CBE—Life Sciences Education
Vol. 9, 417–421, Winter 2010

Essay

Mathematical Biology at an Undergraduate Liberal Arts College

Lisette de Pillis* and Stephen C. Adolph†

Departments of *Mathematics and †Biology, Harvey Mudd College, Claremont, CA 91711

Submitted August 3, 2010; Accepted August 4, 2010
Monitoring Editor: John Jungck

INTRODUCTION

Since 2002 we have offered an undergraduate major in Mathematical Biology at Harvey Mudd College. The major was developed and is administered jointly by the mathematics and biology faculty. In this paper we describe the major, courses, and faculty and student research and discuss some of the challenges and opportunities we have experienced.

BACKGROUND: HARVEY MUDD COLLEGE

Harvey Mudd College is an undergraduate liberal arts college that emphasizes science, engineering, and mathematics. The college enrollment is just under 800 students, and we have 82 full-time tenure-track faculty. Harvey Mudd is located in Claremont, CA, ≈30 miles east of Los Angeles. Harvey Mudd is a member of the Claremont Colleges, which include five undergraduate and two graduate institutions.

Harvey Mudd has seven academic departments: Biology; Chemistry; Computer Science; Engineering; Humanities, Social Sciences, and the Arts; Mathematics; and Physics. Each of these departments offers a major (except Humanities, Social Sciences, and the Arts). Students at Harvey Mudd can also choose one of three interdisciplinary majors: Biology/Chemistry, Mathematics/Computer Science, and Mathematical Biology. A Harvey Mudd student can also choose a major at one of the other Claremont Colleges.

Students take roughly one-third of their courses in their major, one-third in a common core curriculum, and one-third in the humanities, social sciences, and the arts. The common core curriculum includes one semester each of biology, computer science, engineering, and writing. In addition, students take two to three semesters each of physics, chemistry, and mathematics. Thus, all students at the college have a biology background that emphasizes genetics and evolution and a mathematics background that includes linear algebra, probability and statistics, and differential equations. All of the college's majors rely on this broad technical foundation.

DOI: 10.1187/cbe.10–08–0099
Address correspondence to: Lisette de Pillis (depillis@hmc.edu).

MOTIVATION TO START A MATHEMATICAL BIOLOGY MAJOR

Several factors motivated us to develop a mathematical biology major. First, several faculty in mathematics and biology had research projects involving mathematical models in biology. Michael Moody, who was then chair of the Mathematics Department, modeled problems in evolutionary population genetics; Lisette de Pillis modeled the interactions between cancer, chemotherapy, and the immune system; and Steve Adolph modeled the evolution of life histories and phenotypic plasticity. Moody introduced an elective course on mathematical models in biology; this course attracted both biology and mathematics majors and focused on evolution and population genetics. Enrollments in other courses likewise indicated student interest in both subjects: some mathematics majors enrolled in additional biology courses (e.g., ecology), while some biology majors took additional math courses (e.g., discrete mathematics). Several students pursued senior research projects in mathematical biology. For example, one mathematics major completed a senior thesis under the supervision of a biologist, modeling the effects of climate change on geographic distributions. This student went on to earn a Ph.D. in Ecology and Evolution and is now a tenured theoretical ecologist in a biology department. Thus, we felt we had sufficient interest on the part of both students and faculty to develop a mathematical biology major.

Another factor was the common technical background of our students. Because all students had training in both biology and mathematics, we could assume a minimum common background in both fields that could serve as a foundation. In addition, the college had supported several prior experiments involving interdisciplinary teaching; for example, we offered a freshman-level interdisciplinary laboratory course that included modules from physics, engineering, chemistry, and biology (Van Hecke *et al*., 2002). Mathematical biology was the second interdisciplinary major at Harvey Mudd, after the joint major in computer science and mathematics; the college has since added a joint major in chemistry and biology.

Finally, we were encouraged by the growing national interest in rethinking how mathematics and biology should be interconnected at the undergraduate level (National Research Council, 2003).

