Research article

Anisotropic bianchi V cosmological model in Scale Covariant Theory of Gravitation with a time-variable deceleration parameter

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A R T I C L E   I N F O

A B S T R A C T

In this paper we solve the field equations for Scale covariant theory of gravitation which was introduced by Caunato et al. [1], for Bianchi V line element in the presence of perfect fluid medium. Here the deceleration parameter is considered to be time dependent which gives the average scale factor $a(t) = [\sinh(\beta t)]^{1/n}$, where n and β are positive constants. This value of average scale factor is the key expression for solving the field equations. Using the recent observational value of $q_0 = -0.52^{+0.08}_{-0.04}$ and $H_0 = 69.2 \pm 1.2$ derived from BAO/CMB and H(z) data by Santos et al. (2016) [46], we have evaluated three different pairs of (n, β). We observe that the model represents a phase transition from early deceleration to a present accelerating phase for a particular choice of the pair $(n = 2, \beta = 92.75)$. Applying some recently developed diagnostic tools like jerk parameter and statefinders, we find that the derived model is exactly in accordance with standard ΛCDM model. Along with these, many physical, geometric and kinematic properties of the model are thoroughly studied and found consistent with recent observations.

1. Introduction

Canuto et al. [1, 2] in the year 1977, proposed the Scale covariant scalar tensor theory of gravitation. This is one of the finest generalizations of Einstein general theory of gravitation. In the quest for generalizing the general theory of relativity by Einstein, there are various approaches. A scalar gravitational field and non-gravitational field generate the metric in any scalar tensor theories. In a curved space-time the non gravitational field generates the scalar gravitational field through a wave equation. Scalar tensor theories have greater importance nowadays as it solve the problem of dark matter or the missing matter and several other problems which are hardly resolved by General Relativity theory [3, 4, 5, 6, 7, 8, 9, 10, 11]. The distinguish feature of Scale covariant theory is that the gravitational units are applied on Field equations and atomic units are used to measure physical quantities. For this to become possible a conformal transformation $\bar{g}_{ij} = \phi^2(x^k) g_{ij}$ is used. Under this transformation, there is correspondence between the line element $d\bar{s}^2 = \bar{g}_{ij} dx^i dx^j$ in gravitational units and $ds = \phi^{-1}(x) d\bar{s}$ in atomic units (Here quantities with bar denote gravitational units and without bar denotes atomic units). The field equations in Scale covariant theory are invariant under scale transformations. Also, in this theory, the variation of the gravitational constant G is interpreted naturally [12, 13].

In Scale-covariant theory, the gauge function ϕ in its most general formulation may be assumed a function of all space-time coordinates, but in most of the studies and in our study also, it is assumed as a function of time only.

During the last twenty years, the study of cosmology has been revolutionized due to observed astronomical phenomena. These observations indicate two important features of the universe. They are, (i) anisotropy at the early stage and (ii) accelerated expansion at the present epoch. The SNe Ia measurements [14, 15, 16, 17, 18, 19, 20, 21, 22, 23] provide an evidence not only for accelerating expansion of the universe at present, but also for the phase transition from past deceleration to present acceleration. The High-Z Supernova Search (HZSNS) also indicates the transition redshift $z_t = 0.46 \pm 0.13$ at $(1, \sigma)$ c.l. [24] which has been modified to $z_t = 0.43 \pm 0.07$ at $(1, \sigma)$ c.l. [25]. From Supernova Legacy Survey (SNLS) [26] and Davis et al. [27], the transition redshift $z_t \approx 0.6((1, \sigma))$. So, the Deceleration Parameter (DP), which is the measure of the rate of expansion may not be a constant, but time dependent. In fact, it must show a signature flipping from positive to negative [15, 28, 29].

The Scale covariant theory in different contexts have been studied by many researchers [30, 31, 32, 33, 34, 35] from time to time.

2. Basic equations

In Scale covariant theory, the field equations are written as follows:

$$R_{ij} - \frac{1}{2}g_{ij}R + f_{ij}(\phi) = -8\pi GT_{ij} + \Lambda(\phi)g_{ij} \tag{1}$$

$$\phi^2 f_{;ij} = 2\phi\phi_{;ij} - 4\phi_{,i}\phi_{,j} - g_{ij}(\phi\phi^{;i}_{;i} - \phi^{,i}\phi_{,i}). \tag{2}$$

