ia}, Be^{ib}) = K \tag{20}$$

Eq. (20) shows the idempotent property.

So it is proved that

$$CFFEOWA_\eta(K_1, K_2, ..., K_n) = K.$$

Theorem 15 *(Boundedness property). Let us consider a collective set of CFFNs denoted by K_p and defied as $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ with $(p = 1, 2, ..., n)$. If*

$$K^+ = \left(\max_p A_p e^{i\max_p a_p}, \min_p B_p e^{i\min_p b_p}\right), \tag{21}$$

$$K^- = \left(\min_p A_p e^{i\min_p a_p}, \max_p B_p e^{i\max_p b_p}\right)$$

then

$$K^{-} \leq CFFEOWA_{\eta}(K_1, K_2, ..., K_n) \leq K^{+}. \tag{22}$$

Eq. (21) shows the maximum and minimum values of the CFFEOWA operator, where Eq. (22) shows the boundedness property of the CFFEOWA operator.

Theorem 16 *(Monotonicity). Let us consider two collective sets of CFFNs* $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ *and* $K_p^* = (\Psi_p^*, \varpi_p^*) = (A_p^* e^{ia_p^*}, B_p^* e^{ib_p^*})$ *with* $(p = 1, 2, ..., n)$*, if* $A_p \leq A_p^*$*,* $a_p \leq a_p^*$*,* $B_p \geq B_p^*$ *and* $b_p \geq b_p^*$ *then*

$$CFFEOWA_{\eta}(K_1, K_2, ..., K_n) \leq CFFEOWA_{\eta}(K_1^*, K_2^*, ..., K_n^*) \tag{23}$$

Eq. (23) shows the monotonic property of the CFFEOWA operator.

The application of the CFFEWA operator is to weight the CFFNs while arranging the CFFNs in order, the CFFEOWA operator is used. To get the complex Fermatean fuzzy Einstein average (CFFEA) operator, we collect the above two properties as a single operator.

3.3. Complex Fermatean fuzzy Einstein hybrid aggregation operator

Definition 17. Let us consider a collective set of the CFFNs denoted by K_p and defined as $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ with $(p = 1, 2, ..., n)$, so the CFFEHA operator has the following definition:

$$CFFEHA_{\eta,\vartheta}(K_1, K_2, ..., K_n) = \bigoplus_{p=1}^{n} \eta_p \widetilde{K}_{f(p)}, \tag{24}$$

where the connected weight vector is denoted by $\eta = (\eta_1, \eta_2, ..., \eta_n)^T$ with $\Sigma_{p=1}^{n} \eta_p = 1$ and $\eta_p \in [0,1]$, for $(1, 2, 3, ..., n)$, the permutation is denoted by $f(1), f(2), ..., f(n)$ such as $\widetilde{K}_{f(p)} \geq \widetilde{K}_{f(p+1)}$, $\forall p = (1, 2, 3, ..., n-1)$. Here, $\widetilde{K}_{f(p)} = n_{\vartheta} K_p$ with $(p = 1, 2, 3, ..., n)$, here n denotes a balancing coefficient. Here $\vartheta = (\vartheta_1, \vartheta_2, \vartheta_3, ..., \vartheta_n)$ is the weight vector $\ni \Sigma_{p=1}^{n} \vartheta_p = 1$ and $\vartheta_p \in [0,1]$. Eq. (24) represents the CFFEHA operator.

Theorem 18. *Let us consider a collective set of the CFFNs denoted by* K_p *and defined as* $K_p = (\Psi_p, \varpi_p) = (A_p e^{ia_p}, B_p e^{ib_p})$ *with* $(p = 1, 2, ..., n)$*, making the use of* $CFFEHA$ *operator, we get the following precise value:*

$$\begin{aligned} CFFEHA_{\eta,\vartheta}(K_1, K_2, ..., K_n) &= \bigoplus_{p=1}^{n} \eta_p \widetilde{K}_{f(p)} \\ &= \left(\begin{array}{c} \sqrt[3]{\dfrac{\prod_{p=1}^{n}\left(1+\widetilde{A}_{f(p)}^{3}\right)^{\eta_p} - \prod_{p=1}^{n}(1-\widetilde{A}_{f(p)}^{3})^{\eta_p}}{\prod_{p=1}^{n}\left(1+\widetilde{A}_{f(p)}^{3}\right)^{\eta_p} + \prod_{p=1}^{n}(1-\widetilde{A}_{f(p)}^{3})^{\eta_p}}}\, e^{i2\pi\left(\sqrt[3]{\dfrac{\prod_{p=1}^{n}\left(1+(\widetilde{a}_{f(p)}/2\pi)^3\right)^{\eta_p} - \prod_{p=1}^{n}\left(1-(\widetilde{a}_{f(p)}/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{n}\left(1+\widetilde{a}_{f(p)}/2\pi)^3\right)^{\eta_p} + \prod_{p=1}^{n}\left(1-\widetilde{a}_{f(p)}/2\pi)^3\right)^{\eta_p}}}\right)}, \\ \dfrac{\sqrt[3]{2}\prod_{p=1}^{n}(\widetilde{B}_{f(p)})^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+(1-\widetilde{B}_{f(p)}^{3})\right)^{\eta_p} + \prod_{p=1}^{n}(\widetilde{B}_{f(p)}^{3})^{\eta_p}}}\, e^{i2\pi\left(\dfrac{\sqrt[3]{2}\prod_{p=1}^{n}(\widetilde{b}_{f(p)}/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+\left(1-(\widetilde{b}_{f(p)}/2\pi)^3\right)\right)^{\eta_p} + \prod_{p=1}^{n}(\widetilde{b}_{f(p)}/2\pi)^{3\eta_p}}}\right)} \end{array} \right) \end{aligned} \tag{25}$$

where the connected weight vector is denoted by $\eta = (\eta_1, \eta_2, ..., \eta_n)^T$ *with* $\Sigma_{p=1}^{n} \eta_p = 1$ *and* $\eta_p \in [0,1]$*, for* $(1, 2, 3, ..., n)$ *the permutation is denoted by* $f(1), f(2), ..., f(n)$*, such as,* $\widetilde{K}_{f(p)} \geq \widetilde{K}_{f(p+1)}$*,* $\forall p = (1, 2, 3, ..., n-1)$*. Here,* $\widetilde{K}_{f(p)} = n_{\vartheta} K_p$ *with* $(p = 1, 2, 3, ..., n)$*, here* n *denotes a balancing coefficient. Here* $\vartheta = (\vartheta_1, \vartheta_2, \vartheta_3, ..., \vartheta_n)$ *is the weight vector* $\ni \Sigma_{p=1}^{n} \vartheta_p = 1$ *and* $\vartheta_p \in [0,1]$*. Where the output is always a* $CFFN$*. Eq. (25) shows the value of the CFFEHA operator.*

Proof. Here we use some special cases. □

Case (1). Suppose that $\eta = (1/n, 1/n, ..., 1/n)^T$, so the CFFEHA operator converts to the CFFEWA operator.

Case (2). Suppose that $\vartheta = (1/n, 1/n, ..., 1/n)$, so the CFFEHA operator converts to the CFFEOWA operator.

Case (3). Suppose that $a_p = b_p = 0\ \forall p$, so the CFFEHA operator is given below:

$$CFFEHA_{\eta,\vartheta}(K_1, K_2, ..., K_n) = \sqrt[3]{\frac{\prod_{p=1}^{n}\left(1+\widetilde{A}^3_{f(p)}\right)^{\eta_p} - \prod_{p=1}^{n}(1-\widetilde{A}^3_{f(p)})^{\eta_p}}{\prod_{p=1}^{n}\left(1+\widetilde{A}^3_{f(p)}\right)^{\eta_p} + \prod_{p=1}^{n}(1-\widetilde{A}^3_{f(p)})^{\eta_p}}}, \frac{\sqrt[3]{2}\prod_{p=1}^{n}(\widetilde{B}_{f(p)})^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{n}\left(1+(1-\widetilde{B}^3_{f(p)})\right)^{\eta_p} + \prod_{p=1}^{n}(\widetilde{B}^3_{f(p)})^{\eta_p}}} \tag{26}$$

Eq. (26) shows the Fermatean fuzzy Einstein Averaging Aggregation operator.

Example 19. Let us consider three CFFNs $K_1 = \left(0.79e^{i2\pi(0.83)}, 0.81e^{i2\pi(0.77)}\right)$, $K_2 = \left(0.82e^{i2\pi(0.73)},\ 0.73e^{i2\pi(0.90)}\right)$, and $K_3 = (0.74e^{i2\pi(0.86)}, 0.86e^{i2\pi(0.78)})$. Suppose that the weight vector is $\eta = (0.35, 0.2, 0.45)^T$, so we get:

$$\widetilde{K}_1 = \left(\begin{array}{c} \sqrt[3]{\frac{\left(1+A_1^3\right)^{n\vartheta_1}-(1-A_1^3)^{n\vartheta_1}}{\left(1+A_1^3\right)^{n\vartheta_1}+(1-A_1^3)^{n\vartheta_1}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\left(1+(a_1^3/2\pi)^3\right)^{n\vartheta_1}-\left(1-(a_1^3/2\pi)^3\right)^{n\vartheta_1}}{\left(1+(a_1^3/2\pi)^3\right)^{n\vartheta_1}+\left(1-(a_1^3/2\pi)^3\right)^{n\vartheta_1}}}\right)}, \\ \frac{\sqrt[3]{2}(B_1)^{n\vartheta_1}}{\sqrt[3]{\left(1+(1-B_1^3)\right)^{n\vartheta_1}+(B_1^3)^{n\vartheta_1}}}\, e^{i2\pi\left(\frac{\sqrt[3]{2}(b_1/2\pi)^{n\vartheta_1}}{\sqrt[3]{\left(1+\left(1-(b_1/2\pi)^3\right)\right)^{n\vartheta_1}+(b_1/2\pi)^{3n\vartheta_1}}}\right)} \end{array}\right)$$

$$= 0.3519354072e^{i2\pi(0.1039863070)}, 0.9563545178e^{i2\pi(0.5632075528)}$$

$$\widetilde{K}_2 = \left(\begin{array}{c} \sqrt[3]{\frac{\left(1+A_2^3\right)^{n\vartheta_2}-(1-A_2^3)^{n\vartheta_2}}{\left(1+A_2^3\right)^{n\vartheta_2}+(1-A_2^3)^{n\vartheta_2}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\left(1+(a_2/2\pi)^3\right)^{n\vartheta_2}-\left(1-(a_2/2\pi)^3\right)^{n\vartheta_2}}{\left(1+(a_2/2\pi)^3\right)^{n\vartheta_2}+\left(1-(a_2/2\pi)^3\right)^{n\vartheta_2}}}\right)}, \\ \frac{\sqrt[3]{2}(B_2)^{n\vartheta_2}}{\sqrt[3]{\left(1+(1-B_2^3)\right)^{n\vartheta_2}+(B_2^3)^{n\vartheta_2}}}\, e^{i2\pi\left(\frac{\sqrt[3]{2}(b_2/2\pi)^{n\vartheta_2}}{\sqrt[3]{\left(1+\left(1-(b_2/2\pi)^3\right)\right)^{n\vartheta_2}+(b_2/2\pi)^{3n\vartheta_2}}}\right)} \end{array}\right)$$