RECRUITING FOR THE MAJOR

The mathematical biology major was developed and implemented in response to a perceived demand for such a program. The numbers of students in the major fluctuate annually, ranging from between one and seven majors per year, but has averaged 3.25 graduates per year since 2003 and around 10 major advisees per year. For comparison, the college has graduated an average of approximately seven biology majors per year and 19 mathematics majors per year since 2003. There have been essentially no efforts to recruit more students into the mathematical biology major. One reason for this is the genesis of the major, having come into being in response to a perceived need, so there was little consideration of implementing active recruitment strategies. Another reason has to do with limited faculty time and resources. The two advisors to the major are also the faculty that have the responsibility for the development and implementation of the mathematical biology courses, and those same faculty would also be solely responsible for developing recruitment activities. In practice, resources at current levels would likely become strained were the numbers of majors to increase, so there has not been strong motivation to develop recruitment strategies. However, were growth in the mathematical biology major a goal, recruitment strategies that have been successful in other contexts would likely also work well in this context. For example, a few years ago the mathematics major enjoyed significant growth through deliberate recruitment activities. Included in those activities were several informational "Math Nights" in which students were invited to food-and-drink receptions at which they learned about the mathematics major through presentations by several mathematics faculty and got to meet alums who had impressive stories to tell about how they made use of their degrees after graduating. There was also significant effort put toward redesigning and maintaining the Mathematics Department website to provide comprehensive information about the major as well as up-to-date news about the activities of the students and faculty in the department.

COMPONENTS OF THE MATHEMATICAL BIOLOGY MAJOR

One unique aspect of the mathematical biology major at Harvey Mudd as compared with other major programs across the country is the fact that it was jointly developed and is jointly administered by the Mathematics and Biology Departments. This joint ownership gives the students two departmental homes, one in Mathematics and one in Biology. Feedback from our students has shown that they do identify strongly with both disciplines while part of the mathematical biology program. Of significant concern early on was that we not develop a major program that would produce students who only had a partial set of skills in either mathematics or biology. Our aim was to create a program that would train mathematical biology students sufficiently well in both disciplines so that they would be well-prepared to continue graduate studies specializing in just mathematics or biology alone yet still be well-versed in the connections between the two disciplines. Within each department, we chose a subset of courses from our major requirements. Biologists chose the biology courses and mathematicians chose the math courses, but all of this was done in a context of constant communication between the departments. When developing the major we needed to ensure that its requirements did not exceed the college's upper limit for course credits in a major; this was important because curricular initiatives (such as new majors) are subject to approval by the entire faculty, not just individual departments. The mathematical biology major currently requires 41 units (U); by comparison, the mathematics major requires at least 33 U and the biology major requires at least 39 U (including 7 U of chemistry). We emphasize that these unit counts are in addition to the 12 U of math and 3 U of biology in the college's core curriculum; without this core curriculum, we would have added this additional background to the joint major.

Table 1 shows the current major requirements. Flexibility is a key feature: beyond the common core curriculum, the only absolutely required courses are one semester each of introductory biology lab, discrete mathematics, analysis, and mathematical biology. For their other coursework the students choose from a menu of options, with some constraints. For example, there is a computation requirement that can be fulfilled by taking either scientific computing, numerical analysis, algorithms, or computational biology. Similarly, students choose an upper-division biology lab course, an upper-division seminar course, and two upper-division mathematics electives. As a result of this electivity, no two mathematical biology majors to date have taken an identical set of courses to fulfill their major. The major is flexible enough to allow students to switch to a straight mathematics or biology major as late as their senior year.

Students choose their courses in consultation with their academic advisors. Each mathematical biology major has two academic advisors: a math professor and a biology professor. Dual advising ensures that students are taking appropriate classes in both areas while still allowing flexibility.