Here all the terms have their usual meaning. The space-time metric in Bianchi V is given by

$$ds^2 = dt^2 - a_1^2 dx^2 - e^{2mx}[a_2^2 dy^2 + a_3^2 dz^2] \tag{3}$$

where a_1, a_2, a_3 are time dependent and m is taken as an arbitrary constant. The expression for an energy momentum tensor T_{ij} in perfect fluid medium is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{4}$$

where ρ, p and u^i are energy-density, the pressure and four dimensional velocity vector respectively. This velocity vector follows the relation $u^i u_j = 1$. The following non linear differential equations can be obtained by translation above equations together as follows:

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} - \frac{m^2}{a_1^2} - 2\frac{\dot{a}_1}{a_1}\frac{\dot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\frac{\dot{V}}{V} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} = -8\pi Gp \tag{5}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} - \frac{m^2}{a_1^2} - 2\frac{\dot{a}_2}{a_2}\frac{\dot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\frac{\dot{V}}{V} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} = -8\pi Gp \tag{6}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{m^2}{a_1^2} - 2\frac{\dot{a}_3}{a_3}\frac{\dot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\frac{\dot{V}}{V} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} = -8\pi Gp \tag{7}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} - \frac{3m^2}{a_1^2} + \frac{\dot{\phi}}{\phi}\frac{\dot{V}}{V} - \frac{\ddot{\phi}}{\phi} + 3\frac{\dot{\phi}^2}{\phi^2} = 8\pi G\rho \tag{8}$$

$$2\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} = 0. \tag{9}$$

The continuity equation $T^i_{j;i}u^j = 0$ reads,

$$\dot{\rho} + (\rho + p)\frac{\dot{V}}{V} + \rho\left(\frac{\dot{\phi}}{\phi} + \frac{\dot{G}}{G}\right) + 3p\frac{\dot{\phi}}{\phi} = 0. \tag{10}$$

From Eq. (9), we have

$$a_1^2 = a_2 a_3. \tag{11}$$

The following set of equations can be obtained by combining the above equations (5), (6), (7) and (8) by following the approach of different researchers [36, 37, 38, 39, 40, 41].

$$\frac{a_2}{a_1} = k_1 \exp\left(\ell_1 \int \frac{dt}{a^3\phi^2}\right) \tag{12}$$

$$\frac{a_3}{a_2} = k_2 \exp\left(\ell_2 \int \frac{dt}{a^3\phi^2}\right) \tag{13}$$

$$\frac{a_3}{a_1} = k_3 \exp\left(\ell_3 \int \frac{dt}{a^3\phi^2}\right) \tag{14}$$

where ak_1, k_2, k_3 and ℓ_1, ℓ_2, ℓ_3 are constants. With further calculations, the quadrature solution of the metric functions a_1, a_2 and a_3 are given as follows:

$$a_1(t) = c_1 a \exp\left(M_1 \int \frac{dt}{a^3\phi^2}\right) \tag{15}$$

$$a_2(t) = c_2 a \exp\left(M_2 \int \frac{dt}{a^3\phi^2}\right) \tag{16}$$

$$a_3(t) = c_3 a \exp\left(M_3 \int \frac{dt}{a^3\phi^2}\right) \tag{17}$$

with $c_1 = \sqrt[3]{(k_1^2 k_2)^{-1}}$, $c_2 = \sqrt[3]{k_1 k_2^{-1}}$, $c_3 = \sqrt[3]{k_1 k_2^2}$ and $M_1 = -\frac{1}{3}(2\ell_1 + \ell_2)$, $M_2 = -\frac{1}{3}(\ell_2 - \ell_1)$, $M_3 = \frac{1}{3}(\ell_1 + \ell_2)$.

Also, we have

$$M_1 + M_2 + M_3 = 0 \quad \text{and} \quad c_1 c_2 c_3 = 1. \tag{18}$$

Substituting equation (11) into equations (12), (13) and (14), we have $M_1 = 0$, $M_2 = -M_3 = h$ (say) and $c_1 = 1$, $c_2 = c_3^{-1} = d$ (say). Substituting these results into equations (15), (16) and (17), we obtain

$$a_1(t) = a \tag{19}$$

$$a_2(t) = d.a.\exp\left(h \int \frac{dt}{a^3\phi^2}\right) \tag{20}$$

$$a_3(t) = \frac{1}{d}.a.\exp\left(-h \int \frac{dt}{a^3\phi^2}\right). \tag{21}$$

In view of the line element equation (3), the important parameters such as Volume V, Average scale factor a, Expansion scalar θ, Shear scalar σ^2, and Hubble parameter H can be written as follows:

$$V = a_1 a_2 a_3 \tag{22}$$

$$a = (a_1 a_2 a_3)^{1/3} \tag{23}$$

$$\theta = u^\mu_{;\mu} = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \tag{24}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{2}\left[\left(\frac{\dot{a}_1}{a_1}\right)^2 + \left(\frac{\dot{a}_3}{a_2}\right)^2 + \left(\frac{\dot{a}_3}{a_3}\right)^2\right] - \frac{\theta^2}{6} \tag{25}$$

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(H_1 + H_2 + H_3\right). \tag{26}$$

The anisotropy parameter is given by the following relationship as:

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{27}$$

where $\Delta H_i = H_i - H$, $(i = 1, 2, 3)$.