$$= 0.7621121775e^{i2\pi(0.1206210029)}, 0.8227024826e^{i2\pi(0.4347093067)}$$

$$\widetilde{K}_3 = \left(\begin{array}{c} \sqrt[3]{\frac{\left(1+A_3^3\right)^{n\vartheta_3}-(1-A_3^3)^{n\vartheta_3}}{\left(1+A_3^3\right)^{n\vartheta_3}+(1-A_3^3)^{n\vartheta_3}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\left(1+(a_3/2\pi)^3\right)^{n\vartheta_3}-\left(1-(a_3/2\pi)^3\right)^{n\vartheta_3}}{\left(1+(a_3/2\pi)^3\right)^{n\vartheta_3}+\left(1-(a_3/2\pi)^3\right)^{n\vartheta_3}}}\right)}, \\ \frac{\sqrt[3]{2}(B_3)^{n\vartheta_3}}{\sqrt[3]{\left(1+(1-B_3^3)\right)^{n\vartheta_3}+(B_3^3)^{n\vartheta_3}}}\, e^{i2\pi\left(\frac{\sqrt[3]{2}(b_3/2\pi)^{n\vartheta_3}}{\sqrt[3]{\left(1+\left(1-(b_3/2\pi)^3\right)\right)^{n\vartheta_3}+(b_3/2\pi)^{3n\varepsilon_3}}}\right)} \end{array}\right)$$

$$= 0.09928770717e^{i2\pi(0.06812658535)}, 0.9496746077e^{i2\pi(0.8186759338)}$$

Where the score grades are determined below:

$$s(\widetilde{K}_1) = (0.35)^3 - (0.95)^3 + (0.10)^3 - (0.56)^3 = -0.989116$$

$$s(\widetilde{K}_2) = (0.76)^3 - (0.82)^3 + (0.12)^3 - (0.43)^3 = -0.190171$$

$$s(\widetilde{K}_3) = (0.99)^3 - (0.94)^3 + (0.06)^3 - (0.81)^3 = -0.391510$$

so we have,

$$s(\widetilde{K}_2) \succ s(\widetilde{K}_3) \succ s(\widetilde{K}_1).$$

Now we can write

$$\widetilde{K}_{f(2)} = \widetilde{K}_2 = \left(0.76e^{i2\pi(0.12)}, 0.82e^{i2\pi(0.43)}\right)$$

$$\widetilde{K}_{f(3)} = \widetilde{K}_3 = \left(0.99e^{i2\pi(0.06)}, 0.94e^{i2\pi(0.81)}\right)$$

$$\widetilde{K}_{f(1)} = \widetilde{K}_1 = \left(0.35e^{i2\pi(0.10)}, 0.95e^{i2\pi(0.56)}\right)$$

Now put the values of $\widetilde{K}_1, \widetilde{K}_2$ and $\widetilde{K}_3$ in the given equation

$$CFFEOWA_\eta(\widetilde{K}_1, \widetilde{K}_2, \widetilde{K}_3) = \bigoplus_{p=1}^{3} \eta_p \widetilde{K}_p$$

$$= \left(\begin{array}{c} \sqrt[3]{\dfrac{\prod_{p=1}^{3}\left(1+\widetilde{A}^3_{f(p)}\right)^{\eta_p} - \prod_{p=1}^{3}(1-\widetilde{A}^3_{f(p)})^{\eta_p}}{\prod_{p=1}^{3}\left(1+\widetilde{A}^3_{f(p)}\right)^{\eta_p} + \prod_{p=1}^{3}(1-\widetilde{A}^3_{f(p)})^{\eta_p}}} e^{i2\pi\left(\sqrt[3]{\dfrac{\prod_{p=1}^{3}\left(1+(\widetilde{a}_{f(p)}/2\pi)^3\right)^{\eta_p} - \prod_{p=1}^{3}\left(1-(\widetilde{a}_{f(p)}/2\pi)^3\right)^{\eta_p}}{\prod_{p=1}^{3}\left(1+(\widetilde{a}_{f(p)}/2\pi)^3\right)^{\eta_p} + \prod_{p=1}^{3}\left(1-(\widetilde{a}_{f(p)}/2\pi)^3\right)^{\eta_p}}}\right)}, \\ \dfrac{\prod_{p=1}^{3}\sqrt[3]{2}(\widetilde{B}_{f(p)})^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{3}\left(1+(1-\widetilde{B}^3_{f(p)})\right)^{\eta_p} + \prod_{p=1}^{3}(\widetilde{B}^3_{f(p)})^{\eta_p}}} e^{i2\pi\left(\dfrac{\prod_{p=1}^{3}\sqrt[3]{2}(\widetilde{b}_{f(p)}/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{p=1}^{3}\left(1+\left(1-(\widetilde{b}_{f(p)}/2\pi)^3\right)\right)^{\eta_p} + \prod_{p=1}^{3}(\widetilde{b}_{f(p)}/2\pi)^{3\eta_p}}}\right)} \end{array} \right)$$

$$= 0.9151424793e^{i2\pi(0.02222526438)}, 0.9339538724e^{i2\pi(0.1044026993)}$$

4. Multi-attribute group decision-making model

In order to find the best choice, here we present the MAGDM model to utilize the CFFEWA operator and the CFFEOWA operator. The following form is the general form of a MAGDM model.

4.1. For complex Fermatean fuzzy Einstein hybrid aggregation operator

Let us consider the set of alternatives by $G = \{G_1, G_2, G_3, ..., G_g\}$. If $l_1, l_2, l_3, ..., l_c$ represent the decision standard numbers. The aim is to choose the optimal choice from the set of alternatives on the basis of decision standard numbers. A policymaker's basic need for the MAGDM problem is to examine it. If $\eta = (\eta_1, \eta_2, ..., \eta_w)^T$ is the weight vector then it is the normalized weight vector of the decision standard. According to the performance, the specialist judges the power of the alternative G_g respective to the given criteria l_c to allocate it as a CFFN that can be represented as $D_{qr} = (\Psi_{qr}, \varpi_{qr}) = (A_{qr}e^{ia_{qr}}, B_{qr}e^{ib_{qr}})$. A complex Fermatean fuzzy decision matrix (CFFDM) φ is organized in the given form.

$$\varphi = \begin{pmatrix} \Psi_{11}, \varpi_{11} & \Psi_{12}, \varpi_{12} & \dots & \Psi_{1c}, \varpi_{1c} \\ \Psi_{21}, \varpi_{21} & \Psi_{22}, \varpi_{22} & \dots & \Psi_{2c}, \varpi_{2c} \\ \vdots & \vdots & \vdots & \vdots \\ \Psi_{g1}, \varpi_{g1} & \Psi_{g2}, \varpi_{g2} & \dots & \Psi_{gc}, \varpi_{gc} \end{pmatrix}$$

In order to solve the MAGDM problem, the given procedure is adopted:

Step (1): Arrange the CFFNs allocated by policy maker specialists in the form of a matrix called a CFFDM.

Step (2): Perceive the value of $D_q = (\Psi_q, \varpi_q) = (A_q e^{ia_q}, B_q e^{ib_q})$ of each choice O_q by applying the given operator.

Put $O = \{O_1, O_2, O_3, ..., O_j\}$ represent alternatives set with criteria numbers $l_1, l_2, l_3, ..., l_c$ and the weight vector $\eta = (\eta_1, \eta_2, ..., \eta_w)^T$.

$$O_q = CFFEWA_\eta(V_{q1}, V_{q2}, ..., V_{qw}) = \bigoplus_{r=1}^{w} \eta_r K_{qr} =$$

$$\left(\begin{array}{l}\sqrt[3]{\frac{\prod_{r=1}^{w}\left(1+A_{qr}^{3}\right)^{\eta_p}-\prod_{r=1}^{w}(1-A_{qr}^{3})^{\eta_p}}{\prod_{r=1}^{w}\left(1+A_{qr}^{3}\right)^{\eta_p}+\prod_{r=1}^{w}(1-A_{qr}^{3})^{\eta_p}}}\,e^{i2\pi\left(\sqrt[3]{\frac{\prod_{r=1}^{w}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_p}-\prod_{r=1}^{w}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_p}}{\prod_{r=1}^{w}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_p}+\prod_{r=1}^{w}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_p}}}\right)},\\ \frac{\prod_{r=1}^{w}\sqrt[3]{2}(B_{qr})^{\eta_p}}{\sqrt[3]{\prod_{r=1}^{w}\left(1+(1-B_{qr}^{3})\right)^{\eta_p}+\prod_{r=1}^{w}(B_{qr}^{3})^{\eta_p}}}\,e^{i2\pi\left(\frac{\prod_{r=1}^{w}\sqrt[3]{2}(b_{qr}/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{r=1}^{w}\left(1+\left(1-(b_{qr}/2\pi)^3\right)\right)^{\eta_p}+\prod_{r=1}^{w}(b_{qr}/2\pi)^{3\eta_p}}}\right)}\end{array}\right)$$

Where for CFFEOWA operator, the formula is given below:

$$O_q = CFFEOWA_\eta(V_{q1}, V_{q2}, \dots, V_{qw}) = \bigoplus_{r=1}^{w}\eta_p K_{qf(r)} =$$

$$\left(\begin{array}{l}\sqrt[3]{\frac{\prod_{r=1}^{w}\left(1+A_{qf(r)}^{3}\right)^{\eta_p}-\prod_{r=1}^{w}(1-A_{qf(r)}^{3})^{\eta_p}}{\prod_{r=1}^{w}\left(1+A_{qf(r)}^{3}\right)^{\eta_p}+\prod_{r=1}^{w}(1-A_{qf(r)}^{3})^{\eta_p}}}\,e^{i2\pi\left(\sqrt[3]{\frac{\prod_{r=1}^{w}\left(1+(a_{qf(r)}/2\pi)^3\right)^{\eta_p}-\prod_{r=1}^{w}\left(1-(a_{qf(r)}/2\pi)^3\right)^{\eta_p}}{\prod_{r=1}^{w}\left(1+(a_{qf(r)}/2\pi)^3\right)^{\eta_p}+\prod_{r=1}^{w}\left(1-(a_{qf(r)}/2\pi)^3\right)^{\eta_p}}}\right)},\\ \frac{\prod_{r=1}^{w}\sqrt[3]{2}(B_{qf(r)})^{\eta_p}}{\sqrt[3]{\prod_{r=1}^{w}\left(1+(1-B_{qf(r)}^{3})\right)^{\eta_p}+\prod_{r=1}^{w}(B_{qf(r)}^{3})^{\eta_p}}}\,e^{i2\pi\left(\frac{\prod_{r=1}^{w}\sqrt[3]{2}(b_{qf(r)}/2\pi)^{\eta_p}}{\sqrt[3]{\prod_{r=1}^{w}\left(1+\left(1-(b_{qf(r)}/2\pi)^3\right)\right)^{\eta_p}+\prod_{r=1}^{w}(b_{qf(r)}/2\pi)^{3\eta_p}}}\right)}\end{array}\right)$$

Here $(1, 2, 3, \dots, w)$ has the permutation $f(1), f(2), \dots, f(w)$ where $V_{qf(r)} \geq V_{qf(r+1)}$ for all $r = (1, 2, 3, \dots, w-1)$.

Step (3): The formula for score functions $s(O_q)$ of predilection value for each option O_q is $s(O_q) = A_q^3 - B_q^3 + (\frac{a_q}{2\pi})^3 - (\frac{b_q}{2\pi})^3$.

Step (4): If two options have the same score functions, then apply the formula of accuracy degrees, that is: $f(O_q) = A_q^3 + B_q^3 + (\frac{a_q}{2\pi})^3 + (\frac{b_q}{2\pi})^3$.

Result: The solution to a MAGDM problem is the alternative of a higher score function among all the alternatives.

4.2. For complex Fermatean fuzzy Einstein TOPSIS method

In this section, we aim to develop a new idea named CFFE-TOPSIS to comprise the CFFE data by solving a MAGDM problem. The proposed concept of the CFFE-TOPSIS method is a good option to find the alternative nearer to the positive ideal solution PIS and more away from the negative ideal solution NIS.

