



# Bianchi type-V dark energy cosmological model in general relativity in the presence of massive scalar field



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## ABSTRACT

In this paper, we discuss spatially homogeneous and anisotropic Bianchi type-V dark energy cosmological model in the presence of an attractive massive scalar field in general relativity. We have solved the field equations using (i) the shear scalar of the metric is proportional to the expansion scalar which results a relationship between metric potentials and (ii) a power law between the massive scalar field and the average scale factor. We have computed the cosmological parameters like dark energy density, equation of state parameter, skewness parameters, deceleration parameter and statefinder parameters of our dark energy model with massive strings and discussed their physical significance in the light of the recent scenario of accelerated expansion of the universe and cosmological observations.

## 1. Introduction

General Relativity (GR) is a geometric theory which describes gravitational phenomena. It is also useful in constructing mathematical models in cosmology which deals with large scale structure of the universe. The modern cosmological observations have confirmed the accelerated expansion of the universe [1, 2, 3]. It has also been confirmed that the reason for this late time acceleration is an exotic force with negative pressure dubbed as 'dark energy' (DE). In order to explain this DE, various DE models in GR and in modified theories of gravitation have been investigated. Cosmological constant, which represents energy density associated with quantum vacuum, was considered to be the simplest candidate to produce this cosmic acceleration. But this simple DE model is plagued with the coincidence and other serious problems in general relativity. Hence, different dynamical DE models have been investigated to explain this cosmic acceleration of the universe. Noteworthy among them are the scalar field models such as quintessence and k-essence [4, 5]. It may be noted that there is another class of modified matter models based on perfect fluids so-called generalized Chaplygin gas models [6], pilgrim DE models [7, 8, 9] and holographic DE models [10, 11]. There exists another class of DE models that modify general relativity. The DE models corresponding to  $f(R)$  gravity [12],  $f(R, T)$  gravity [13] and scalar-tensor theories of gravity proposed by Brans and

Dicke [14] and Saez and Ballester [15].

There exist several dynamical DE models, in literature, presented by various authors both in GR and in modified theories of gravitation. Among them we are interested in Bianchi type spatially homogeneous and anisotropic DE models which are very important to study the evolution of our universe at its early stages and to describe the small amounts of anisotropy at the beginning of the universe. A host of authors have investigated spatially homogeneous and anisotropic DE models both in general relativity and in modified theories of gravitation [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26] (we mention only some of them). Also it is useful to construct viable Bianchi type scalar field DE models in the framework of particle physics that is DE models in the presence scalar-meson fields and massive scalar fields in general relativity.

In cosmology, scalar fields play a vital role as they describe matter fields with spin less quanta and represent gravitational fields. Scalar fields are classified into two types-zero mass scalar fields which describe long range interactions and massive scalar fields which represent short range interactions. It is this physical importance that has attracted the attention of many researchers to study the scalar fields. It is also supposed that the scalar fields cause the accelerated expansion of the universe and help to solve the horizon problem in cosmology.

Cosmological models in the presence of mass less and massive scalar fields coupled with different physical systems have been extensively

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studied in the past [27, 28, 29, 30, 31, 32, 33]. Since our main aim is to discuss dynamical DE models in the presence of scalar fields, we will mention some investigations of DE models in the presence of massive and mass less scalar fields. Reddy [34] has studied Bianchi type-V (B-V) DE model with scalar meson fields in general relativity. Naidu [35] discussed Bianchi type-II modified holographic Ricci DE model with attractive massive scalar field. Recently, Aditya and Reddy [36] have discussed the dynamics of Bianchi type-III cosmological model in the presence of anisotropic DE and an attractive massive scalar meson field.

The above discussion has inspired us to investigate the dynamics of spatially homogeneous anisotropic DE model in B-V space-time. This is because of the fact that B-V models contain isotropic models as special cases and allow arbitrary small anisotropies at some instant of cosmic time. Also FRW models are particular cases of Bianchi type-I, V and IX universes. Following is the plan of this paper: In Sec. 2, we derive the Einstein explicit field equations in the presence of anisotropic DE and attractive massive scalar field. Sec. 3 deals with the solution of the field equations and presentation of DE model. Evaluation and physical discussion of dynamical parameters of the model are presented in Sec. 4. Conclusions are presented in Sec.5.