CAPSTONE COURSE IN MATHEMATICAL BIOLOGY

The required Mathematical Biology course is taught as two half-semester courses; this allows nonmajors to sample mathematical biology without needing to make a full semes-

Table 1. Mathematical biology degree requirements at Harvey Mudd College

Common core requirements (taken by all students at the college)	
Introductory biology (3 U; genetics and evolution, without lab)	
Mathematics (12 U; including calculus, linear algebra, differential equations, probability, and statistics)	
Mathematics (10 U)	
Discrete Math (3 U)	
Analysis I (3 U)	
Two electives (4–6 U); examples include:	
Probability	Mathematical Statistics
Stochastic Processes	Advanced Linear Algebra
Operations Research	Dynamical Systems
Partial Differential Eqns.	Abstract Algebra
Biology (15 U)	
Introductory Laboratory (1 U)	
Choose three (9 U): Comparative Physiology; Ecology and Environmental Biology; Evolutionary Biology; Molecular Biology	
Two upper-division electives (one seminar and one lab, 5 U)	
Computation (3 U)	
Choose one: Scientific Computing; Numerical Analysis; Algorithms; Computational Biology	
Mathematical Biology (4 U)	
Mathematical Biology I & II	
Additional Requirements (9 U)	
One additional elective (3 U). Examples: Biostatistics; Principles of Computer Science; Physical Chemistry	
Senior Thesis Research in either Biology or Mathematics, or clinic (2 semesters; 6 U)	
Colloquium: Biology (2 semesters) and Math (2 semesters, including 1 semester of Math Forum)	

U, semester units

ter's commitment. Course prerequisites include linear algebra, differential equations, and introductory biology. These courses are taken by Harvey Mudd students as part of their required core curriculum. Typical enrollment ranges from 8 to 12 students; this includes not only mathematical biology majors but also students from a variety of other majors, including biology, math, engineering, and physics. In addition, students from the other Claremont Colleges commonly enroll in the course. Most students taking the course are juniors and seniors; occasionally a highly qualified sophomore or first-year student is enrolled.

The course is cotaught by two professors, one from biology and one from mathematics. Both teachers attend all class sessions, although normally each class session is led by one professor. The course emphasizes mathematical models in biology and includes a variety of mathematical approaches and biological problems. Our textbook is *Dynamic Models in Biology,* by Stephen Ellner and John Guckenheimer (2006). Class sessions include a mix of lectures, discussion, and in-class problem-solving. The course is not heavily homework-driven but we do assign weekly problem sets, including some from the text.

The course content reflects our belief that there is not a single canon in mathematical biology; rather, our goal is to expose students to a range of examples and modeling approaches. The course includes some conventional topics such as single-population models, including discrete, continuous, and age-structured models. We also cover coupled differential equation models, including classic competition and infectious disease models, as well as a recent model of gene regulatory networks (Elowitz and Leibler, 2000). Some of these topics closely follow the textbook. Other topics include evolutionarily stable strategies, allometry and scaling in physiology, and metapopulation models. We also include topics that spring from our own research interests, particularly cancer-immune system interactions (de Pillis and Radunskaya, 2001) and the evolution of phenotypic plasticity (Padilla and Adolph, 1996). In addition to the textbook, students read some papers from the primary literature. At the end of each half-semester, students choose a pair of published models in some area of mathematical biology and present an overview of these papers to the rest of the class.

During the first several years of the class, we made extensive use of guest speakers who presented their own research; this was facilitated by our grant from the W. M. Keck Foundation. At present, we try to include several guest speakers each semester. Guest speakers have yielded multiple benefits, as described below.

MATHEMATICAL BIOLOGY STUDENTS: AFTER GRADUATION

After they graduate from Harvey Mudd, the most common activity for mathematical biology majors is to attend graduate school, usually in some field of biology. Students have chosen diverse fields, including animal behavior, neuroscience, ecology, systems biology, epidemiology, biostatistics, plant biology, molecular and developmental biology, and environmental engineering. The second most common postgraduation activity is to work as a laboratory research assistant, usually as a prelude to future graduate study.

Students who have studied some mathematical biology but majored in another subject also are likely to attend graduate programs. In particular, a number of mathematics majors have gone on to pursue graduate work in mathematical biology. Finally, a few of our former students have begun careers in human medicine or veterinary medicine.

FACULTY AND UNDERGRADUATE RESEARCH IN MATHEMATICAL BIOLOGY

The existence of the mathematical biology major and the accompanying specialized coursework has catalyzed new collaborations and research activity in several areas of mathematical biology, linking Harvey Mudd and the other Claremont Colleges to other institutions.