Let us consider the set of alternatives by $G = \{G_1, G_2, G_3, \dots, G_g\}$. If $l_1, l_2, l_3, \dots, l_c$ represent the decision criteria numbers. The aim is to choose the optimal choice from the set of alternatives on the basis of decision standard numbers. A policymaker is the basic need of the multi-attribute decision-making matrix MADM to examine it. Here the set of the policy makers is denoted by $\check{E} = \{\check{E}_1, \check{E}_2, \check{E}_3, \dots, \check{E}_n\}$. If $\eta = (\eta_1, \eta_2, \dots, \eta_x)^T$ is the weight vector, where the weight vector must satisfy the condition $\sum_{i=1}^{w}\eta_i = 1$. According to the performance, the specialist judges the power of the alternative G_g respective to the given criteria l_c to allocate it as a CFFEN that can be represented as $\mathbb{U}^{(x)} = (u_{qr}^{(x)})_{g\times c}$, where

$$u_{qr}^{(x)} = (\Psi_{qr}^{(x)}, \varpi_{qr}^{(x)}, \tau_{qr}^{(x)}) = (A_{qr}^{(x)}e^{ia_{qr}^{(x)}}, B_{qr}^{(x)}e^{ib_{qr}^{(x)}}, Z_{qr}^{(x)}e^{iz_{qr}^{(x)}}).$$

The evaluation of hesitancy is

$$\tau_{qr}^{(x)} = \sqrt[3]{1-(A_{qr}^{(x)})^3-(B_{qr}^{(x)})^3}\,e^{i2\pi\left(\sqrt[3]{1-(\frac{a_{qr}^{(x)}}{2\pi})^3-(\frac{b_{qr}^{(x)}}{2\pi})^3}\right)}$$

The CFFEDM of the policymakers $\check{E}_x$, denoted by $\mathbb{U}^{(x)}$, is given as:

$$\mathbb{U}^{(x)} = \begin{pmatrix} \Psi_{11}^{(x)}, \varpi_{11}^{(x)}, \tau_{11}^{(x)} & \Psi_{12}^{(x)}, \varpi_{12}^{(x)}, \tau_{12}^{(x)} & \dots & \Psi_{1c}^{(x)}, \varpi_{1c}^{(x)}, \tau_{1c}^{(x)} \\ \Psi_{21}^{(x)}, \varpi_{21}^{(x)}, \tau_{21}^{(x)} & \Psi_{22}^{(x)}, \varpi_{22}^{(x)}, \tau_{22}^{(x)} & \dots & \Psi_{2c}^{(x)}, \varpi_{2c}^{(x)}, \tau_{2c}^{(x)} \\ \vdots & \vdots & \vdots & \vdots \\ \Psi_{g1}^{(x)}, \varpi_{g1}^{(x)}, \tau_{g1}^{(x)} & \Psi_{g2}^{(x)}, \varpi_{g2}^{(x)}, \tau_{g2}^{(x)} & \dots & \Psi_{gc}^{(x)}, \varpi_{gc}^{(x)}, \tau_{gc}^{(x)} \end{pmatrix}$$

Our aim is to choose the most appropriate option. So we develop a procedure for the CFFE-TOPSIS method.

Step (1): compute the weights of the DMs.

The judgment value of policymakers is to be assigned in the form of linguistic terms and should be a CFFNs. If $U_x = (\Psi_x, \varpi_x, \tau_x) = (A_x e^{ia_x}, B_x e^{ib_x}, Z_x e^{iz_x})$ denotes the integrity of decision maker, then the weight of xth decision makers can be computed as:

$$\eta_x = \frac{\left(A_x + Z_x\left(\frac{A_x}{A_x+B_x}\right)\right) + \frac{a_x}{2\pi} + \frac{z_x}{2\pi}\left(\frac{\frac{a_x}{2\pi}}{\frac{a_x}{2\pi}+\frac{b_x}{2\pi}}\right)}{\sum_{x=1}^{n}\left(A_x + Z_x\left(\frac{A_x}{A_x+B_x}\right)\right) + \frac{a_x}{2\pi} + \frac{z_x}{2\pi}\left(\frac{\frac{a_x}{2\pi}}{\frac{a_x}{2\pi}+\frac{b_x}{2\pi}}\right)} \tag{27}$$

where the condition $\sum_{x=1}^{n} \eta_x = 1$ must be satisfied by $\eta_x \in [0,1]$. Eq. (27) shows the weight of expert.

Step (2): As $\mathbb{U}^{(x)} = (u_{qr}^{(x)})_{g\times c}$ represents the matrix of CFFDM model of each policymaker $\check{E}_x$ with weight η_x. To tabulate the matrix, every alternative has an aggregated value by all the experts regarding specific criteria. This matrix is called a complex Fermatean fuzzy Einstein decision matrix denoted by $\mathbb{U} = (u_{qr}^{(x)})_{g\times c}$. Now using the CFFEWA operator

$$u_{qr} = CFFEWA_\eta(u_{qr}^{(1)}, u_{qr}^{(2)}, \dots, u_r^{(n)})$$

$$u_{qr} = \eta_1 u_{qr}^{(1)} \oplus \eta_2 u_{qr}^{(2)} \oplus \dots \oplus \eta_n u_{qr}^{(n)}$$

$$u_{qr} = \left(\begin{array}{l} \sqrt[3]{\frac{\prod_{x=1}^{n}\left(1+A_{qr}^3\right)^{\eta_x} - \prod_{x=1}^{n}(1-A_{qr}^3)^{\eta_x}}{\prod_{x=1}^{n}\left(1+A_{qr}^3\right)^{\eta_x} + \prod_{x=1}^{n}(1-A_{qr}^3)^{\eta_x}}}\, e^{i2\pi\left(\sqrt[3]{\frac{\prod_{x=1}^{n}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_x} - \prod_{x=1}^{n}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_x}}{\prod_{x=1}^{n}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_x} + \prod_{x=1}^{n}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_x}}}\right)}, \\ \frac{\prod_{x=1}^{n}\sqrt[3]{2}(B_{qr})^{\eta_x}}{\sqrt[3]{\prod_{x=1}^{n}\left(1+(1-B_{qr}^3)\right)^{\eta_x} + \prod_{x=1}^{n}(B_{qr}^3)^{\eta_x}}}\, e^{i2\pi\left(\frac{\prod_{x=1}^{n}\sqrt[3]{2}(b_{qr}/2\pi)^{\eta_x}}{\sqrt[3]{\prod_{x=1}^{n}\left(1+\left(1-(b_{qr}/2\pi)^3\right)\right)^{\eta_x} + \prod_{x=1}^{n}(b_{qr}/2\pi)^{3\eta_x}}}\right)} \end{array} \right) \tag{28}$$

Eq. (28) shows the value of aggregated CFFEWA operator.

$$\tau_{G_q}(l_r) = \left(\begin{array}{l} \sqrt[3]{1 - \frac{\prod_{x=1}^{n}\left(1+A_{qr}^3\right)^{\eta_x} - \prod_{x=1}^{n}(1-A_{qr}^3)^{\eta_x}}{\prod_{x=1}^{n}\left(1+A_{qr}^3\right)^{\eta_x} + \prod_{x=1}^{n}(1-A_{qr}^3)^{\eta_x}} - \frac{\left(\prod_{x=1}^{n}2(B_{qr})^{\eta_x}\right)^3}{\prod_{x=1}^{n}\left(1+(1-B_{qr}^3)\right)^{\eta_x} + \prod_{x=1}^{n}(B_{qr}^3)^{\eta_x}}} \\ e^{i2\pi\left(\sqrt[3]{1-\frac{\prod_{x=1}^{n}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_x} - \prod_{x=1}^{n}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_x}}{\prod_{x=1}^{n}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_x} + \prod_{x=1}^{n}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_x}} - \frac{\left(\prod_{x=1}^{n}2(b_{qr}/2\pi)^{\eta_x}\right)^3}{\prod_{x=1}^{n}\left(1+\left(1-(b_{qr}/2\pi)^3\right)\right)^{\eta_x} + \prod_{x=1}^{n}(b_{qr}/2\pi)^{3\eta_x}}}\right)}, \end{array} \right) \tag{29}$$

where $u_{qr} = (\Psi_{G_q}(l_r), \varpi_{G_q}(l_r), \tau_{G_q}(l_r)) = (A_{G_q}(l_r)e^{ia_{G_q}(l_r)}, B_{G_q}(l_r)e^{ib_{G_q}(l_r)}, Z_{G_q}(l_r)e^{iz_{G_q}(l_r)})$, $q = 1,2,3,\dots,g$, and $r = 1,2,3,\dots,c$. Eq. (29) shows degree of indeterminacy.

So, $\mathbb{U} = (u_{qr})_{g\times c}$ is tabulated as:

$$\mathbb{U} = \begin{pmatrix} (\Psi_{G_1}(l_1), \varpi_{G_1}(l_1), \tau_{G_1}(l_1)) & (\Psi_{G_1}(l_2), \varpi_{G_1}(l_2), \tau_{G_1}(l_2)) & \dots & (\Psi_{G_1}(l_c), \varpi_{G_1}(l_c), \tau_{G_1}(l_c)) \\ (\Psi_{G_2}(l_1), \varpi_{G_2}(l_1), \tau_{G_2}(l_1)) & (\Psi_{G_2}(l_2), \varpi_{G_2}(l_2), \tau_{G_2}(l_2)) & \dots & (\Psi_{G_2}(l_c), \varpi_{G_2}(l_c), \tau_{G_2}(l_c)) \\ \vdots & \vdots & \vdots & \vdots \\ (\Psi_{G_g}(l_1), \varpi_{G_g}(l_1), \tau_{G_g}(l_1)) & (\Psi_{G_g}(l_2), \varpi_{G_g}(l_2), \tau_{G_g}(l_2)) & \dots & (\Psi_{G_g}(l_c), \varpi_{G_g}(l_c), \tau_{G_g}(l_c)) \end{pmatrix}$$

Step (3): The criteria have different values according to their importance. Each decision-making expert $\check{E}_x$ evaluates all the criteria and allocates weight in the form of complex Fermatean fuzzy Einstein number (CFFEN) to each criterion. Suppose that $S_r^{(x)} =$

$(\Psi_r^{(x)}, \varpi_r^{(x)}, \tau_r^{(x)}) = (A_r^{(x)} e^{i a_r^{(x)}}, B_r^{(x)} e^{i b_r^{(x)}}, Z_r^{(x)} e^{i z_r^{(x)}})$ is the CFFE weight allocated to the criteria l_r by the decision maker $\breve{E}_x$. The discrete point of view of the decision makers are collected in the form of matrix called weight matrix, denoted by S using CFFEWA operator. $S = (s_1, s_2, ..., s_r)^T$ where $r = 1, 2, 3, ..., c$.

