4π Radiotherapy Using a Linear Accelerator: A Misnomer in Violation of the Solid Geometric Boundary Conditions in Three-Dimensional Euclidean Space

Biplab Sarkar, Tharmarnadar Ganesh, Anusheel Munshi, Arjunan Manikandan[1], Bidhu Kalyan Mohanti

Department of Radiation Oncology, Manipal Hospitals, Dwarka, New Delhi, [1]Department of Medical Physics, Apollo Proton Cancer Center, Chennai, Tamil Nadu, India

Abstract

Purpose: The concept of $4\pi^c$ radiotherapy is a radiotherapy planning technique receiving much attention in recent times. The aim of this article is to disprove the feasibility of the 4π radiotherapy using a cantilever-type linear accelerator or any other external-beam delivery machines. **Materials and Methods:** A surface integral-based mathematical derivation for the maximum achievable solid angle for a linear accelerator was carried out respecting the rotational boundary conditions for gantry and couch in three-dimensional Euclidean space. The allowed movements include a gantry rotation of 0–$2\pi^c$ and a table rotation of $\left(\frac{\pi^c}{2} - 0 - \frac{3\pi^c}{2}\right)$. **Results:** Total achievable solid angle by cantilever-type linear accelerator (or any teletherapy machine employing a cantilever design) is $\frac{1}{r^2}\int_{\theta=0}^{\pi}\int_{\varphi=0}^{\pi}(rSin\theta d\varphi).(rd\theta) = 2\pi^c$, which is applicable only for the foot and brain radiotherapy where the allowed table rotation is 90°–0°–270°. For other sites such as pelvis, thorax, or abdomen, achievable solid angle as the couch rotation comes down significantly. Practically, only suitable couch angle is 0° by avoiding gantry–couch–patient collision. **Conclusions:** Present cantilever design of linear accelerator prevents achieving a 4π radian solid angle at any point in the patient. Even the most modern therapy machines like CyberKnife which has a robotic arm also cannot achieve 4π geometry. Maximum achievable solid angle under the highest allowable boundary condition(s) cannot exceed $2\pi^c$, which is restricted for only extremities such as foot and brain radiotherapy. For other parts of the body such as pelvis, thorax, and abdomen, the solid angle is reduced to $1/5^{th}$ (maximum value) of the $4\pi^c$. To obtain a $4\pi^c$ solid angle in a three-dimensional Euclidean space, the patient has to be a zero-dimensional point and X-ray head of the linear accelerator has a freedom to rotate in every point of a hypothetical sphere of radius 1 m. This article establishes geometrically why it is not possible to achieve a $4\pi^c$ solid angle.

Keywords: 4π, 4π radiotherapy, Euclidean space, linear accelerator, solid angle

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Introduction

This has reference to the concept called as "4π radiotherapy," which in recent times has attracted much attention both scientifically and commercially.[1] The concept of 4π radiotherapy was developed in the University of California, Los Angeles (UCLA), during 2013.[2,3] Subsequently, it was commercially adopted by Varian Medical Systems in their RapidArc planning approach.[3] Since the "4π radiotherapy" solution is commercially available, it is used by several researchers in their planning studies.[2-15] The studies include sites such as thorax (lung stereotaxy),[2,3,7,12,15] brain,[12-16] head and neck,[5,10] abdomen (liver stereotactic body radiotherapy),[7,11,15] pelvis (prostate),[9] and phantom[6] in knowledge-based planning[17] besides several other discussions in scientific forums.[8,12] As claimed by Dong *et al*., in 2013, they have achieved a $4\pi^c$ (steradian) solid angle at the tumor center for thorax cases and subsequently similar claims were made by rest of the authors until the last month (June 2019).[2-15,17,18] However, the question

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Address for correspondence: Dr. Biplab Sarkar, Manipal Hospitals, Dwarka, New Delhi - 110 075, India. E-mail: biplabphy@gmail.com

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"whether it is possible to obtain a $4\pi^c$ solid angle using a linear accelerator in three-dimensional Euclidean space?" is an interesting one to probe into against the backdrop of these mushrooming scientific literature and commercial products. The simple answer is "no". It is not possible to obtain a $4\pi^c$ solid angle using the current cantilever design of linear accelerator at any point in space with or without the patient on the couch. One may add some more variables such as vertical patient motion during therapy delivery or surface rendering using infrared to avoid gantry–patient collision, but these will not help in achieving $4\pi^c$ solid angle at any point as explained below.

The aim of this article is to mathematically establish the infeasibility of $4\pi^c$ radiotherapy with the present cantilever design of linear accelerator or any other teletherapy machines.