Early in the development of the major, our funding allowed us to invite guest faculty to teach short modules in the mathematical biology course. Visiting faculty involvement turned out to be a very enriching aspect of the program. In addition to inviting a visiting researcher to deliver a week's worth of lectures in the new mathematical biology course, local Claremont Colleges faculty were invited to sit in on the course. One of the earliest new research collaborations emerged as a result of this visiting faculty and course auditing activity. This collaboration subsequently received National Science Foundation (NSF) funding and involved two campuses, three faculty members, and a total of 12 undergraduate students over the course of the 3-yr project.

Outside of the course itself, there has been funding to support summer research, and students are also expected to complete either a senior thesis or be part of a Harvey Mudd research-clinic project in their senior year. A sampling of research collaborations that have taken place on campus since the institution of the major are listed in Table 2. Some of these projects have been funded summer research projects involving faculty and undergraduate students, others are senior thesis topics, and yet others are ongoing faculty–faculty collaborations.

Another current project is being run as a collaboration among several of the Claremont Colleges and is being funded through the NSF program in Mathematical Biology for Undergraduates. The five-year program, "Research Experiences at the Biology Mathematics Interface (REBMI)," is open to undergraduate students from all of the Claremont Colleges. One of the project PIs (L.d.P.) is a developer of the Harvey Mudd mathematical biology course, and the lead PI on the REBMI project (John Milton) first came to Claremont as a visiting researcher sponsored by the early Harvey Mudd mathematical biology program. Milton subsequently was recruited as a tenured faculty member in Claremont. The REBMI project provides research experiences and courses to students in a way that is meant to enrich, not replace, their chosen course of major study. REBMI is described further by Milton *et al*. (2010).

It is certainly the case that funding dedicated to supporting such research endeavors is a strong catalyst to the formation of these collaborations. However, it should be noted that the successful maintenance of the research programs also involves more than just funding. Institutional support must exist in the form of sufficient time allotted to engaged faculty as well as evaluation procedures that reward interdisciplinary activities.

Table 2. Examples of collaborative research in mathematical biology at Harvey Mudd and the Claremont Colleges

Optimally Controlling Combination Immuno-Chemotherapy in Cancer Models
Statistical Estimation of Physiological Performance
Mathematical Modeling of Nutational Movement of Plant Cotyledons
Modeling Ecological Invasions on Dynamic Habitats
Multidisciplinary Study of Structural Development in Tendril-Bearing Plants
Identifying Oscillating Gene Expression Transcripts in Microarray Time Series Data
Modeling HIV/AIDS: Preferential Anti-Retroviral Treatment Distribution in Resource-Constrained Countries
An ODE Model of Tumor Growth and Effect of Immunotherapy and Chemotherapy Treatment in Colorectal Cancer
Stochastic-Deterministic Mathematical Models of the Immune Response to HIV Infection

CHALLENGES IN IMPLEMENTATION

In an institution whose primary mission is teaching, there still remain significant challenges to implementing interdisciplinary teaching and research programs like the mathematical biology program at Harvey Mudd. Both authors of this paper have served as chairs of their respective departments and have had to struggle with the challenges of constrained resources. However, both have also experienced the challenges of developing and delivering truly interdisciplinary courses. We will briefly discuss how we found to best address the competing needs of the department and the faculty engaged in interdisciplinary work.

Among the most common challenges is how departments will "count" teaching loads. This is an area that should be explored carefully. Here quantitative and qualitative evaluation of teaching loads come into conflict. From a numerical perspective, it may not make economic sense to place two professors simultaneously in one interdisciplinary classroom for a semester. There is strong economic resistance to this model, and the preference of the "bean counters" is either to assign only one professor to teach an interdisciplinary course or to give only partial teaching credit for teaching a full course. However, if the institution wishes to encourage the development of interdisciplinary courses, it will have to recognize the overhead costs of developing cross-disciplinary collaborative research and teaching. From the perspective of a faculty member, the total time spent in preparing and presenting an interdisciplinary course in collaboration with another can be equivalent to or more than the time invested in solo-teaching a traditional course in one's own field of specialty. We believe that the qualitative benefit to the students of being taught by research professors in different fields and of experiencing the different approaches to thinking about research problems is significant but difficult to quantify. Frankly stated, one of the most effective ways to encourage faculty to deliver jointly taught interdisciplinary courses is to assign teaching credit that reflects the amount of time each faculty member actually spends in the classroom. This, then, allows for multiple models of joint teaching. In some models, the course may be so modular as to allow one faculty member at a time to deliver a self-contained module for only a few weeks in the semester, with no other faculty presence necessary. In that case, it is reasonable to assign teaching credit for those few weeks. On the other hand, if the course is truly interdisciplinary at every level, and the mathematical and biology aspects are delivered in an interwoven and well-coordinated way, then the course is not as naturally modular. In such a case, it makes sense for both faculty members to be present in the classroom throughout the semester and, as such, it is reasonable to assign full teaching credit to both faculty.