$$s_r = CFFEWA_\eta(s_r^{(1)}, s_r^{(2)}, ..., s_r^{(n)})$$

$$s_r = \eta_1 s_r^{(1)}, \oplus \eta_2 s_r^{(2)} \oplus ... \oplus \eta_n s_r^{(n)}$$

$$s_r = \left(\begin{array}{c} \sqrt[3]{\dfrac{\prod_{x=1}^{n}\left(1+A_{qr}^3\right)^{\eta_x} - \prod_{x=1}^{n}(1-A_{qr}^3)^{\eta_x}}{\prod_{x=1}^{n}\left(1+A_{qr}^3\right)^{\eta_x} + \prod_{x=1}^{n}(1-A_{qr}^3)^{\eta_x}}}\, e^{i2\pi\left(\sqrt[3]{\dfrac{\prod_{x=1}^{n}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_x} - \prod_{x=1}^{n}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_x}}{\prod_{x=1}^{n}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_x} + \prod_{x=1}^{n}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_x}}}\right)}, \\ \dfrac{\prod_{x=1}^{n}\sqrt[3]{2}(B_{qr})^{\eta_x}}{\sqrt[3]{\prod_{x=1}^{n}\left(1+(1-B_{qr}^3)\right)^{\eta_x} + \prod_{x=1}^{n}(B_{qr}^3)^{\eta_x}}}\, e^{i2\pi\left(\dfrac{\prod_{x=1}^{n}\sqrt[3]{2}(b_{qr}/2\pi)^{\eta_x}}{\sqrt[3]{\prod_{x=1}^{n}\left(1+\left(1-(b_{qr}/2\pi)^3\right)\right)^{\eta_x} + \prod_{x=1}^{n}(b_{qr}/2\pi)^{3\eta_x}}}\right)} \end{array} \right) \tag{30}$$

Eq. (30) shows the weight allocated to every criteria by the decision makers.

$$\tau_s(l_r) = \left(\begin{array}{c} \sqrt[3]{1 - \dfrac{\prod_{x=1}^{n}\left(1+A_{qr}^3\right)^{\eta_x} - \prod_{x=1}^{n}(1-A_{qr}^3)^{\eta_x}}{\prod_{x=1}^{n}\left(1+A_{qr}^3\right)^{\eta_x} + \prod_{x=1}^{n}(1-A_{qr}^3)^{\eta_x}} - \dfrac{\left(\prod_{x=1}^{n}2(B_{qr})^{\eta_x}\right)^3}{\prod_{x=1}^{n}\left(1+(1-B_{qr}^3)\right)^{\eta_x} + \prod_{x=1}^{n}(B_{qr}^3)^{\eta_x}}} \\ e^{i2\pi\left(\sqrt[3]{1-\dfrac{\prod_{x=1}^{n}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_x} - \prod_{x=1}^{n}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_x}}{\prod_{x=1}^{n}\left(1+(a_{qr}/2\pi)^3\right)^{\eta_x} + \prod_{x=1}^{n}\left(1-(a_{qr}/2\pi)^3\right)^{\eta_x}} - \dfrac{\left(\prod_{x=1}^{n}2(b_{qr}/2\pi)^{\eta_x}\right)^3}{\prod_{x=1}^{n}\left(1+\left(1-(b_{qr}/2\pi)^3\right)\right)^{\eta_x} + \prod_{x=1}^{n}(b_{qr}/2\pi)^{3\eta_x}}}\right)} \end{array} \right), \tag{31}$$

here $S = (s_1, s_2, ..., s_c)^T$ and $s_r = (\Psi_s(l_r), \varpi_s(l_r), \tau_s(l_r)) = (A_s(l_r)e^{i a_s(l_r)},\ B_s(l_r)e^{i b_s(l_r)},\ Z_s(l_r)e^{i z_s(l_r)})$ where $r = 1, 2, 3, ..., c$. Eq. (31) shows the degree of indeterminacy for each expert.

Step (4): By using the aggregated complex Fermatean fuzzy Einstein decision matrix (ACFFEDM) and weighted matrix S, the aggregated weighted complex Fermatean fuzzy Einstein decision matrix (AWCFFEDM) $\mathbb{U}^* = (u_{qr}^*)_{g\times c}$ can be obtained. Here $u_{qr}^* = u_{qr} \otimes s_r$.

$$u_{qr}^* = A_{G_q}(l_r)A_s(l_r)e^{i2\pi\left(\frac{a_{G_q}(l_r)}{2\pi}\right)\left(\frac{a_s(l_r)}{2\pi}\right)}, \sqrt[3]{B_{G_q}^3(l_r) + B_s^3(l_r) - B_{G_q}^3(l_r)B_s^3(l_r)}\, e^{i2\pi\left(\sqrt[3]{\left(\frac{b_{G_q}(l_r)}{2\pi}\right)^3 + \left(\frac{b_s(l_r)}{2\pi}\right)^3 - \left(\frac{b_{G_q}(l_r)}{2\pi}\right)^3\left(\frac{b_s(l_r)}{2\pi}\right)^3}\right)} \tag{32}$$

Eq. (31) shows the values of aggregated complex Fermatean fuzzy Einstein decision matrix.

The following matrix is the construction of AWCFFEDM:

$$\mathbb{U}^* = \begin{pmatrix} (\Psi_{G_1}^*(l_1), \varpi_{G_1}^*(l_1), \tau_{G_1}^*(l_1)) & (\Psi_{G_1}^*(l_2), \varpi_{G_1}^*(l_2), \tau_{G_1}^*(l_2)) & \dots & (\Psi_{G_1}^*(l_c), \varpi_{G_1}^*(l_c), \tau_{G_1}^*(l_c)) \\ (\Psi_{G_2}^*(l_1), \varpi_{G_2}^*(l_1), \tau_{G_2}^*(l_1)) & (\Psi_{G_2}^*(l_2), \varpi_{G_2}^*(l_2), \tau_{G_2}^*(l_2)) & \dots & (\Psi_{G_2}^*(l_c), \varpi_{G_2}^*(l_c), \tau_{G_2}^*(l_c)) \\ \vdots & \vdots & \vdots & \vdots \\ (\Psi_{G_g}^*(l_1), \varpi_{G_g}^*(l_1), \tau_{G_g}^*(l_1)) & (\Psi_{G_g}^*(l_2), \varpi_{G_g}^*(l_2), \tau_{G_g}^*(l_2)) & \dots & (\Psi_{G_g}^*(l_c), \varpi_{G_g}^*(l_c), \tau_{G_g}^*(l_c)) \end{pmatrix}$$

Where $u_{qr}^* = (\Psi_{G_q}^*(l_r), \varpi_{G_q}^*(l_r), \tau_{G_q}^*(l_r)) = (A_{G_q}^*(l_r)e^{i a_{G_q}^*(l_r)}, B_{G_q}^*(l_r)e^{i b_{G_q}^*(l_r)}, Z_{G_q}^*(l_r)e^{i z_{G_q}^*(l_r)})$, $q = 1, 2, 3, ..., g$ and $r = 1, 2, 3, ..., c$. Here

$$\tau_{G_q}^*(l_r) = \sqrt[3]{1 - (A_{G_q}^*(l_r))^3 - (B_{G_q}^*(l_r))^3}\, e^{i2\pi\left(\sqrt[3]{1-\left(\frac{a_{G_q}^*(l_r)}{2\pi}\right)^3 - \left(\frac{b_{G_q}^*(l_r)}{2\pi}\right)^3}\right)}$$

Step (5): As each CFFN has complex degrees of satisfaction and non-satisfaction and two complex numbers cannot be compared with each other. So in order to get the positive ideal solution PIS and negative ideal solution NIS, the AWCFFED matrix $\mathbb{U}^*$ is not helpful.

In such a situation, the formula of score degrees of CFFNs is applicable. For this requirement, we need to find the score matrix $\mathbb{u}^{\{} = (u^{\{}_{qr})_{g\times c}$ by computing the score degrees of each entry in $\mathbb{u}^{*}$.

Here we have,

$$u^{\{}_{qr} = \left(A^{\{}_{G_q}(l_r)\right)^3 - \left(B^{\{}_{G_q}(l_r)\right)^3 + (\frac{a^{\{}_{G_q}(l_r)}{2\pi})^3 - (\frac{b^{\{}_{G_q}(l_r)}{2\pi})^3 \tag{33}$$

$$\mathbb{u}^{\{} = \begin{pmatrix} u^{\{}_{11} & u^{\{}_{12} & \cdots & u^{\{}_{1c} \\ u^{\{}_{21} & u^{\{}_{22} & \cdots & u^{\{}_{2c} \\ \vdots & \vdots & \vdots & \vdots \\ u^{\{}_{g1} & u^{\{}_{g2} & \cdots & u^{\{}_{gc} \end{pmatrix}$$

Eq. (33) represents the score degree of each entry.

Step (6): If the set of benefit criteria is represented by $\Re_{\lambda}$ and the set of cost criteria is represented by $\Im_{\varrho}$. So the PIS G^{+} and NIS G^{-} are the following:

$$G^{+} = \{G^{+}(l_1), G^{+}(l_2), ..., G^{+}(l_c)\}$$

$$G^{+} = \{G^{-}(l_1), G^{-}(l_2), ..., G^{-}(l_c)\}$$

Where, on the basis of criteria l_r the PIS and NIS can be assessed as:

$$G^{+}(l_r) = \{(\max_q u^{\{}_{qr} | l_r \in \Re_{\lambda}), (\min_q u^{\{}_{qr} | l_r \in \Im_{\varrho}) | q = 1, 2, 3, ..., g\} \tag{34}$$

$$G^{-}(l_r) = \{(\min_q u^{\{}_{qr} | l_r \in \Re_{\lambda}), (\max_q u^{\{}_{qr} | l_r \in \Im_{\varrho}) | q = 1, 2, 3, ..., g\} \tag{35}$$

Eq. (34) and (35) represent the PIS and NIS respectively.

Step (7): In real-world MAGDM, it is impossible to find the most suitable option (Positive ideal solution), and the worst option (negative ideal solution). To handle such a problem, we determine distance measures and calculate the distance of PIS $d(G_q, G^{+})$ and NIS $d(G_q, G^{-})$ from each alternative G_q. For this requirement, the formula of Euclidean distance between the two sets can be utilized. So the Euclidean distance of PIS $d(G_q, G^{+})$ and Euclidean distance of NIS $d(G_q, G^{-})$ can be defined as:

$$d(G_q, G^{+}) = \sqrt[3]{\sum_{r=1}^{c} \left(G_q(l_r) - G^{+}(l_r)\right)^3} \tag{36}$$

$$d(G_q, G^{-}) = \sqrt[3]{\sum_{r=1}^{c} \left(G_q(l_r) - G^{-}(l_r)\right)^3} \tag{37}$$

Eq. (36) and (37) represent the Ecluidean distance of PIS and NIS respectively.

Step (8): The following formula can be utilized to calculate the relative closeness index of an alternative G_q.

$$\mathbb{R}_{q^{+}} = \frac{d(G_q, G^{-})}{d(G_q, G^{+}) + d(G_q, G^{-})} \tag{38}$$

where $q = 1, 2, 3, ..., g$. Eq. (38) represents the relative closeness index.

Later on, it was proved by the two researchers Hadi-Vencheh and Mirajberi [56] that there are some specific situations, in which the closeness cannot give proper output. To handle such kinds of problems, the revised closeness index Æ was introduced by them. Which is given as:

$$Æ\left(G_q\right) = \frac{d(G_q, G^{-})}{d_{\max}(G_q, G^{-})} - \frac{d(G_q, G^{+})}{d_{\min}(G_q, G^{+})} \tag{39}$$

Eq. (39) shows the revised closeness index.

Where $q = 1, 2, 3, ..., g$.

The separation from NIS and closeness to the positive ideal solution be measured by the revised closeness index formula of an alternative. The most suitable option (alternative) is with the most revised closeness index.