Using equations (5), (6), (7) and (8), the expressions for ρ and p can be obtained as,

$$8\pi G\rho = 3H^2 - \sigma^2 - \frac{3m^2}{a_1^2} - \frac{\ddot{\phi}}{\phi} + 3\left(\frac{\dot{\phi}}{\phi}\right)^2 + 3H\frac{\dot{\phi}}{\phi} \tag{28}$$

$$8\pi Gp = H^2(2q - 1) - \sigma^2 + \frac{m^2}{a_1^2} - \frac{\ddot{\phi}}{\phi} + \left(\frac{\dot{\phi}}{\phi}\right)^2 - H\frac{\dot{\phi}}{\phi}. \tag{29}$$

3. Exact solutions of the field equations and other calculations

In the above section, the quadrature solutions for scale factors A, B and C are obtained with average scale factor a and gauge function ϕ as unknown integrand. To find the exact solutions we need the appropriate expressions for these terms.

As discussed earlier, the universe is undergoing an accelerated expansion which was decelerating in past. So, we can not accept a constant DP. The DP q is defined as:

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{30}$$

The above equation (30) can be transformed to

$$\frac{\ddot{a}}{a} + q\frac{\dot{a}^2}{a^2} = 0. \tag{31}$$

As the average scale factor a is time dependent and if we assume that there is one to one correspondence between t and a, we can further assume the following relationship

$$q = q(t) = q(a(t)). \tag{32}$$

As t and a are increasing function, there is possibility of cosmological bounce or turnaround. The above idea is valid if there is no such cosmological bounce or turnaround. From equations (31) and (32), we obtain

$$\int e^{\int \frac{q}{a}da} da = t + q_0 \tag{33}$$

where q_0 is an integrating constant. Without any loss of generality, we choose $\int \frac{q}{a}da$ in the following manner so that the above equation (33) can be integrable,

$$\int \frac{q}{a}da = \ln f(a). \tag{34}$$

The above equation doesn't violate the nature of generality of solutions. Hence from equations (33) and (34), we get

$$\int f(a)da = t + q_0. \tag{35}$$

For the physically viable solution to be consistent with observations, we assume $f(a)$ in the above equation as follows:

$$f(a) = \frac{1}{\beta} \cdot \frac{na^{n-1}}{\sqrt{1 + (a^n)^2}} \tag{36}$$

where β and n are arbitrary constant and a positive constant respectively. Now integrating equation (35) by taking the value of $f(a)$ from equation (36), we get average scale factor a as

$$a(t) = [\sinh(\beta t)]^{\frac{1}{n}}. \tag{37}$$

The above approach for a time dependent DP is used in many studies [42, 43, 44, 45].

From equation (37), the time varying deceleration parameter comes out to be

$$q(t) = -\frac{a\ddot{a}}{\dot{a}^2} = n.[sech(\beta t)]^2 - 1 = n[1 - (\tanh(\beta t))^2] - 1. \tag{38}$$

We observe from the above equation that $q > 0$ for $t < \frac{1}{\beta}\tanh^{-1}(1 - \frac{1}{n})^{1/2}$, and $q < 0$ for $t > \frac{1}{\beta}\tanh^{-1}(1 - \frac{1}{n})^{1/2}$.

Also, using the relation

$$a = \frac{a_0}{(1 + z)} \tag{39}$$

we get the relation between DP q and redshift z as

$$q(z) = \frac{n}{(\frac{a_0}{1+z})^{2n} - 1} - 1. \tag{40}$$

Next, as far as gauge function is concerned, it is also assumed to be time dependent. Here we take the following relationship between the gauge function ϕ and average scale factor a as

$$\phi = \phi_0 a^\alpha = \phi_0[\sinh(\beta t)]^{\alpha/n} \tag{41}$$

where α is any constant and ϕ_0 is an arbitrary constant (See ref. [45]). Substituting the values of a and ϕ from equations (37) and (41) into equations (19), (20) and (21) and integrating to get the exact solutions for a_1, a_2 and a_3 as,