## 2. Theory

### 2.1. Einstein field equations for DE

The space-time represented by Bianchi type –V metric is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2x} dy^2 - C^2 e^{-2x} dz^2 \tag{1}$$

where A, B and C are functions of cosmic time t.

The combined stress-energy tensor in the presence of anisotropic DE and an attractive massive scalar field is given by

$$T_{ij} = T_{ij}^{(de)} + T_{ij}^{(s)} \tag{2}$$

where

$$T_{ij}^{(de)} = (\rho_{de} + p_{de}) - p_{de} g_{ij} \tag{3}$$

$$T_{ij}^{(s)} = \varphi_{,i} \varphi_{,j} - \frac{1}{2} (\varphi_{,k} \varphi^{,k} - M^2 \varphi^2) \tag{4}$$

Here  $\rho_{de}$  is the DE density,  $p_{de}$  is the DE pressure, M is the mass of the scalar field  $\varphi$  which satisfies the Klein-Gordon equation

$$g^{ij} \varphi_{,ij} + M^2 \varphi = 0 \tag{5}$$

and comma and semi colon denote ordinary and covariant differentiation respectively.

The energy-momentum tensor of anisotropic DE fluid given by Eq. (3) can be parameterized as

$$T_{ij}^{(de)} = \text{diag}[1, \omega_{de}, -(\omega_{de} + \gamma), -(\omega_{de} + \delta)] \rho_{de} \tag{6}$$

where

$$\omega_{de} = \frac{p_{de}}{\rho_{de}} \tag{7}$$

Here  $\omega_{de}$  is the EoS parameter of DE and the skewness parameters  $\gamma$  and  $\delta$  are the deviations from  $\omega_{de}$  along y and z-axes respectively.

Now, Einstein field equations in the presence of anisotropic DE fluid and attractive massive scalar fields are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = - \left( T_{ij}^{(de)} + T_{ij}^{(s)} \right) \tag{8}$$

Here we choose units such that  $8\pi G = c = 1$ . Using comoving coordinates the explicit form of field Eq. (8) for the metric (1), by the use of

Eqs. (3), (4), (5), (6), and (7), can be written as

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} - \frac{\dot{\varphi}^2}{2} - \frac{M^2 \varphi^2}{2} = \rho_{de} \tag{9}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} + \frac{\dot{\varphi}^2}{2} - \frac{M^2 \varphi^2}{2} = -\omega_{de} \rho_{de} \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} + \frac{\dot{\varphi}^2}{2} - \frac{M^2 \varphi^2}{2} = -(\omega_{de} + \gamma) \rho_{de} \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} + \frac{\dot{\varphi}^2}{2} - \frac{M^2 \varphi^2}{2} = -(\omega_{de} + \delta) \rho_{de} \tag{12}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2 \frac{\dot{A}}{A} = 0 \tag{13}$$

and the Klein-Gordon Eq. (5) for the metric (1) takes the form

$$\ddot{\varphi} + \dot{\varphi} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + M^2 \varphi = 0 \tag{14}$$

The conservation law for the energy-momentum tensor of DE fluid yields

$$\dot{\rho}_{de} + \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) (\rho_{de} + p_{de}) = 0 \tag{15}$$

where an over head dot indicates derivative with respect to cosmic time t.

We shall now define cosmological parameters which would help in solving the above field equations. The volume V and the average scale factor a(t) are given by

$$V = a^3 = ABC. \tag{16}$$

The average Hubble parameter H, scalar expansion  $\theta$  and the shear scalar  $\sigma^2$  are defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right). \tag{17}$$

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{18}$$

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{1}{2} \left[ \left( \frac{\dot{A}}{A} \right)^2 + \left( \frac{\dot{B}}{B} \right)^2 + \left( \frac{\dot{C}}{C} \right)^2 - \frac{\dot{A}\dot{B}}{A} - \frac{\dot{B}\dot{C}}{B} - \frac{\dot{C}\dot{A}}{C} \right]. \tag{19}$$