Step (9): At the end, the alternatives of revised closeness index $Æ(G_q)$ are ranked in descending order. The final result is selecting the alternative $\widetilde{G}$ with the highest value of the revised closeness index.

$$\widetilde{G} = \{G_q : (q, Æ\left(G_q\right) = \max_{1\le y\le g} Æ\left(G_y\right))\} \tag{40}$$

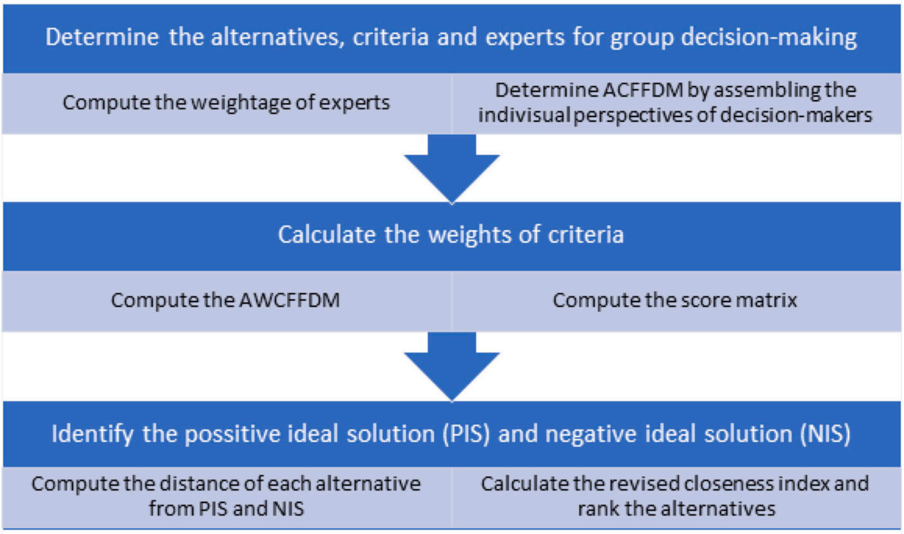

Fig. 1. The Flow-chart of Proposed Fuzzy TOPSIS Method.

Eq. (40) shows the alternative with the highest value as an optimal choice.
Fig. 1 shows the flow chart for the proposed fuzzy TOPSIS method.

5. Mathematical model for multi-attribute group decision-making

5.1. Selection of an English language instructor

An announcement for a job posting for an English language instructor who teaches English has been made by a private school. The business owner hopes to employ a qualified teacher for this position. For the position of teacher, a total of 5 applicants have submitted applications. The approach utilized in the MCDM problem mentioned above is applied here. The purpose of this MCDM problem is to choose the best qualified applicant for the position of principle. In this intricate Fermatean fuzzy model, we apply the proposed named, operators CFFEWA and CFFEOWA. The options in a model are the following contenders:

G_1: Shafiq Khan
G_2: Shakeela Bibi
G_3: Afaq Khan
G_4: Sadia Alam
G_5: Usman Khan

The business owner has assembled a team to conduct interviews and choose the best candidate for the open position. The best qualified applicant for this position will be chosen by a specially organized group of decision-makers and DMs. The experts will consider a candidate's skills when choosing an English language teacher (criteria).

$l_1 =$ Qualification
$l_2 =$ Accent
$l_3 =$ English communication skill
$l_4 =$ English grammar

6. TOPSIS method under CFFS information

In order to get benefit from the CFFE-TOPSIS method, we solve the above MAGDM problem by the CFFS-TOPSIS method.

Step (1): The linguistic terms of experts and criteria are shown in Table 1 given below:

Table 1
The experts and criterion in the form of linguistic terms and their weights.

Linguistic terms	CFFNs	weights (η_x)
Very very intelligent (VVI)	$[0.93e^{i2\pi(0.87)}, 0.54e^{i2\pi(0.63)}, 0.33e^{i2\pi(0.45)}]$	0.2196917951
Very intelligent (VI)	$[0.85e^{i2\pi(0.83)}, 0.60e^{i2\pi(0.69)}, 0.55e^{i2\pi(0.46)}]$	0.2239733444
Normal (N)	$[0.77e^{i2\pi(0.74)}, 0.64e^{i2\pi(0.71)}, 0.65e^{i2\pi(0.61)}]$	0.2152769679
Fair (F)	$[0.66e^{i2\pi(0.62)}, 0.77e^{i2\pi(0.79)}, 0.63e^{i2\pi(0.64)}]$	0.1822907122
Less intelligent (LI)	$[0.60e^{i2\pi(0.58)}, 0.85e^{i2\pi(0.87)}, 0.55e^{i2\pi(0.52)}]$	0.1587671803

Table 1 shows the linguistic terms and weights of alternatives.

Where the weights are calculated by Eq. (26).

To calculate the weight vector, we take the set of experts denoted by $\breve{E} = \{\breve{E}_1, \breve{E}_2, \breve{E}_3\}$ and assign value to every expert in the form of linguistic terms. Thus by using Eq. (26), we get the weight of experts as:

Table 2
The importance of experts and their weights.

Experts	Linguistic terms	CFFNs	weights (η_x)
$\breve{E}_1$	Very very intelligent (VVI)	$[0.93e^{i2\pi(0.87)}, 0.54e^{i2\pi(0.63)}, 0.33e^{i2\pi(0.45)}]$	0.1078578256
$\breve{E}_2$	Intelligent (I)	$[0.85e^{i2\pi(0.83)}, 0.60e^{i2\pi(0.69)}, 0.55e^{i2\pi(0.46)}]$	0.4549025631
$\breve{E}_3$	Normal (N)	$[0.77e^{i2\pi(0.74)}, 0.64e^{i2\pi(0.71)}, 0.65e^{i2\pi(0.61)}]$	0.4372396113

Table 2 shows the linguistic term and weight vector for each expert.

Step (2): The candidates relative to their capability are rated in the form of linguistic terms in Table 3:

Table 3
Linguistic terms for rating the researchers.

Linguistic terms	CFFNs
Very very intelligent (VVI)	$[0.93e^{i2\pi(0.87)}, 0.51e^{i2\pi(0.58)}, 0.40e^{i2\pi(0.52)}]$
Very intelligent (VI)	$[0.89e^{i2\pi(0.84)}, 0.62e^{i2\pi(0.68)}, 0.38e^{i2\pi(0.45)}]$
Intelligent (I)	$[0.86e^{i2\pi(0.80)}, 0.67e^{i2\pi(0.70)}, 0.40e^{i2\pi(0.52)}]$
Normal good (NG)	$[0.80e^{i2\pi(0.78)}, 0.69e^{i2\pi(0.75)}, 0.54e^{i2\pi(0.47)}]$
Normal (N)	$[0.77e^{i2\pi(0.74)}, 0.64e^{i2\pi(0.71)}, 0.65e^{i2\pi(0.61)}]$
Less normal (LN)	$[0.71e^{i2\pi(0.70)}, 0.73e^{i2\pi(0.71)}, 0.63e^{i2\pi(0.67)}]$
Very less normal (VLN)	$[0.70e^{i2\pi(0.68)}, 0.76e^{i2\pi(0.73)}, 0.60e^{i2\pi(0.67)}]$
Normal bad (NB)	$[0.60e^{i2\pi(0.54)}, 0.85e^{i2\pi(0.79)}, 0.55e^{i2\pi(0.70)}]$
Bad (B)	$[0.50e^{i2\pi(0.48)}, 0.90e^{i2\pi(0.87)}, 0.47e^{i2\pi(0.61)}]$
Very bad (VB)	$[0.44e^{i2\pi(0.39)}, 0.95e^{i2\pi(0.94)}, 0.38e^{i2\pi(0.48)}]$

In order to form a collective result, the judgment of individual policymakers are collected and forms a matrix named, the CFFDMs. The judgment of every policymaker is shown in the following Tables 4, 5, and 6.

Table 4
The value assessed by $\breve{E}_1$.

Ψ^1	l_1	l_2
G_1	$[0.88e^{i2\pi(0.84)}, 0.60e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.54)}]$	$[0.81e^{i2\pi(0.80)}, 0.62e^{i2\pi(0.64)}, 0.61e^{i2\pi(0.61)}]$
G_2	$[0.80e^{i2\pi(0.79)}, 0.68e^{i2\pi(0.75)}, 0.56e^{i2\pi(0.44)}]$	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$
G_3	$[0.91e^{i2\pi(0.89)}, 0.59e^{i2\pi(0.56)}, 0.34e^{i2\pi(0.50)}]$	$[0.81e^{i2\pi(0.77)}, 0.60e^{i2\pi(0.56)}, 0.63e^{i2\pi(0.72)}]$
G_4	$[0.76e^{i2\pi(0.79)}, 0.72e^{i2\pi(0.75)}, 0.57e^{i2\pi(0.44)}]$	$[0.80e^{i2\pi(0.79)}, 0.68e^{i2\pi(0.75)}, 0.56e^{i2\pi(0.44)}]$
G_5	$[0.81e^{i2\pi(0.80)}, 0.62e^{i2\pi(0.64)}, 0.61e^{i2\pi(0.61)}]$	$[0.92e^{i2\pi(0.88)}, 0.55e^{i2\pi(0.54)}, 0.38e^{i2\pi(0.54)}]$
Ψ^1	l_3	l_4
G_1	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$	$[0.88e^{i2\pi(0.84)}, 0.60e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.54)}]$
G_2	$[0.93e^{i2\pi(0.89)}, 0.62e^{i2\pi(0.59)}, 0.48e^{i2\pi(0.48)}]$	$[0.80e^{i2\pi(0.79)}, 0.68e^{i2\pi(0.75)}, 0.56e^{i2\pi(0.44)}]$
G_3	$[0.90e^{i2\pi(0.89)}, 0.63e^{i2\pi(0.59)}, 0.27e^{i2\pi(0.48)}]$	$[0.86e^{i2\pi(0.84)}, 0.63e^{i2\pi(0.61)}, 0.48e^{i2\pi(0.56)}]$
G_4	$[0.80e^{i2\pi(0.79)}, 0.68e^{i2\pi(0.75)}, 0.56e^{i2\pi(0.44)}]$	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$
G_5	$[0.80e^{i2\pi(0.77)}, 0.68e^{i2\pi(0.71)}, 0.56e^{i2\pi(0.57)}]$	$[0.93e^{i2\pi(0.89)}, 0.62e^{i2\pi(0.59)}, 0.48e^{i2\pi(0.48)}]$

Table 5
The value assessed by $\check{E}_2$.

$\Bbbk^2$	l_1	l_2
G_1	$[0.86e^{i2\pi(0.84)}, 0.63e^{i2\pi(0.61)}, 0.48e^{i2\pi(0.56)}]$	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$
G_2	$[0.79e^{i2\pi(0.79)}, 0.69e^{i2\pi(0.75)}, 0.56e^{i2\pi(0.44)}]$	$[0.93e^{i2\pi(0.89)}, 0.62e^{i2\pi(0.59)}, 0.48e^{i2\pi(0.48)}]$
G_3	$[0.88e^{i2\pi(0.84)}, 0.60e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.54)}]$	$[0.86e^{i2\pi(0.84)}, 0.63e^{i2\pi(0.61)}, 0.48e^{i2\pi(0.56)}]$
G_4	$[0.81e^{i2\pi(0.77)}, 0.60e^{i2\pi(0.56)}, 0.63e^{i2\pi(0.72)}]$	$[0.82e^{i2\pi(0.80)}, 0.61e^{i2\pi(0.59)}, 0.61e^{i2\pi(0.66)}]$
G_5	$[0.76e^{i2\pi(0.71)}, 0.69e^{i2\pi(0.68)}, 0.41e^{i2\pi(0.69)}]$	$[0.88e^{i2\pi(0.84)}, 0.60e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.54)}]$
$\Bbbk^2$	l_3	l_4
G_1	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$	$[0.92e^{i2\pi(0.88)}, 0.55e^{i2\pi(0.54)}, 0.38e^{i2\pi(0.54)}]$
G_2	$[0.88e^{i2\pi(0.84)}, 0.60e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.54)}]$	$[0.90e^{i2\pi(0.89)}, 0.63e^{i2\pi(0.59)}, 0.27e^{i2\pi(0.48)}]$
G_3	$[0.81e^{i2\pi(0.77)}, 0.60e^{i2\pi(0.56)}, 0.63e^{i2\pi(0.72)}]$	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$
G_4	$[0.93e^{i2\pi(0.89)}, 0.62e^{i2\pi(0.59)}, 0.48e^{i2\pi(0.48)}]$	$[0.86e^{i2\pi(0.84)}, 0.63e^{i2\pi(0.61)}, 0.48e^{i2\pi(0.56)}]$
G_5	$[0.85e^{i2\pi(0.80)}, 0.65e^{i2\pi(0.63)}, 0.48e^{i2\pi(0.62)}]$	$[0.90e^{i2\pi(0.89)}, 0.63e^{i2\pi(0.59)}, 0.27e^{i2\pi(0.48)}]$

Table 6
The value assessed by $\check{E}_3$.