Materials and Methods

The detailed mathematical derivation is provided elsewhere; however, for the completeness, we are presenting the summary of the result.[19,20] Total solid angle (angle in three-dimension) in Euclidean space is defined by,

$d\Omega = \frac{ds}{r^2}$ where ds is the surface area and r is the radius vector.

A $4\pi^c$ solid angle can be achieved only at the center of a sphere.

And hence, solid angle as $\Omega = \frac{Area}{r^2} = \frac{1}{r^2}\left[\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}(rSin\theta d\varphi).(rd\theta)\right] = \frac{4\pi r^2}{r^2} = 4\pi^c\ (= 12.56^c)$.

A linear accelerator with its accelerating arm attached to a vertical frame can geometrically be considered as a cantilever. The allowed movements include a gantry rotation of $0–2\pi^c$ and a table rotation of $(\frac{\pi^c}{2} - 0 - \frac{3\pi^c}{2})$. Therefore, the total solid angle obtained by a linear accelerator (or any teletherapy machine employing a cantilever design) is $\frac{1}{r^2}\int_{\theta=0}^{\pi}\int_{\varphi=0}^{\pi}(rSin\theta d\varphi).(rd\theta) = 2\pi^c$.

Results

The total allowed solid angle is reduced to $2\pi^c$ under maximum allowed boundary condition for a linear accelerator. Achievability of $2\pi^c$ solid angle is only limited to the treatment of the extremities such as foot and brain radiotherapy under the condition that each point in the hemisphere created by gantry $[-\pi^c–0–+\pi^c]$ and couch $[-\frac{\pi^c}{2} -– 0–+\frac{\pi^c}{2}]$ sees the target volume.

For example, total solid angle obtained by a 40 cm × 40 cm field size when gantry is rotated over a $0–2\pi^c$ arc is only 2.51^c. Total solid angle at the tumor center is reduced further with blocked or multileaf collimator-shaped fields as the total surface area becomes less due to shaping or blocking.

To elaborate the dimension of the solid angle encountered in radiotherapy, we provide a simple example of 4-field box technique or full-arc VMAT technique. With a 20 cm × 20 cm open field size, the solid angle calculated as:

$$\frac{4\pi^c \times 400}{4 \times 3.142 \times 100 \times 100}$$

$= 0.0127\pi^c$ solid angle

$= 0.04^c$ solid angle

For four beams $= 0.16^c$ solid angle.

Similarly, total solid angle for a single-arc VMAT technique with 20 cm × 20 cm open field is following. Surface area created by 20 cm × 20 cm open field over a path length created by single full gantry rotation of 360° is = 2π × 20 cm × 100 cm. Therefore, the solid angle calculated as $= \frac{4\pi^c \times 20 \times 2\pi \times 100}{4\pi \times 100 \times 100} = 0.4\pi^c$.

A typical CyberKnife plan with 100 static segments having each field opening of 4 cm × 4 cm yields a total solid angle $= \frac{4\pi^c \times 1600}{4 \times \pi \times 100 \times 100} = 0.05\pi^c$, considering no overlap between beams, which is lesser than the typical VMAT/linear accelerator-based solid angle.

Discussion

Several investigators have presented the pictorial representation of "4π radiotherapy" in their articles; however, none could achieve the complete $4\pi r^2$ surface area in any one of those studies.[2-15,17,18]

Furthermore, patient–gantry–table collision is an additional potential risk when trying to achieve so-called "$4\pi^c$ solid angle." Only the iPlan stereotactic planning system (BrainLab AG, Feldkirchen, Germany) offers a collision map, and other planning systems such as Eclipse (Varian Medical Systems, Palo Alto, CA) or Monaco (CMS Elekta, Sunnyvale, CA, USA) cannot generate a collision map. Figure 1 presents a head and neck case planned with impractical combination of couch positions and gantry rotation; nevertheless, treatment planning system (TPS) did not warn about any impending collision. It was obvious that the plan was not deliverable as the gantry cannot cross 90° position even with a couch rotation as little as 20° [Figure 2]. Unless otherwise a surface-rendering technique is used, "4π radiotherapy" and planning with many noncoplanar beams is an inefficient and error-prone process.[6] All beams/arcs need to be verified for collision manually by therapists when the patient is on the couch. If it is found that the plan is nonexecutable due to collision issues, it would require a replanning, leading to delay in patient treatment.