Thus, the priorities of an institution must be carefully discussed and articulated. If properly planned, appropriate teaching load policies as well as time for research development can be established in such a way as to encourage faculty to engage in the development of cross-disciplinary teaching and research work.

FUTURE DIRECTIONS

A broader view of quantitative biology should certainly include the rapidly increasing connections between computer science and biology. During the past several years we have observed a growing interest among undergraduate students in combining these fields in a way that is different from traditional mathematical biology. We also have several faculty in computer science and biology whose research is at the intersection of these fields. We are exploring ways to expand our existing mathematical biology major to accommodate computational biology. This effort involves our computer science faculty as well as the mathematics and biology faculty, and we envision a program that is jointly administered by all three departments. We have developed a major in computational and mathematical biology that has been approved by all three departments; the next step is approval by the college. This new unified major includes the existing mathematical biology major as one track and adds a track in computational biology.

In a related effort, several Harvey Mudd faculty have been teaching an integrated year-long introductory computer science and biology course that replaces the separately taught introductory courses in these two fields. This course, which is jointly taught by faculty from computer science and biology, was developed with support from the Howard Hughes Medical Institute.

ACKNOWLEDGMENTS

We thank the present and former faculty of the mathematics and biology departments for their continued support of the mathematical biology major. Jon Jacobsen and Elissa Schwartz have taught the Mathematical Biology course several times, and Ali Nadim and John Milton contributed substantially to teaching the course. In addition, our numerous guest speakers have greatly enriched our program. We also thank Sheldon Wettack, former Dean of Faculty at Harvey Mudd, for his enthusiastic support of the new major. We thank the W.M. Keck Foundation, the Howard Hughes Medical Institute, and the NSF for funding the curricular and research components of our program. We thank the editors of *CBE—Life Sciences Education* for the opportunity to describe our program, and Pat Marsteller and John Jungck for continuing to promote the cross-pollination of biology and mathematics. This paper is dedicated to the memory of Michael Moody (1952–2010), pioneering mathematical biologist at Harvey Mudd College.

REFERENCES

de Pillis, L. G., and Radunskaya, A. E. (2001). A mathematical tumor model with immune resistance and drug therapy: an optimal control approach. J. Theor. Med. *3*, 79–100.

Ellner, S. P., and Guckenheimer, J. (2006). Dynamic Models in Biology, Princeton, NJ: Princeton University Press.

Elowitz, M. B., and Leibler, S. (2000). A synthetic oscillatory network of transcriptional regulators. Nature *403*, 335–338.

Milton, J. G., Radunskaya, A. E., Lee, A. H., dePillis, L. G., and Bartlett, D. F. (2010). Team research at the biology–mathematics interface: project management perspectives. CBE Life Sci. Educ. 93, 316–322.

National Research Council (2003). BIO 2010: Transforming Undergraduate Education for Future Research Biologists, Washington, DC: National Academies Press.

Padilla, D. K., and Adolph, S. C. (1996). Plastic inducible morphologies are not always adaptive: the importance of time delays in a stochastic environment. Evol. Ecol. *10*, 105–117.

Van Hecke, G. R., Karukstis, K. K., Haskell, R. C., McFadden, C. S., and Wettack, F. S. (2002). An integration of chemistry, biology, and physics: the interdisciplinary laboratory. J. Chem. Educ. *79*, 837–844.