$\Bbbk^3$	l_1	l_2
G_1	$[0.82e^{i2\pi(0.80)}, 0.61e^{i2\pi(0.59)}, 0.61e^{i2\pi(0.66)}]$	$[0.94e^{i2\pi(0.91)}, 0.50e^{i2\pi(0.49)}, 0.35e^{i2\pi(0.50)}]$
G_2	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$
G_3	$[0.88e^{i2\pi(0.84)}, 0.60e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.54)}]$	$[0.91e^{i2\pi(0.89)}, 0.59e^{i2\pi(0.56)}, 0.34e^{i2\pi(0.50)}]$
G_4	$[0.81e^{i2\pi(0.77)}, 0.60e^{i2\pi(0.56)}, 0.63e^{i2\pi(0.72)}]$	$[0.76e^{i2\pi(0.79)}, 0.72e^{i2\pi(0.75)}, 0.57e^{i2\pi(0.44)}]$
G_5	$[0.82e^{i2\pi(0.80)}, 0.61e^{i2\pi(0.59)}, 0.61e^{i2\pi(0.66)}]$	$[0.81e^{i2\pi(0.80)}, 0.62e^{i2\pi(0.64)}, 0.61e^{i2\pi(0.61)}]$
$\Bbbk^3$	l_3	l_4
G_1	$[0.88e^{i2\pi(0.84)}, 0.60e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.54)}]$	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$
G_2	$[0.80e^{i2\pi(0.79)}, 0.68e^{i2\pi(0.75)}, 0.56e^{i2\pi(0.44)}]$	$[0.88e^{i2\pi(0.84)}, 0.60e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.54)}]$
G_3	$[0.86e^{i2\pi(0.84)}, 0.63e^{i2\pi(0.61)}, 0.48e^{i2\pi(0.56)}]$	$[0.81e^{i2\pi(0.77)}, 0.60e^{i2\pi(0.56)}, 0.63e^{i2\pi(0.72)}]$
G_4	$[0.90e^{i2\pi(0.89)}, 0.63e^{i2\pi(0.59)}, 0.27e^{i2\pi(0.48)}]$	$[0.92e^{i2\pi(0.88)}, 0.55e^{i2\pi(0.54)}, 0.38e^{i2\pi(0.54)}]$
G_5	$[0.77e^{i2\pi(0.78)}, 0.70e^{i2\pi(0.71)}, 0.58e^{i2\pi(0.55)}]$	$[0.90e^{i2\pi(0.89)}, 0.63e^{i2\pi(0.59)}, 0.27e^{i2\pi(0.48)}]$

Now, we use the linguistic terms assigned by Expert $\check{E}_1$, Expert $\check{E}_2$, and Expert $\check{E}_3$ of each alternative relative to it's criteria shown in Table 7.

Table 7
Assigning the linguistic terms.

	l_1					l_2				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
$\check{E}_1$	VI	NG	VVI	N	NG	NG	N	NG	NG	VVI
$\check{E}_2$	I	NG	VI	NG	N	N	VVI	I	NG	VI
$\check{E}_3$	NG	N	VI	NG	NG	VVI	N	VVI	N	NG
	l_3					l_4				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
$\check{E}_1$	N	VVI	VI	NG	NG	VI	NG	I	N	VVI
$\check{E}_2$	N	VI	NG	VVI	I	VVI	VI	N	I	VI
$\check{E}_3$	VI	NG	I	VI	N	N	VI	NG	VVI	VI

Table 7 shows the linguistic terms assigned by Expert $\check{E}_1$, Expert $\check{E}_2$, and Expert $\check{E}_3$ of each alternative relative to it's criteria.

Now to find the ACFFED matrix, we aggregate the opinion of policymakers about each alternatives. In this case, we use a special operator defined in Eq. (27). Table 8 shows the aggregated complex Fermatean fuzzy Einstein decision matrix $\Bbbk$.

Table 8
ACFFEDM ⋓.

⋓	l_1
G_1	$[0.8463e^{i2\pi(0.1647)}, 0.4904e^{i2\pi(0.0762)}, 0.6510e^{i2\pi(0.9983)}]$
G_2	$[0.7826e^{i2\pi(0.1573)}, 0.5502e^{i2\pi(0.0924)}, 0.7074e^{i2\pi(0.9984)}]$
G_3	$[0.8836e^{i2\pi(0.1693)}, 0.4753e^{i2\pi(0.0785)}, 0.5874e^{i2\pi(0.9982)}]$
G_4	$[0.8051e^{i2\pi(0.1546)}, 0.4860e^{i2\pi(0.0730)}, 0.7135e^{i2\pi(0.9986)}]$
G_5	$[0.7937e^{i2\pi(0.1525)}, 0.5134e^{i2\pi(0.0802)}, 0.7917e^{i2\pi(0.9986)}]$
⋓	l_2
G_1	$[0.8744e^{i2\pi(0.1689)}, 0.4755e^{i2\pi(0.0754)}, 0.6072e^{i2\pi(0.9982)}]$
G_2	$[0.8659e^{i2\pi(0.1679)}, 0.5262e^{i2\pi(0.0824)}, 0.5896e^{i2\pi(0.9982)}]$
G_3	$[0.8806e^{i2\pi(0.1713)}, 0.4869e^{i2\pi(0.0735)}, 0.5864e^{i2\pi(0.9981)}]$
G_4	$[0.7936e^{i2\pi(0.1591)}, 0.5275e^{i2\pi(0.0849)}, 0.7070e^{i2\pi(0.9984)}]$
G_5	$[0.8595e^{i2\pi(0.1657)}, 0.4786e^{i2\pi(0.0788)}, 0.6344e^{i2\pi(0.9983)}]$
⋓	l_3
G_1	$[0.8270e^{i2\pi(0.1616)}, 0.5201e^{i2\pi(0.0851)}, 0.6647e^{i2\pi(0.9983)}]$
G_2	$[0.8213e^{i2\pi(0.1651)}, 0.5051e^{i2\pi(0.0852)}, 0.6819e^{i2\pi(0.9982)}]$
G_3	$[0.8448e^{i2\pi(0.1633)}, 0.4891e^{i2\pi(0.0738)}, 0.6542e^{i2\pi(0.9984)}]$
G_4	$[0.9082e^{i2\pi(0.1761)}, 0.5005e^{i2\pi(0.0764)}, 0.5006e^{i2\pi(0.9980)}]$
G_5	$[0.8135e^{i2\pi(0.1578)}, 0.5356e^{i2\pi(0.0849)}, 0.6753e^{i2\pi(0.9984)}]$
⋓	l_4
G_1	$[0.8672e^{i2\pi(0.1610)}, 0.4909e^{i2\pi(0.0781)}, 0.6122e^{i2\pi(0.9984)}]$
G_2	$[0.8832e^{i2\pi(0.1718)}, 0.4936e^{i2\pi(0.0787)}, 0.5757e^{i2\pi(0.9981)}]$
G_3	$[0.7995e^{i2\pi(0.1567)}, 0.5141e^{i2\pi(0.0795)}, 0.7067e^{i2\pi(0.9985)}]$
G_4	$[0.8842e^{i2\pi(0.1705)}, 0.4772e^{i2\pi(0.0742)}, 0.5848e^{i2\pi(0.9982)}]$
G_5	$[0.9037e^{i2\pi(0.1781)}, 0.4991e^{i2\pi(0.0745)}, 0.5163e^{i2\pi(0.9979)}]$

Step (3): Here we will find the weight assigned to each criterion l_c by each policy maker $\check{E}_n$, and then we compute the decision of each criteria by each policy maker $\check{E}_n$ and form a new matrix named, weight matrix constructed as $ß = \{ß_1, ß_2, ß_3, ..., ß_w\}^T$. Now using Eq. (29) we get:

Table 9
Weighted matrix $\check{E}_1$.

Criteria	$\check{E}_1$
l_1	$0.8456e^{i2\pi(0.1653)}, 0.6398e^{i2\pi(0.1051)}, 0.5110e^{i2\pi(0.9981)}$
l_2	$0.8254e^{i2\pi(0.1605)}, 0.6321e^{i2\pi(0.1014)}, 0.5699e^{i2\pi(0.9982)}$
l_3	$0.8594e^{i2\pi(0.1666)}, 0.6597e^{i2\pi(0.1052)}, 0.4275e^{i2\pi(0.9980)}$
l_4	$0.8561e^{i2\pi(0.1656)}, 0.6552e^{i2\pi(0.1047)}, 0.4502e^{i2\pi(0.9981)}$

Table 9 shows the aggregated values assigned by Expert $\check{E}_1$ to each criteria.

Table 10
Weighted matrix $\check{E}_2$.

Criteria	$\check{E}_2$
l_1	$0.8292e^{i2\pi(0.1598)}, 0.6403e^{i2\pi(0.1025)}, 0.5510e^{i2\pi(0.9982)}$
l_2	$0.8633e^{i2\pi(0.1666)}, 0.6342e^{i2\pi(0.0995)}, 0.4664e^{i2\pi(0.9981)}$
l_3	$0.8563e^{i2\pi(0.1635)}, 0.6329e^{i2\pi(0.0991)}, 0.4913e^{i2\pi(0.9982)}$
l_4	$0.8788e^{i2\pi(0.1715)}, 0.6263e^{i2\pi(0.0964)}, 0.4229e^{i2\pi(0.9980)}$

Table 10 shows the aggregated values assigned by Expert $\check{E}_2$ to each criteria.

Table 11
Weighted matrix $\check{E}_3$.

Criteria	$\check{E}_3$
l_1	$0.8240e^{i2\pi(0.1600)}, 0.6255e^{i2\pi(0.0983)}, 0.5806e^{i2\pi(0.9983)}$
l_2	$0.8630e^{i2\pi(0.1683)}, 0.6200e^{i2\pi(0.0983)}, 0.4917e^{i2\pi(0.9980)}$
l_3	$0.8508e^{i2\pi(0.1662)}, 0.6451e^{i2\pi(0.1042)}, 0.4872e^{i2\pi(0.9980)}$
l_4	$0.8620e^{i2\pi(0.1661)}, 0.6164e^{i2\pi(0.0965)}, 0.5003e^{i2\pi(0.9981)}$

Table 11 shows the aggregated values assigned by Expert $\check{E}_3$ to each criteria.

Table 12
The significance of criteria in the form of linguistic terms and their weights.