$$a_1 = [\sinh(\beta t)]^{1/n} \tag{42}$$

$$a_2 = d\,[\sinh(\beta t)]^{1/n} \exp\left[(-1)^{\frac{2\alpha+n+3}{2n}} m_0 \cosh(\beta t) K(t)\right] \tag{43}$$

$$a_3 = \frac{1}{d}\,[\sinh(\beta t)]^{1/n} \exp\left[(-1)^{\frac{2\alpha+3n+3}{2n}} m_0 \cosh(\beta t) K(t)\right] \tag{44}$$

provided $\beta \neq 0$. Here $m_0 = 2h/\beta\phi_0$ and $K(t) = K\left(\frac{1}{2}, \frac{2\alpha+n+3}{n}; \frac{3}{2}; \cosh^2(\beta t)\right)$.

The solution for expansion scalar and shear scalar can be obtained as

$$\theta = \frac{3\beta}{n}\coth(\beta t). \tag{45}$$

$$\sigma^2 = \left(\frac{h}{[\sinh(\beta t)]^{\frac{3+2\alpha}{n}}}\right)^2. \tag{46}$$

The parameters $H_1 = \frac{\dot{a_1}}{a_1}$, $H_2 = \frac{\dot{a_2}}{a_2}$ and $H_3 = \frac{\dot{a_3}}{a_3}$ which are directional Hubble parameters, can be given as

$$H_1 = \frac{\beta}{n}\coth(\beta t), \quad H_2 = \frac{\beta}{n}\coth(\beta t) + \frac{h}{[\sinh(\beta t)]^{\frac{3+2\alpha}{n}}},$$

$$H_3 = \frac{\beta}{n}\coth(\beta t) - \frac{h}{[\sinh(\beta t)]^{\frac{3+2\alpha}{n}}}. \tag{47}$$

The parameters H and V are calculated as

$$H = \frac{\beta}{n}.\coth(\beta t). \tag{48}$$

$$V = \sinh^{3/n}(\beta t). \tag{49}$$

The Anisotropy parameter A_m can be obtained as

$$A_m = \frac{2}{3}\left(\frac{nh}{\beta\coth(\beta t)[\sinh(\beta t)]^{\frac{3+2\alpha}{n}}}\right)^2. \tag{50}$$

The energy density ρ can be calculated as

$$\rho = \frac{\beta^2}{n^2}\left[3(\alpha + 1) + 2\alpha^2\right]\coth^2(\beta t) - \left(\frac{h}{[\sinh(\beta t)]^{\frac{3+2\alpha}{n}}}\right)^2 - \frac{3m^2}{[\sinh(\beta t)]^{2/n}}$$

$$+ \frac{\alpha\beta^2}{n[\sinh(\beta t)]^2} \tag{51}$$

$$p = \frac{(2+\alpha)\beta^2}{n[\sinh(\beta t)]^2} + \frac{m^2}{[\sinh(\beta t)]^{2/n}} - \frac{(3+\alpha)\beta^2}{n^2}\coth^2(\beta t)$$

$$- \left(\frac{h}{[\sinh(\beta t)]^{\frac{3+2\alpha}{n}}}\right)^2 \tag{52}$$

4. Results and discussions

Here we study the behavior of various physical and kinematic parameters obtained above. Using the recent observational value of $q_0 = -0.52^{+0.08}_{-0.04}$ and $H_0 = 69.2 \pm 1.2$ derived from BAO/CMB and H(z) data [46], we have evaluated three different pairs of (n, β) for plotting the graphs. We have used $\alpha = 2$, $h = 1$ for plotting. The above results are discussed in the following subsections.

4.1. Deceleration parameter

The variation of the average scale factor a can clearly be seen in Fig. 1. The behavior of a can clearly be analyzed with the following figures.