The anisotropy parameter and deceleration parameter are, respectively, given by

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2 \tag{20}$$

$$q = -1 + \frac{d}{dt} \left( \frac{1}{H} \right) \tag{21}$$

## 3. Calculation

### 3.1. Solution of field equations and DE model

In order to present a DE model in the presence of massive scalar field, here, we solve the field Eqs. (9), (10), (11), (12), (13), (14), and (15). Integrating Eq. (13), we get

$$A^2 = k_1 BC \tag{22}$$

Without any loss of generality, the constant of integration  $k_1$  can be chosen as unity so that we have

$$A^2 = BC \tag{23}$$

The set of field Eqs. (9), (10), (11), and (12) together with Eqs. (14) and (15) are a system of five independent equations [Eq. (15), being the conservation equation] in eight unknowns  $A, B, C, \rho_{de}, \omega_{de}, V, \delta, \gamma$ . Hence to find a determinate solution we use the following physically significant conditions:

i. The shear scalar of the space-time is proportional to the expansion scalar so that we have [37].

$$C = B^n \tag{24}$$

where  $n \neq 1$  is a positive constant which preserves the anisotropy of the space time.

ii. In order to reduce the mathematical complexity of the system we use [39].

$$3 \frac{\dot{A}}{A} = - \frac{\dot{\varphi}}{\varphi} \tag{25}$$

Now from Eqs. (14), (23), and (25), we obtain

$$\varphi = \exp\left(\varphi_0 t - \frac{M^2 t^2}{2} + \varphi_1\right). \tag{26}$$

Eqs. (25) and (26) will, together, give us

$$A = a_1 \exp\left(\frac{\frac{1}{2}M^2 t^2 - \varphi_0 t - \varphi_1}{3}\right) \tag{27}$$

where  $a_1 = a_0^{\frac{1}{3}}$ ,  $a_0$ ,  $\varphi_0$  and  $\varphi_1$  are constants of integration. From Eqs. (23), (24), and (25) we get

$$B = \left[ a_1 \exp\left(\frac{\frac{1}{2}M^2 t^2 - \varphi_0 t - \varphi_1}{3}\right) \right]^{\frac{2}{n+1}} \tag{28}$$

$$C = \left[ a_1 \exp\left(\frac{\frac{1}{2}M^2 t^2 - \varphi_0 t - \varphi_1}{3}\right) \right]^{\frac{2n}{n+1}} \tag{29}$$

Now using Eqs. (27), (28), and (29), we can write the B-V model (1) as

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$$\omega_{de} = \frac{-1}{\rho_{de}} \left[ \frac{M^2}{3} + \frac{4(n^2 + 1)(M^2 t - \varphi_0)^2}{(n + 1)^2} - \left( \frac{1}{a_1^2} - \frac{(M^2 t - \varphi_0)^2}{18} \right) \exp\left(\frac{2\varphi_0 t - 2\varphi_1 - M^2 t^2}{3}\right) - \frac{M^2}{2} \exp\left(\frac{2\varphi_0 t - M^2 t^2 + 2\varphi_1}{3}\right) \right] \tag{40}$$


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$$ds^2 = dt^2 - \left[ a_1 \exp\left(\frac{\frac{1}{2}M^2 t^2 - \varphi_0 t - \varphi_1}{3}\right) \right]^2 dx^2 - \left[ a_1 \exp\left(\frac{\frac{1}{2}M^2 t^2 - \varphi_0 t - \varphi_1}{3}\right) \right]^{\frac{4}{n+1}} e^{-2x} dy^2 - \left[ a_1 \exp\left(\frac{\frac{1}{2}M^2 t^2 - \varphi_0 t - \varphi_1}{3}\right) \right]^{\frac{4n}{n+1}} e^{-2x} dz^2 \tag{30}$$

with the massive scalar field given by Eq. (26).