Criteria	$\check{E}_1$	$\check{E}_2$	$\check{E}_3$	CFFE weights
l_1	I	I	I	$0.8288e^{i2\pi(0.0321)}, 0.5030e^{i2\pi(0.0127)}, 0.6719e^{i2\pi(0.9999882917)}$
l_2	I	I	I	$0.8595e^{i2\pi(0.0334)}, 0.4982e^{i2\pi(0.0125)}, 0.6226e^{i2\pi(0.9999869289)}$
l_3	I	I	I	$0.8542e^{i2\pi(0.0330)}, 0.5088e^{i2\pi(0.0128)}, 0.6257e^{i2\pi(0.9999873218)}$
l_4	I	I	I	$0.8693e^{i2\pi(0.0337)}, 0.4960e^{i2\pi(0.0122)}, 0.6046e^{i2\pi(0.9999866370)}$

Table 12 shows the linguistic terms of each Expert $\check{E}_1$, $\check{E}_2$, and $\check{E}_3$ assigned to each criteria as well as the aggregated values assigned by each Expert $\check{E}_1$, $\check{E}_2$, and $\check{E}_3$ to each criteria.

Where the weights of Table 12 can be calculated by using Eq. (27) which are given below:

$$\left(\eta_x\right) = \{0.2478136647, 0.2493897429, 0.2530980770, 0.2496985154\}$$

Step (4): Find aggregated weighted complex Fermatean fuzzy decision matrix (AWCFFDM) $\mathbb{U}^{\{}$.

In this step, the required matrix can be found by using Eq. (31).

Table 13
AWCFFEDM $\mathbb{U}^{\{}$.

$\mathbb{U}^{\{}$	l_1
G_1	$0.7014e^{i2\pi(0.00013)}, 0.6128e^{i2\pi(0.01214)}, 0.7516e^{i2\pi(0.9999994036)}$
G_2	$0.6486e^{i2\pi(0.00012)}, 0.6484e^{i2\pi(0.01471)}, 0.7688e^{i2\pi(0.9999989390)}$
G_3	$0.7323e^{i2\pi(0.00013)}, 0.6045e^{i2\pi(0.01251)}, 0.7282e^{i2\pi(0.9999993474)}$
G_4	$0.6672e^{i2\pi(0.00012)}, 0.8687e^{i2\pi(0.01163)}, 0.3614e^{i2\pi(0.9999994757)}$
G_5	$0.6578e^{i2\pi(0.00012)}, 0.6260e^{i2\pi(0.01278)}, 0.7774e^{i2\pi(0.9999993042)}$
$\mathbb{U}^{\{}$	l_2
G_1	$0.7515e^{i2\pi(0.00014)}, 0.6017e^{i2\pi(0.01201)}, 0.7098e^{i2\pi(0.9999994226)}$
G_2	$0.7442e^{i2\pi(0.00014)}, 0.6310e^{i2\pi(0.01312)}, 0.6955e^{i2\pi(0.9999992472)}$
G_3	$0.7568e^{i2\pi(0.00014)}, 0.6080e^{i2\pi(0.01171)}, 0.6990e^{i2\pi(0.9999994648)}$
G_4	$0.6820e^{i2\pi(0.00013)}, 0.6318e^{i2\pi(0.01352)}, 0.7550e^{i2\pi(0.9999991762)}$
G_5	$0.7387e^{i2\pi(0.00014)}, 0.6034e^{i2\pi(0.01224)}, 0.7224e^{i2\pi(0.9999993887)}$
$\mathbb{U}^{\{}$	l_3
G_1	$0.7064e^{i2\pi(0.00013)}, 0.6331e^{i2\pi(0.01355)}, 0.7328e^{i2\pi(0.9999991707)}$
G_2	$0.7015e^{i2\pi(0.00013)}, 0.6245e^{i2\pi(0.01357)}, 0.7435e^{i2\pi(0.9999991671)}$
G_3	$0.7216e^{i2\pi(0.00013)}, 0.6156e^{i2\pi(0.01176)}, 0.7311e^{i2\pi(0.9999994579)}$
G_4	$0.7757e^{i2\pi(0.00014)}, 0.6219e^{i2\pi(0.01217)}, 0.6638e^{i2\pi(0.9999993992)}$
G_5	$0.6948e^{i2\pi(0.00013)}, 0.6424e^{i2\pi(0.01352)}, 0.7364e^{i2\pi(0.9999991762)}$
$\mathbb{U}^{\{}$	l_4
G_1	$0.7538e^{i2\pi(0.00013)}, 0.6090e^{i2\pi(0.01244)}, 0.7018e^{i2\pi(0.9999993583)}$
G_2	$0.7677e^{i2\pi(0.00014)}, 0.6105e^{i2\pi(0.01254)}, 0.6838e^{i2\pi(0.9999993427)}$
G_3	$0.6950e^{i2\pi(0.00013)}, 0.6225e^{i2\pi(0.01266)}, 0.7506e^{i2\pi(0.9999993236)}$
G_4	$0.7686e^{i2\pi(0.00014)}, 0.6013e^{i2\pi(0.01182)}, 0.6899e^{i2\pi(0.9999994495)}$
G_5	$0.7855e^{i2\pi(0.00015)}, 0.6137e^{i2\pi(0.01187)}, 0.6573e^{i2\pi(0.9999994425)}$

Table 13 shows the aggregated weighted complex Fermatean fuzzy decision matrix (AWCFFDM).

Step (5): Use Eq. (32) to compute the score function of each CFFN, where the score matrix is denoted by $\Bbbk^*$ and shown in Table 14.

Table 14
Score matrix $\Bbbk^*$.

$\Bbbk^*$	l_1	l_2	l_3	l_4
G_1	0.11488	0.20661	0.09865	0.20252
G_2	0.00024	0.16089	0.10167	0.22495
G_3	0.17176	0.20876	0.14247	0.09438
G_4	−0.35860	0.06505	0.22631	0.23667
G_5	0.03928	0.18343	0.07042	0.25363

Step (6): Eq. (34) is used to find the useful criteria that is positive ideal solution (PIS) and Eq. (35) is used to find the unuseful criteria that is negative ideal solution (NIS).

$$G^+ = \{0.17176, 0.20876, 0.22631, 0.25363\}$$

$$G^- = \{-0.35860, 0.06505, 0.07042, 0.09438\}$$

Step (7): Use Eq. (36) and Eq. (37) to find the distance of each alternative G_q from PIS as well as from NIS, respectively. Also, use Eq. (38) to compute the relative closeness index.

Table 15
Distance of alternatives from PIS and NIS and their ranking.

Alternative	$d(G_q, G^+)$	$d(G_q, G^-)$	Revised closeness index	Ranking
G_1	0.03162	0.33206	−0.45258	2
G_2	0.06975	0.22213	−2.30514	4
G_3	0.02427	0.39054	0	1
G_4	0.38635	0.08166	−15.70973	5
G_5	0.04572	0.26207	−1.21276	3

Table 15 shows the PIS and NIS of each alternative along with revised closeness index and the ranking of each alternative.

Step (8): According to the revised closeness indices, the rank of alternative is as follows:

$$G_3 \succ G_1 \succ G_5 \succ G_2 \succ G_4.$$

From the output, we found that G_3 is the most capable candidate for the required post.

6.1. *Complex Fermatean fuzzy Einstein weighted averaging aggregation operator*

We apply the CFFEWA operator to Table 8 and get the result shown in Table 16.

Table 16
Aggregated values.

G_1	$0.8547944585e^{i2\pi(0.03290493968)}, 0.4941145334e^{i2\pi(0.01251540944)}$
G_2	$[0.8429664160e^{i2\pi(0.03322743331)}, 0.5183241568e^{i2\pi(0.01345134553)}$
G_3	$0.8556726274e^{i2\pi(0.03315169706)}, 0.4912121037e^{i2\pi(0.01213802337)}$
G_4	$0.8564521002e^{i2\pi(0.03320260065)}, 0.4975352617e^{i2\pi(0.01225381105)}$
G_5	$0.8488865802e^{i2\pi(0.03290429589)}, 0.5064307876e^{i2\pi(0.01265762474)}$

Table 16 shows the aggregated values of each alternative.
Now, compute the score function of each alternative G_q shown in Table 17.

Table 17
Score and ranking.

Alternatives	G_1	G_2	G_3	G_4	G_5
Score	0.5039	0.4597	0.5080	0.5050	0.4818
Ranking	3	5	1	2	4

From the output shown in Table 17, we found that G_3 is the best option for the required post.

6.2. For complex Fermatean fuzzy Einstein ordered weighted averaging aggregation operator

To find the required result, first we find the order of Table 8 and arrange the alternatives in descending order according to the score function of each alternative shown in Table 18.

Table 18
ACFFEODM $\Cup$.

$\Cup$	l_1
G_1	$[0.8744e^{i2\pi(0.1689)}, 0.4755e^{i2\pi(0.0754)}, 0.6072e^{i2\pi(0.9982)}]$
G_2	$[0.8832e^{i2\pi(0.1718)}, 0.4936e^{i2\pi(0.0787)}, 0.5757e^{i2\pi(0.9981)}]$
G_3	$[0.8836e^{i2\pi(0.1693)}, 0.4753e^{i2\pi(0.0785)}, 0.5874e^{i2\pi(0.9982)}]$
G_4	$[0.9082e^{i2\pi(0.1761)}, 0.5005e^{i2\pi(0.0764)}, 0.5006e^{i2\pi(0.9980)}]$
G_5	$[0.9037e^{i2\pi(0.1781)}, 0.4991e^{i2\pi(0.0745)}, 0.5163e^{i2\pi(0.9979)}]$
$\Cup$	l_2
G_1	$[0.8672e^{i2\pi(0.1610)}, 0.4909e^{i2\pi(0.0781)}, 0.6122e^{i2\pi(0.9984)}]$
G_2	$[0.8659e^{i2\pi(0.1679)}, 0.5262e^{i2\pi(0.0824)}, 0.5896e^{i2\pi(0.9982)}]$
G_3	$[0.8806e^{i2\pi(0.1713)}, 0.4869e^{i2\pi(0.0735)}, 0.5864e^{i2\pi(0.9981)}]$
G_4	$[0.8842e^{i2\pi(0.1705)}, 0.4772e^{i2\pi(0.0742)}, 0.5848e^{i2\pi(0.9982)}]$
G_5	$[0.8595e^{i2\pi(0.1657)}, 0.4786e^{i2\pi(0.0788)}, 0.6344e^{i2\pi(0.9983)}]$
$\Cup$	l_3
G_1	$[0.8463e^{i2\pi(0.1647)}, 0.4904e^{i2\pi(0.0762)}, 0.6510e^{i2\pi(0.9983)}]$
G_2	$[0.8213e^{i2\pi(0.1651)}, 0.5051e^{i2\pi(0.0852)}, 0.6819e^{i2\pi(0.9982)}]$
G_3	$[0.8448e^{i2\pi(0.1633)}, 0.4891e^{i2\pi(0.0738)}, 0.6542e^{i2\pi(0.9984)}]$
G_4	$[0.8051e^{i2\pi(0.1546)}, 0.4860e^{i2\pi(0.0730)}, 0.7135e^{i2\pi(0.9986)}]$
G_5	$[0.8135e^{i2\pi(0.1578)}, 0.5356e^{i2\pi(0.0849)}, 0.6753e^{i2\pi(0.9984)}]$
$\Cup$	l_4
G_1	$[0.8270e^{i2\pi(0.1616)}, 0.5201e^{i2\pi(0.0851)}, 0.6647e^{i2\pi(0.9983)}]$
G_2	$[0.7826e^{i2\pi(0.1573)}, 0.5502e^{i2\pi(0.0924)}, 0.7074e^{i2\pi(0.9984)}]$
G_3	$[0.7995e^{i2\pi(0.1567)}, 0.5141e^{i2\pi(0.0795)}, 0.7067e^{i2\pi(0.9985)}]$
G_4	$[0.7936e^{i2\pi(0.1591)}, 0.5275e^{i2\pi(0.0849)}, 0.7070e^{i2\pi(0.9984)}]$
G_5	$[0.7937e^{i2\pi(0.1525)}, 0.5134e^{i2\pi(0.0802)}, 0.7917e^{i2\pi(0.9986)}]$

Table 18 shows the order of each order of alternatives.