Fig. 1(a) represents the time dependent deceleration parameter (q) for three different pairs (n, β). We observe that for $(n = 2, \beta = 92.75)$ there is a phase transition from decelerating expansion ($q > 0$) of the universe in the past to accelerating expansion ($q < 0$) at present. This fact is highly supported by Type Ia supernova and CMB data [14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. For this pair of $(n = 2, \beta = 92.75)$ the present value of the DP is calculated as -0.54. The recent observational data supports this value of q. Fig. 1(b) shows the variation of DP versus redshift z for two pairs of (n, a_0). It also shows the transient nature of the DP for $(n = 2, a_0 = 1.5)$. The phase transition from decelerating ($q > 0$) to accelerating ($q < 0$) universe took place at $z \approx 0.50$. SNe type Ia measurements provide the direct empirical proof of the phase transition from the past deceleration to present acceleration. Recent acceleration ($z < 0.5$) and past deceleration ($z > 0.5$) were clearly favored by the SNe data which is established in the preliminary observations. In 2004, the value of $z_t = 0.46 \pm 0.13$ at $(1\,\sigma)$ c.l. [24] was obtained which was further modified to $z_t = 0.43 \pm 0.07$ at $(1\,\sigma)$ c.l. [25] in 2007. This data was provided by the High-z Supernova Search (HZSNS) team. More recently, Santos et al. [46] using SN Ia + BAO/CMB(Planck) + H(z) data, including the 68% and 95% confidence intervals, have obtained z_t in the range 0.66–0.70. So the recent observations also support our results [24, 25, 26, 27, 28, 29, 46].
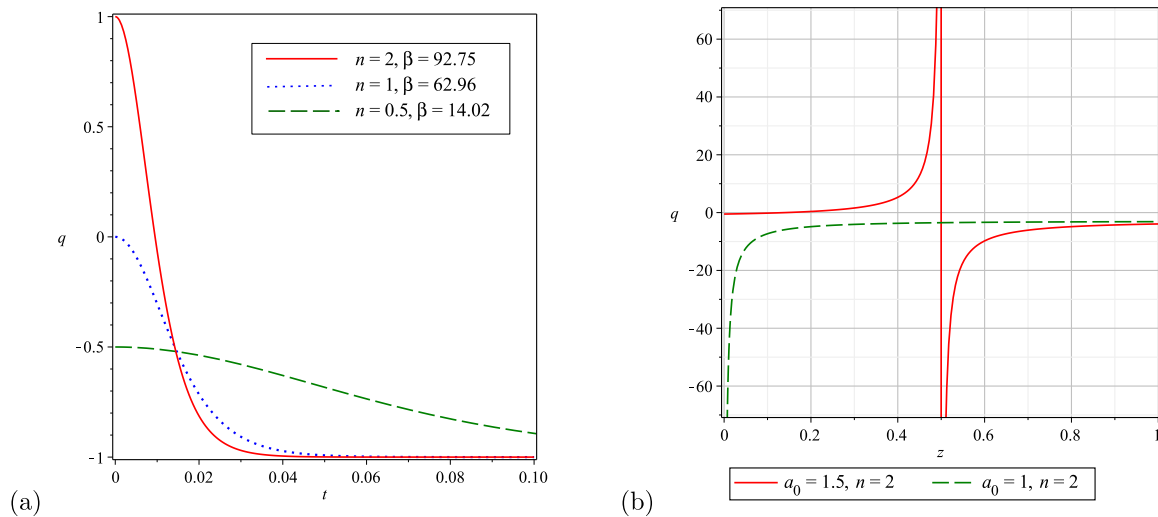
Fig. 1. (a) The graph of deceleration parameter q versus cosmic time t. (b) The graph of deceleration parameter q versus redshift z.

4.2. Jerk parameter

The expansion of the universe is verified by Hubble parameter $H(t) = \frac{\dot{a}}{a}$. The modern cosmology starts with this concept of expanding universe. The measurement of the contemporary deceleration parameter q_0 where $q(t) = -\frac{a\ddot{a}}{\dot{a}^2}$ is an important parameter in the pursuit of observational cosmology in recent times. Considering the third derivative of a is useful as the universe was once decelerating is now accelerating. A dimensionless third derivative of the scale factor $a(t)$ with respect to cosmic time t is jerk parameter j which measures the jerk and is given as follows [47]:

$$j = \frac{\dddot{a}a^2}{\dot{a}^3} \tag{53}$$

An alternative and a convenient method to describe cosmological models close to concordance ΛCDM model is provided by the use of jerk parameter which is clearly described by Blandford et al. [47] and Rapetti et al. [48]. For the ΛCDM model, the value of j is always unity. This is a remarkable feature of j. A non-ΛCDM model occurs if there is any deviation from the value of $j = 1$. This is similar as deviations from the equation of state (EoS) parameter $\omega = -1$ do in more standard dynamical approaches. In our model, the jerk parameter j is obtained as

$$j = 1 + (2n^2 - 3n)(sech(\beta t))^2 \tag{54}$$

From Fig. 2, it is observed that, at the early time j assumes a positive value greater than 1, which shows that our model violates the standard ΛCDM model at the early stage, but at the late time j approaches to 1 i.e. our model approaches to ΛCDM model. Finally, we conclude from all current cosmological observations that the jerk of the universe equals one [49].