### 3.2. Physical and dynamical parameters of the model

The following are the dynamical parameters in the universe given by Eqs. (30) and (26) which are necessary for discussion in cosmology:

$$V = \left[ a_1 \exp\left(\frac{\frac{1}{2}M^2 t^2 - \varphi_0 t - \varphi_1}{3}\right) \right]^3 \tag{31}$$

$$H = \frac{1}{3}(M^2 t - \varphi_0) \tag{32}$$

$$\theta = 3H = (M^2 t - \varphi_0) \tag{33}$$

$$\sigma^2 = \frac{(M^2 t - \varphi_0)^2}{6} \left( \frac{n - 1}{n + 1} \right)^2 \tag{34}$$

$$\Delta = \frac{2}{3} \left( \frac{n - 1}{n + 1} \right)^2 \tag{35}$$

$$q = - \left( 1 + \frac{3M^2}{(M^2 t - \varphi_0)^2} \right) \tag{36}$$

Now using Eqs. (26), (27), (28), and (29) in Eq. (9) we find DE density as

$$\rho_{de} = \frac{(M^2 t - \varphi_0)^2}{18(n + 1)^2} \left( 4(n^2 + 16n + 4) - (n + 1)^2 \exp\left(\frac{2\varphi_0 t - M^2 t^2 - 2\varphi_1}{3}\right) \right) - \frac{1}{a_1^2} \exp\left(\frac{2\varphi_0 t + 2\varphi_1 - M^2 t^2}{3}\right) - \frac{M^2}{2} \exp\left(\frac{2\varphi_0 t - M^2 t^2 + 2\varphi_1}{3}\right) \tag{37}$$

Using Eqs. (10), (11), (27), (28), and (29) we obtain the skewness parameter as

$$\gamma = \frac{1}{\rho_{de}} \left[ \frac{(n - 1)}{3(n + 1)} (M^2 + (M^2 t - \varphi_0)^2) \right] \tag{38}$$

where  $\rho_{de}$  is given by Eq. (37).

From Eqs. (10), (12), (27), (28), and (29), we get another skewness parameter as

$$\delta = \frac{1}{\rho_{de}} \left[ \frac{(1 - n)}{3(1 + n)} (M^2 + (M^2 t - \varphi_0)^2) \right] \tag{39}$$

From Eqs. (10), (26), (27), (28), and (29) the EoS parameter can be obtained as

The statefinder parameters are defined as

$$r = \frac{\overset{\bullet\bullet\bullet}{a}}{aH^3}, s = \frac{r - 1}{3(q - \frac{1}{2})} \tag{41}$$

which, in this case, are found to be

$$r = 1 + \frac{9M^2}{(M^2 t - \varphi_0)^2}, s = \frac{-2M^2}{2M^2 + (M^2 t - \varphi_0)^2} \tag{42}$$

### 4. Results and discussion

Eq. (30) represents B-V DE universe in the presence of attractive massive scalar field given by Eq. (26). The model has no initial singularity, i.e. at  $t = 0$ . It can be seen that the physical parameters  $H, \theta, \sigma^2$  tend to finite values as  $t \rightarrow 0$  and tend to infinity as  $t \rightarrow \infty$ . The spatial volume  $V$  shows an exponential increase with cosmic time which shows that the universe undergoes an exponential expansion from a finite volume. It may be observed that the anisotropy parameter is constant throughout which shows that the universe is spatially homogeneous and uniform. Also at  $n = 1$ , the mean anisotropy parameter, shear scalar and skewness parameters vanish showing that our universe becomes isotropic and shear free at late times which, in fact, should be the case in the light of the recent cosmological observations.

**Scalar field:** The behavior of scalar field  $\phi$  versus cosmic time for different values of  $\phi_0$ . It can be seen that the scalar field is positive throughout the evolution of the universe for all three values of  $\phi_0$ . It increases with time and attains a maximum value at certain point of time and then decreases to attain a constant positive value. Also, we observed that as  $\phi_0$  increases the scalar field increases.

**Deceleration parameter  $q$ :** This plays a significant role in the discussion of the nature of the model obtained. When  $q > 0$ , the model exhibits decelerates in the standard way, when  $q = 0$ , a constant rate of expansion and when  $-1 \leq q < 0$  an accelerated expansion. Also when  $q = -1$ , the universe shows an exponential expansion and when  $q < -1$ , super exponential expansion.

Fig. 2 shows the behavior of deceleration parameter versus cosmic time  $t$  for different values  $\phi_0$ . It can be seen that, for our model,  $q$  is less than  $-1$  and hence we obtain a universe with super exponential expansion. Also it may be observed as  $\phi_0$  increases the super exponential expansion slows down.