Now we aggregated the values of each alternative G_q of the above table which are shown in Table 19.

Table 19
Aggregated values.

G_1	$0.8548001383e^{i2\pi(0.03290604787)}, 0.4940390978e^{i2\pi(0.01251082334)}$
G_2	$0.8427827224e^{i2\pi(0.03322230033)}, 0.5184303918e^{i2\pi(0.01345541503)}$
G_3	$0.8556726274e^{i2\pi(0.03315169706)}, 0.4912121037e^{i2\pi(0.01213802337)}$
G_4	$0.8558553814e^{i2\pi(0.03317906151)}, 0.4974735247e^{i2\pi(0.01225137317)}$
G_5	$0.8486699050e^{i2\pi(0.03289444166)}, 0.5064577510e^{i2\pi(0.01265938376)}$

Now, we compute the score function of each alternative G_q to find the ranking of alternatives given in Table 20.

Table 20
Score and ranking.

Alternatives	G_1	G_2	G_3	G_4	G_5
Score	0.5040	0.4593	0.5080	0.5038	0.4813
Ranking	2	5	1	3	4

From the output shown in Table 20, we found that G_3 is the suitable candidate for the required post.

The final ranking of the decision-making study solved with the help of CFFEWA operator shown in Table 17, CFFEOWA operator shown in Table 20 and CFFE-TOPSIS shown in Table 15 are shown in Table 21.

Table 21
Comparative analysis of proposed operators with CFFE-TOPSIS method.

Method	Ranking	Best alternative
CFFEWA operator	$G_3 > G_4 > G_1 > G_5 > G_2$	G_3
CFFEOWA operator	$G_3 > G_1 > G_4 > G_5 > G_2$	G_3
CFFE-TOPSIS	$G_3 > G_1 > G_5 > G_2 > G_4$	G_3

7. Comparison and advantages of proposed operators

Here we compare the CFFEAA operators with other existing operators to see their importance. For this, we take a decision-making problem MCDM [57]. The purpose of taking an MCDM model is to seek a more affective option with the help of proposed CFFEA AOs. In every result, we found the same alternative as the most affective option. The specialty of the CFFEAA operators is to solve the phenomena of periodicity and result in a more specific form than the existing operators. The CFFEAA operators can usefully be applied to two-dimensional models more clearly because of their phase term. Another specialty of the CFFEAA operators is that they can handle any information on one-dimensional phenomena when its phase term is taken as zero.

1. Here, we use the complex Fermatean fuzzy Einstein weighted average CFFEWA operator, the CFFEOWA operator, and the CFFEHA operator.
2. The comparison between CFFEA AOs and CIFEA AOs show us the consistency of CFFEA AOs and has been represented in the graph in figure (2).
3. Because of cube on it's both functions, the CFFEA AOs work sufficiently in the decision-making information and cover the restrictions of all endure AOs which cannot handle two-dimensional information.
4. Our CFFEA AOs deal with the data of CFFNs, CPFNs, and CIFNS in terms of Einstein generalization in a decision-making problem.
5. Some AOs have been used to solve the data of the CPF model, but they have some restrictions. Sometimes CIFEA AOs cannot handle the data because the sum of the both functions is more than 1. In these conditions, we use CFFEA AOs to give us a suitable answer. So our proposed operators of CFFEWA, CFFEOWA, and CFFEHA are more helpful than CIFEWA operator, CIFEOWA operator, and CIFEHA operator.
6. To secure the parametric structure and generalized structure of Einstein t-conorm and Einstein t-norm, the operators we have proposed lie in a parametric construction.
7. Apart from the selected decision-making scheme, the considered operators observe a large-scale approach in different popular decision-making approaches, considering the aggregation of separate information to assess a group's adequate solution in terms of Einstein t-conorm and Einstein t-norm.

Table 22 shows the comparison between our proposed CFFEAA operators with other existing operators [57] to see their importance. For this, we have taken a multicriteria group decision-making MCGDM problem consisting of three experts. The purpose of an MCGDM model is to seek a more affective infection with the help of CFFEA AOs.

Table 22
Value of alternatives.

θ	Proposed operators	G_1	G_2	G_3	G_4	G_5
1	CIFWA [57]	1.1605	0.8812	0.3491	0.6484	1.2545
	CFFWA	0.3722	0.2144	0.1078	0.1474	0.5647
1	CIFOWA [57]	1.1478	0.9458	0.6687	0.6774	1.2719
	CFFOWA	0.3464	0.1771	0.1995	0.1743	0.5319
1	CIFHA [57]	1.3249	1.0153	0.4290	0.7917	1.3421
	CFFHA	0.3666	0.2289	0.1996	0.2118	0.5714
2	CIFEWA [57]	1.1468	0.8650	0.3096	0.1693	1.2501
	CFFEWA	0.3705	0.2079	0.0994	0.1424	0.5563
2	CIFEOWA [57]	1.1359	0.9306	0.6339	0.6539	1.2675
	CFFEOWA	0.3452	0.1708	0.1992	0.1714	0.5258
2	CIFEHA [57]	1.3352	1.0264	0.3598	0.8005	1.3611
	CFFEHA	0.3644	0.2229	0.1923	0.2073	0.5639

Now, according to Table 22, we find the ranking of each alternative G_q shown in Table 23.

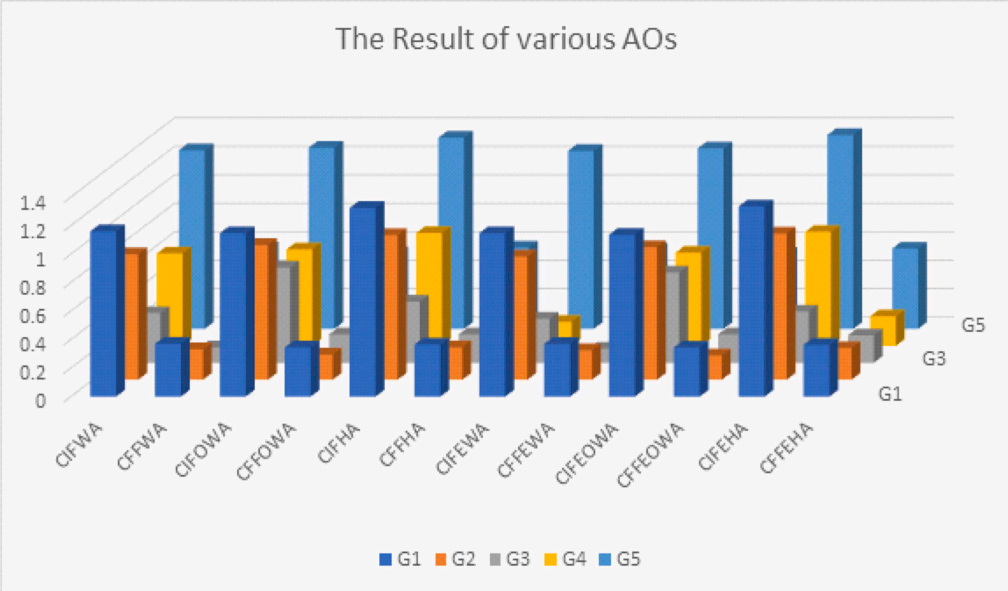

Fig. 2. The graph of comparative study.

Table 23
Rank of alternatives.

θ	Proposed operators	Ranking
1	CIFWA [57]	$G_5 > G_1 > G_2 > G_3 > G_3$
	CFFWA	$G_5 > G_1 > G_2 > G_4 > G_3$
1	CIFOWA [57]	$G_5 > G_1 > G_2 > G_4 > G_3$
	CFFOWA	$G_5 > G_1 > G_3 > G_2 > G_4$
1	CIFHA [57]	$G_5 > G_1 > G_2 > G_4 > G_3$
	CFFHA	$G_5 > G_1 > G_3 > G_2 > G_4$
2	CIFEWA [57]	$G_5 > G_1 > G_2 > G_4 > G_3$
	CFFEWA	$G_5 > G_1 > G_2 > G_4 > G_3$
2	CIFEOWA [57]	$G_5 > G_1 > G_2 > G_3 > G_4$
	CFFEOWA	$G_5 > G_1 > G_3 > G_4 > G_2$
2	CIFEHA [57]	$G_5 > G_1 > G_2 > G_4 > G_3$
	CFFEHA	$G_5 > G_1 > G_3 > G_4 > G_2$

Table 22 represents the values of every each alternatives and Table 23 shows the ranking of each alternatives. Fig. 2 shows the statistical analysis of the comparative study.

8. Conclusion

In case to handle the one-dimensional inexact data, the model of fuzzy set theory FST is conventionally applied. But some real-life information often involves different attributes that have compelled the researchers to expand the models to handle more unclarity. Certainly, a number of decision-making models appear with periodic information that cannot be handled by the models of FS, IFS, PFS, and FFS. Specifically, an extended concept was required to handle such problems. For this purpose, Ramot proposed the idea of CFSs which was helpful to overcome the restriction involved in some real phenomena. In order to overcome the issue of non-satisfaction degree, this concept was further developed into some extended concepts named CIFSs, CPFSs, and CFFSs. Due to finding more uncertainty in two-dimensional phenomena, the CFFSs have a better tendency than CIFSs and CPFSs. With an eye on use in decision-making processes, we have combined the benefits of the Einstein operator and the adaptability of CFFS in this study to develop several AOs.

In this research study, we introduced some AOs called CFFEAA operators to address more unclear information about MAGDM. For the CFFEA operator, we have also created a TOPSIS technique. Our suggested AOs are simple to use with CFF data. For the CFFE-TOPSIS technique and the CFFEAA operators, we have proposed two algorithms. Additionally, we have created a MAGDM issues that use the suggested operators. The CFFEAA operators are more flexible when employed to solve a MAGDM problem due to the cube on their degrees. So that we can produce a more versatile result, we can apply our suggested operators to two-dimensional data.

Future directions

In short, numerous recent extensions of fuzzy sets under Einstein operators provide significant benefits to scenarios involving decision-making. In the future, our aim is to extend our study to

1. Complex Fermatean fuzzy Frank AOs.
2. Complex Fermatean Power AOs.
3. Complex Fermatean fuzzy Hamacher geometric AOs.
4. Complex Fermatean fuzzy Logarithmic AOs.
5. Complex Fermatean fuzzy Yager AOs.
6. Complex Fermatean fuzzy Dombi AOs.
7. We will also develop some methods like GRA, EDAS, CODAS, COPRAS and some other methods for the proposed work.

Besides the highlighted efficiencies of the proposed operators, there is a notable deficiency in these operators owing to the restrictions of the CFFS. Therefore, it will also be a beneficial task to exploit the competency of the Einstein t-norms to construct more practical and flexible operators by employing the theoretical background of the border and generalized model, namely, the complex Fermatean cubic fuzzy set.

Limitation & Discussion

1. Our study is limited to MCDM and MCGDM problems.
2. This study is applicable for unknown weights.
3. Our study is limited to Einstein generalization and required further improvement to deal with other generalization like Hamacher.
4. This research work deals with two-dimensional data.

CRediT authorship contribution statement

All authors listed have significantly contributed to the development and the writing of this article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data were used to support this study.

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