4.3. Statefinder diagnosis

The properties of dark energy in a model can be characterized by the Statefinder which is a geometrical diagnostic. This can be done in an independent manner. A pair of dimensionless parameters (r, s) constitutes the Statefinder. These parameters are defined as follows:

$$r = \frac{\dddot{a}}{aH^2}, \quad s = \frac{r - 1}{3(q - \frac{1}{2})} \tag{55}$$

This pair (r, s) are widely used to check the viability of a variety of dark energy (DE) models. This diagnostic tool is independent of dark energy density which is an important feature of the Statefinder diagnostic. In $r - s$ plane, the region of phantom and quintessence DE eras is described
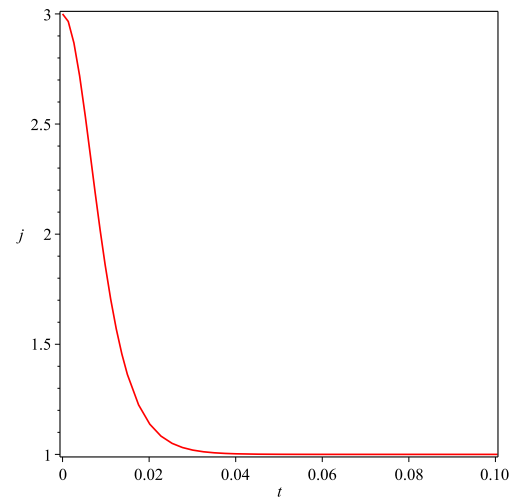

Fig. 2. The plot of jerk parameter j versus cosmic time t for $n = 2$, $\beta = 92.75$.

by s greater than 0 and r less than 1, $(r, s) = (1, 0)$ corresponds to ΛCDM limit, $(r, s) = (1, 1)$ represents CDM limit and s is less than 0 and r is greater than 1 indicates chaplygin gas [50, 51].

In our derived model, the parameters r and s are calculated as follows

$$r = \left(\frac{(\cosh(\beta t))^2 + 2n^2 - 3n}{(\cosh(\beta t))^2 - 1} \right) \cdot (\tanh(\beta t))^2,$$

$$s = -\frac{2}{3} \left(\frac{2n^2 - 3n}{3(\cosh(\beta t))^2 - 2n} \right) \tag{56}$$

From above two equations, we find a relation between r and s as follows

$$r = -\frac{1}{2} \cdot \left[\frac{(4n^2 - 6ns - 6n - 9s)(18ns - 4n - 21s + 6)}{(4n^2 - 6ns - 6n + 9s)(2n - 3s - 3)} \right] \tag{57}$$

Fig. 3 shows the graph of r vs s for $n = 2$. We see that Statefinder parameters in most of the evolution history remains in the region $(s > 0, r \leq 1)$, which corresponds to phantom and quintessence DE eras. It shows a violation in a very short region then again follow the same era. From Eq. (56) and its Fig. 3, we see that at $s \to 0$, $r \to 1$ which corresponds to ΛCDM model. So, our derived model resembles with ΛCDM model in most of the evolution history.

4.4. Anisotropy parameter

From eq. (50) and its corresponding Fig. 4, we observe that the anisotropic parameter is a positive decreasing function of time. At the
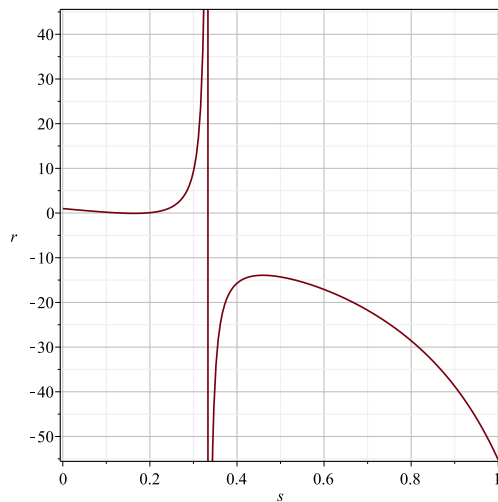
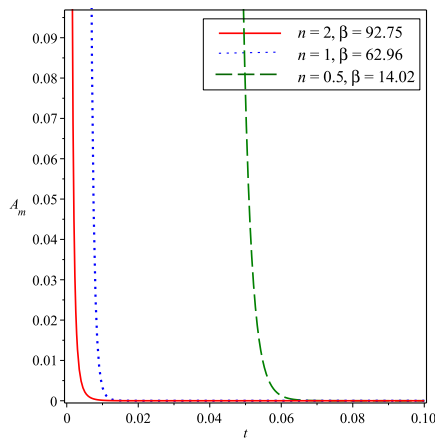**Fig. 3.** The plot of statefinder parameters r versus s.

Fig. 5. The plot of energy density ρ versus cosmic time t.