**Statefinder parameters (r,s):** Sahni et al. [38] have proposed two parameters, known as statefinder parameters, defined by Eq. (41). The main aim of these parameters is to distinguish the different DE models that are being proposed from time to time in modern cosmology. When  $(r, s) = (1,1)$ , we have cold dark matter (CDM) limit while  $(r, s) = (1,0)$  gives  $\Lambda$ CDM limit. Also, when  $r < 1$  we have quintessence DE region and for  $s > 0$  phantom DE regions.

Fig. 3 Represents the statefinder parameters for our model. It may be observed that as  $\phi_0$  increases our model approaches  $\Lambda$ CDM model at certain point of time in future.

**EoS parameter ( $\omega_{de}$ ):** This parameter characterizes DE and is defined by Eq. (7). This was, usually, considered as constant with phase values  $-1, 0, \frac{1}{3}$ , and  $+1$  for vacuum field, dust distribution, radiation and stiff fluid. But this should not be regarded as constant and should be in general function of time or redshift [39]. Several authors have investigated quintessence model involving scalar fields which lead to time dependent

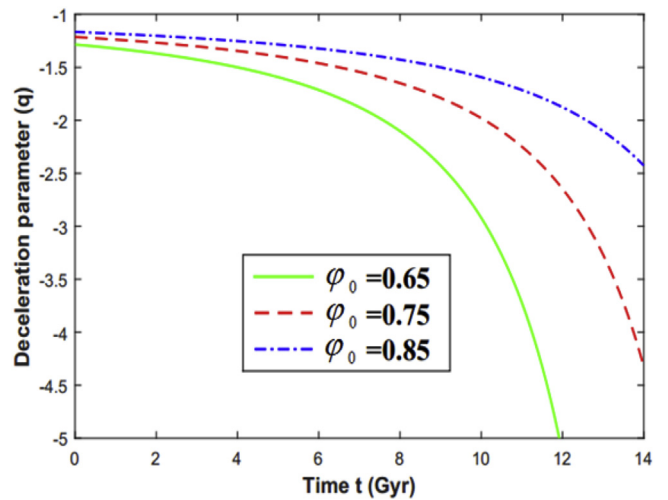


Fig. 2. Plot of deceleration parameter versus cosmic time for  $M=14$ .

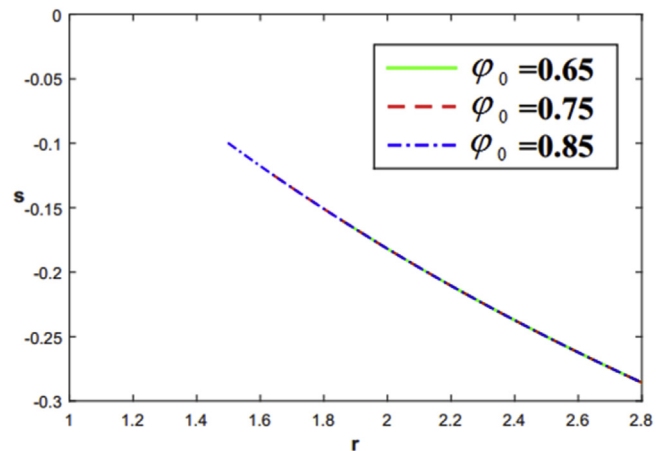


Fig. 3. Plot of r-s plane for  $M=14$ .

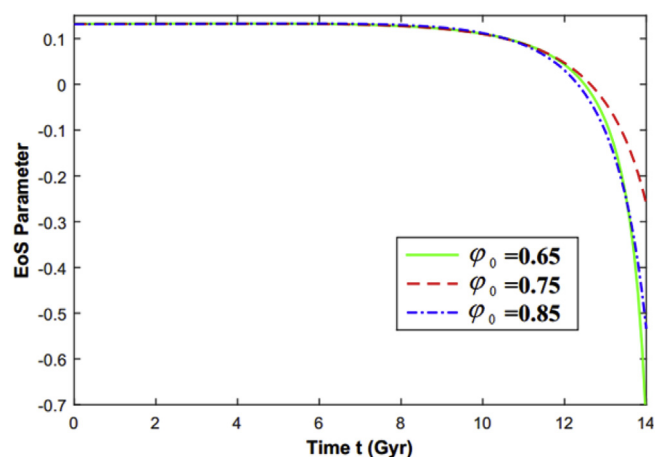


Fig. 4. Plot of EoS parameter versus time  $t$  for  $M=14, n=0.95, a_1=55$  and  $\phi_1 = 5$ .