The term "4π radiotherapy" is a misnomer and does not represent the true geometry the technique is capable of

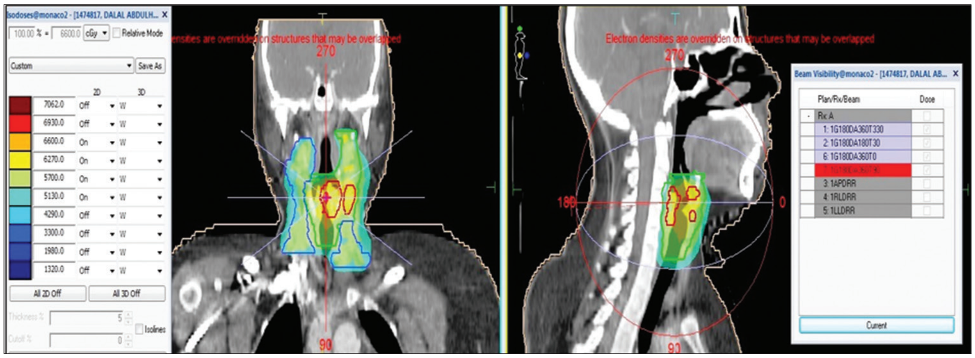

Figure 1: Treatment planning using infeasible gantry and table angles, gantry rotation 0°–360° couch angle 0°, ±30° and 90° for a head and neck case

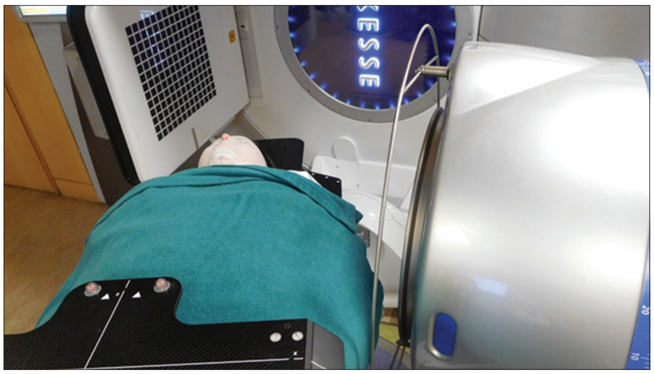

Figure 2: Gantry at 90° position colliding with couch at 20° position for the same head neck patient; no couch position ≥±12° practically possible as it does not allow the gantry to cross ±90° position for Elekta linear accelerator. For Varian accelerator, the accessible couch angle further reduces as the isocenter to gantry end clearance is lesser than Elekta accelerators

achieving. It is impossible to deliver true “4π radiotherapy” using the present cantilever design of a medical linear accelerator or for that matter using other external beam therapy machines such as Tomotherapy (Accuray Inc., Madison, WI) or CyberKnife (Accuray Inc., Madison, WI). The maximum solid angle achievable in treating human individuals is limited to $2\pi^c$ with cantilever-type medical accelerators and may increase, but never can reach 4π, even in advanced machines such as CyberKnife. Further, as the scope of gantry movement is very limited for the complex geometry beams from such advanced machines, only static intensity-modulated beams may be possible with a significant increase in the treatment time.

Among all the therapy delivery techniques, only brachytherapy comes closer to true “4π radiotherapy” if one considers the source as a point.

Therefore, solid angle encountered in radiotherapy is much less than $4\pi^c$; although it is theoretically possible to achieve maximum $2\pi^c$ solid angle in brain or foot radiotherapy, it is not required clinically.

As a corollary of this study, while reviewing a selective list of six major journals in the field of radiation oncology and medical physics ([1] Physics in Medicine and Biology, [2] International Journal of Radiation Oncology-Biology-Physics, [3] Medical Physics, [4] Radiation Oncology, [5] Acta Oncologica, and [6] Journal of Applied Clinical Medical Physics), we found that a large number (30%; 5/15)[5,6,9,13,18] of these research activities were aided by the vendors, as stated in their financial disclosure statements. Further, the question of nonfeasibility of “4π radiotherapy” has been raised against two previous 4π articles, which was either not at all or answered unsatisfactorily.[19-21] For example, UCLA group ambiguously responded to the question regarding the feasibility of “4π radiotherapy;” however, it failed to establish mathematically (or geometrically) the achievability of $4\pi^c$ solid angle using a linear accelerator.[17] Therefore, it is evident that efforts have been made to establish the superiority of “4π” technique over the standard noncoplanar treatment technique by identifying it with a fancy unscientific name and with commercial interests, which is not a healthy practice in terms of dignity of our profession.[22]

Conclusion

We propose that the scientific and commercial use of the misnomer “4π radiotherapy” should stop forthwith because it is not possible to obtain $4\pi^c$ solid angle at any point with an existing accelerator design. Further, peer reviewers should be also cautious in recommending articles on “4π radiotherapy” for publication.

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Conflicts of interest

There are no conflicts of interest.

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