Fig. 4. The plot of anisotropic parameter A_m versus cosmic time t.
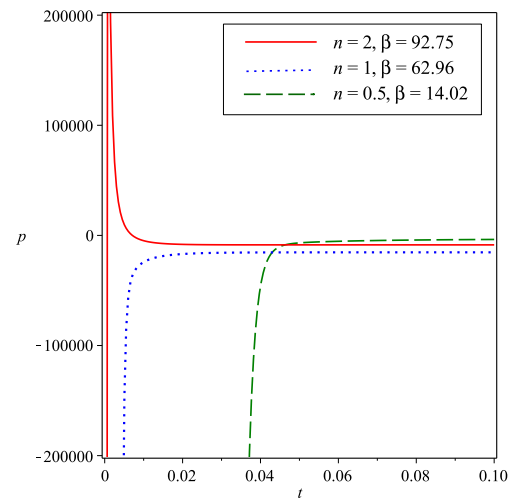
Fig. 6. The plot of isotropic pressure p versus cosmic time t.

early phase of the universe our derived model is highly anisotropic, which attains isotropy in due course of time. At late time $A_m \to 0$, i.e. the desired isotropy of the universe is attained. Modern cosmology also supports that the early universe was highly anisotropic, which in due course of time attains isotropy, and at present the Universe is isotropic. This is also evident in our model. So our model is consistent with observations in this respect.

4.5. Energy density

From eq. (51) and its corresponding Fig. 5, we observe that the energy density of our derived model is a positive decreasing function of time. At the early phase of the universe it possesses singularity ($\rho \to \infty$), and this high energy density is responsible for the big-bang. It decreases sharply thereafter, which corresponds to the period of rapid expansion of the Universe. Then the rate of decrease becomes moderate, indicating that the rate of expansion becomes slower. At late time (i.e. as $t \to \infty$) the energy density tends to zero, which shows that the universe is expanding and will keep on expanding forever. So or model favors the expanding Universe.

4.6. Isotropic pressure

The isotropic pressure p is calculated in Eq. (52). We have plotted it against t in Fig. 6 for three pairs of (n, β). All the three starts with highly negative pressure. For $(n = 2, \beta = 92.75)$, it acquires a high positive value and then starts decreasing. Also, $p \to 0$ as $t \to \infty$. The negative pressure
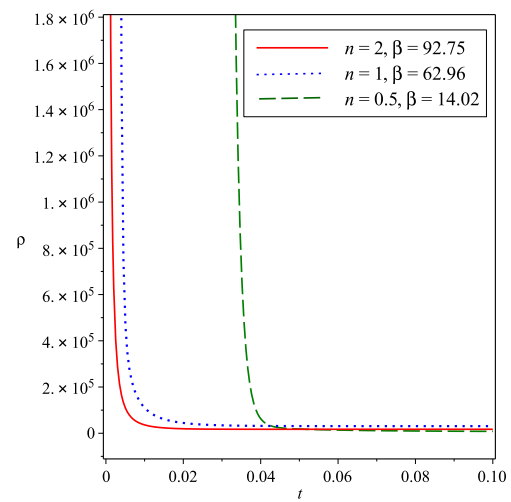
may be a possible cause of the accelerated expansion of the universe. The red (continuous) graph in Fig. 6 indicates that at the early phase the expansion is accelerated (p – ve), then it is decelerated (p + ve), while the other two curves may correspond to accelerated expansion for entire evolution history. Also, $p \to 0$ as $t \to \infty$ for all the three cases. This also validate our expanding Universe model.