EoS parameter.

Fig. 4 depicts the variation of EoS parameter for our model. It may be observed that for different values of  $\phi_0$  the model starts in the matter dominated region, varies in the quintessence region ( $-1 < \omega_{de} < -\frac{1}{3}$ )

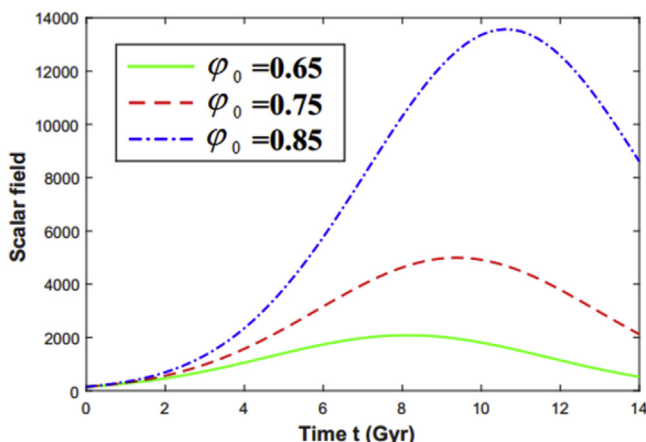


Fig. 1. Plot of scalar field versus cosmic time  $t$  for  $M=14$  and  $\phi_1 = 5$ .

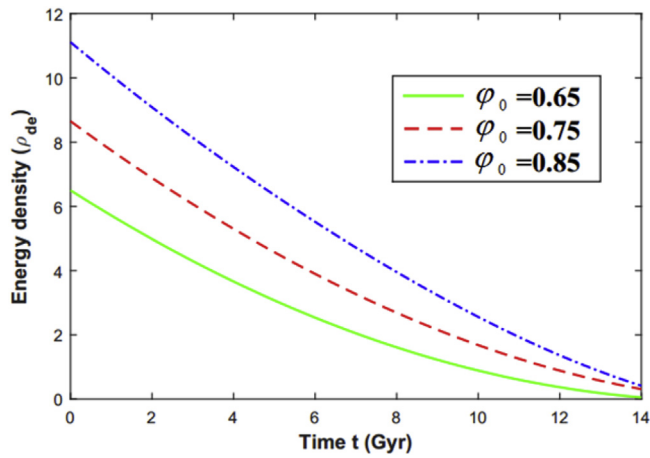


Fig. 5. Plot of energy density versus cosmic time  $t$  for  $M=14$ ,  $n=0.95$ ,  $a_1=55$  and  $\phi_1 = 5$ .

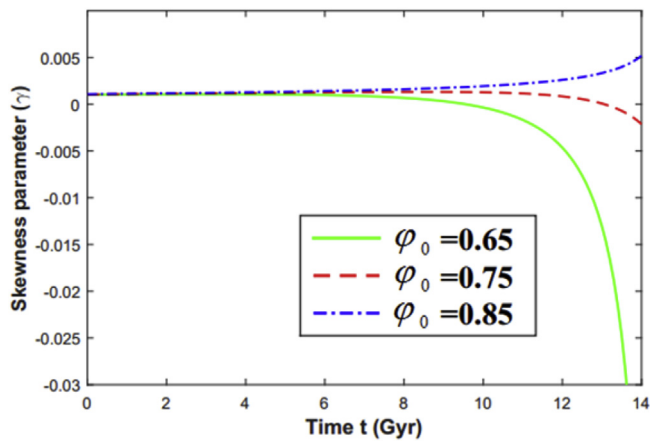


Fig. 6. Plot of skewness parameter ( $\gamma$ ) versus cosmic time. for  $M=14$ ,  $n=0.95$ ,  $a_1=55$  and  $\phi_1 = 5$ .