4.7. Other physical and geometric parameters

This model corresponds to big-bang singular model of the universe as the volume scalar is initially zero and it tends to infinity for the large time for $t \to \infty$. This means that the universe is expanding continuously. The other parameters like θ, σ^2, H, H_1, H_2 and H_3 are all infinity initially at $t = 0$ and all these parameters will vanish for the late time as $t \to \infty$. This means that the model is inhomogeneous and anisotropic in early time of evolution and will become homogeneous and isotropic in due course of time, which shows consistency with most of the observational results. The gauge function has accelerating and decelerating nature. It will behave exactly like average scale factor as they are proportional. The behavior of all the above parameters suggest that the model is exactly behaving like a singular model of the universe. That is the universe starts with singularity. There is a transition in the model from initial anisotropy to isotropy at present.
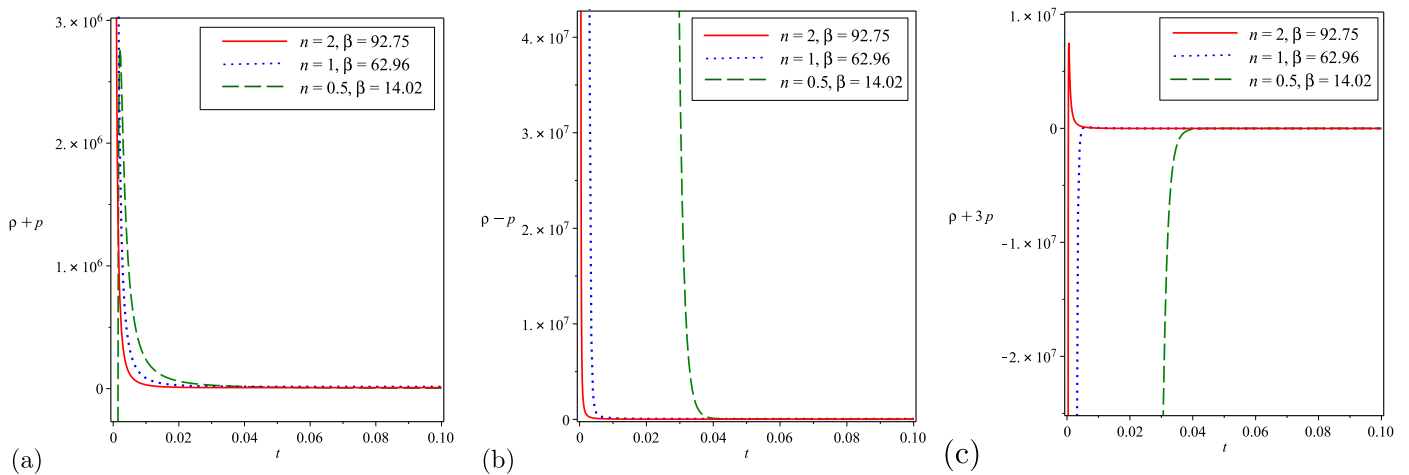
Fig. 7. (a) Presentation of energy condition of $\rho + p$ versus t for different values of n and β. (b) Presentation of energy condition of $\rho - p$ versus t for different values of n and β. (c) Presentation of energy condition of $\rho + 3p$ versus t for different values of n and β.

4.8. Energy conditions

With the help of energy density ρ and isotropic pressure p, there are two approaches to diagnose a cosmological model. The first is by finding equations of state $\omega = \frac{p}{\rho}$. Depending on the values of ω, the cosmological models can be classified as quintessence, standard Λ-CDM, phantom etc. This approach is basically used in dark energy models. In another approach, we test the energy conditions. The null energy condition (NEC) and weak energy conditions (WEC) are given by (i) $\rho \geq 0$ and (ii) $\rho + p \geq 0$, dominant energy conditions (DEC) are given by (iii) $|p| \leq \rho$ (i.e. $\rho - p \geq 0$ and $\rho + p \geq 0$). Whereas (iv) $\rho + 3p \geq 0$ is the strong energy condition (SEC). Based on Eqs. (51) and (52), the left hand side of energy conditions have been plotted in Figs. 7(a), 7(b) and 7(c). From these figures and Fig. 5, it is observed that (i) $\rho \geq 0$; (ii) $\rho + p \geq 0$; (iii) $\rho - p \geq 0$, meaning that our model satisfies the null, weak as well as the dominant energy conditions. From Fig. 7(c) we observe that $\rho + 3p \leq 0$ at most of the time of evolution history, meaning that the strong energy condition is violated, which is acceptable for our model as our model resembles with the dark energy models which approaches to standard ΛCDM model.

5. Conclusion

Here we have obtained an exact solution for Scale Covariant theory in perfect fluid medium with Bianchi type-V as the line element. For finding the solution, we have used a well established time dependent deceleration parameter. The justification for the variable deceleration parameter is fully explained with several given references. By using this DP, we have obtained the average scale factor $a(t) = [\sinh(\beta t)]^{1/n}$, where n and β are positive constants. The model represents a transit model for the pair $(n = 2, \ \beta = 92.75)$, showing transition from early decelerating $(q > 0)$ to the present accelerating $(q < 0)$ universe. Since the model has finite singularity, this model represents a big bang singular model $(V \to 0$ at $t \to 0)$ of the universe. Our model resembles with ΛCDM model with jerk $j \to 1$ and statefinders $(r, s) \to (1, 0)$. In addition, different physical, geometric and kinematic parameters have been obtained and studied. The results obtained are found to be in good agreement with recent observations and established theories.

Declarations

Author contribution statement

Mohammad Zeyauddin, Rashid Zia, C V Rao: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

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