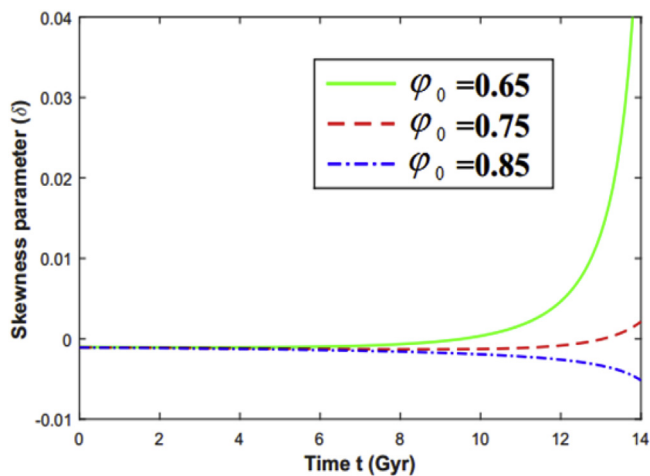


Fig. 7. Plot of skewness parameter ( $\delta$ ) versus cosmic time. for  $M=14$ ,  $n=0.95$ ,  $a_1=55$  and  $\phi_1 = 5$ .

and proceeds to the phantom region ( $\omega_{de} < -1$ ).

Energy density and skewness parameters.

Fig. 5 gives the energy density of our model for different values of  $\phi_0$ . We observe that it is always positive and increases as  $\phi_0$  increases.

Skewness parameter describes the amount of anisotropy in the dark energy fluid. These are represented in Figs. 6 and 7 for different values of  $\phi_0$ . It may be observed that the effect of scalar field on the skewness parameters, initially is negligible and it influences at the present epoch.

### 5. Conclusions

This investigation is about the determination of a spatially homogeneous and anisotropic DE cosmological model in the presence of attractive massive scalar field in the framework of B-V space-time. In order to obtain a deterministic model we have used a relation between metric potentials and a power law between the average scale factor and the scalar field. We have computed all the cosmological and kinematical parameters and discussed their physical significance in the light of the present cosmological scenario and observations. The following are the results in brief:

- Our model describes spatially homogeneous and anisotropic B-V dark energy model with an attractive massive scalar field in general relativity.
- Our model is non-singular and undergoes an exponential expansion from finite volume leading to early inflation.
- Since the anisotropy parameter is constant, the model is homogeneous and uniform throughout. However, at late times the model is isotropic and shear free which is in accordance with the present cosmological scenario.
- All the physical quantities of the model are finite initially and tend to infinity for sufficiently large values of cosmic time.
- We observe that the scalar field is positive throughout the evolution of the universe for all three values of  $\phi_0$ . Also, it is observed that as  $\phi_0$  increases the scalar field increases. (Fig. 1)
- It can be seen that, for our model,  $q$  is less than  $-1$  and hence we obtain a universe with super exponential expansion. Also it may be observed as  $\phi_0$  increases the super exponential expansion slows down (Fig. 2).
- Study of statefinders plane ( $r$ - $s$  plane) shows that as  $\phi_0$  increases our model approaches  $\Lambda$ CDM model at certain point of time in future (Fig. 3).
- We observed that for different values of  $\phi_0$  the model starts in the matter dominated region, varies in the quintessence region ( $-1 < \omega_{de} < \frac{-1}{3}$ ) and proceeds to the phantom region ( $\omega_{de} < -1$ ) (Fig. 4). Also, the energy density of our model is always positive and increases as  $\phi_0$  increases.
- Skewness parameter describes the amount of anisotropy in the dark energy fluid. In our model the effect of scalar field on the skewness parameters, initially is negligible and it influences at the present epoch.
- It may be observed that the scalar field in the model influences all the physical parameters of the model.

Finally, we may conclude that all the above results are in good agreement with the present cosmological observations. Our DE model will help for a better understanding of DE driving the universe acceleration and which is, even today, is a cosmological mystery.

### Declarations

#### Author contribution statement

- R. L. Naidu: Analyzed and interpreted the data.
- Y. Aditya: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.
- D. R. K. Reddy: Conceived and designed the analysis; Wrote the paper.

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The authors declare no conflict of interest.

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