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#### Research article

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# Adaptive urban drinking water supply model using the effect of node elevation and head loss formula: A case study

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#### ABSTRACT

Along with population growth and health improvement, water demand due to urbanization is increasing and creating a need to develop a strategy for handling water supply networks (WSNs). In the last decade, software modeling of WSNs has been developed to evaluate the state of networks in terms of pressure control, leakage analysis, and overall demand determination. In the case of very complex and extremely large networks, it is very difficult to manage the water supply. Metropolitan Waterworks Authority (MWA) in Thailand has to supply drinking water to the three densely populated cities; Bangkok, Nonthaburi, and Samut Prakan, that cover an area of 2944.05 km<sup>2</sup>. Hence, MWA has developed a main pipe model using EPANET software as a managing tool. This tool can offer a good solution for the water supply, but there is approximately a 14 percent error, mainly due to not having the elevation data of the pipe network. The current research is based on demand and pressure modeling analysis with utilizing two important parameters, node elevations, and head loss. The first trial model was an initial revision of the node elevation based on a road surface map. It was found that the model with elevation data could offer a better solution and was 3.95% more accurate than the existing model. The result was significantly improved, but another error, which may have been caused by using an inappropriate head loss model, was found. As the introduced model is based on the Hazen-William model, it cannot offer an accurate solution for all Reynolds number ranges. Even though Darcy-Weisbach is more complex to use, it could provide a better solution. The results indicate the Darcy-Weisbach model produces results that are 8.65% more accurate than the Hazen-William model.

#### 1. Introduction

It is very challenging to manage water supply for a big city such as Bangkok Metropolitan, as it is not only known as one of the famous tourist destinations, but it is also the 20th densest populated city in the world. The Bangkok Metropolitan Consensus in 2021 reveals the population is roughly 11.5 million people or 136 people/km<sup>2</sup>. The Metropolitan Waterworks Authority (MWA) is Thailand's national government agency that is responsible for supplying drinking water to all of Bangkok Metropolitan including Non-thaburi and Samut Prakan. As the public utilities of this metropolitan area were initially set up without good urban planning, the utilities of Bangkok have grown haphazardly. The main water supply network is one good example of this, as it has a very complex pipeline system, making it difficult to manage the water supply to serve 2.52 million customers or an average of 5.965 million cubic meters per day. Moreover, it could be seen that this service area is increasing. Providing enough water at the required pressure is

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challenging for any water supply plan. Nowadays, computers are very advanced, and software has been invented as a decision tool for water distribution management. One of the most popular software is EPANET, which is freeware and also an open-source code that can be developed by software specialists.

#### 2. WDS software

Large and complex water supply systems, such as urban water supply systems are difficult to calculate, therefore, an algorithm has been developed for solving the water supply problem. There are many algorithms for solving the water distribution network. Global Gradient Algorithms (GGA), defined by Toddini and Pilati (1988), are one of the most commonly used in research [1]. There are many advantages of this GGA technique, including no loop definition, less computational time, and quick convergence. Moreover, this technique can offer an accurate answer by allowing to use wider threshold for the initial value. All water network algorithms use the same initial principle; flow continuity and energy conservation equation. The Toddini and Pilati algorithms were developed into the hydraulic software EPANET by Lewis A. Rossman, United States Environmental Protection Agency (EPA) [2]. EPANET is a hydraulic behavior and concentration of chemicals simulation software for water distribution networks. The hydraulic calculation of EPANET is just like any other software that requires the necessary parameters for the software, such as water demand, head of water supply, pipe diameter, pipe length, node elevation, and pipe roughness. Determining pipe roughness or resistance factor depends on the formulas used; Hazen-Williams, Darcy-Weisbach, or Chezy Manning. The accuracy of software results depends on the accuracy of the input parameters and the simulation results must be calibrated with actual field measurements [3]. Based on AWWA Manual M32 instructions, the software simulation results must be at least 80% accurate, so that they can be a reliable and predictable water supply model in other conditions. The first use of EPANET in the pipeline network of the MWA had many errors. It has been updated frequently to be more accurate. At present, the pipeline network model has an accuracy of 83.676%, which is acceptable. But there are some errors that are caused by several reasons, one of them is the error caused by the elevation of the node. Now, no elevation is defined by assuming that every node has an elevation equal to 0. Although the MWA service areas of the three provinces are not very different, the elevations are 0-2 m above sea level. So, the model should be corrected because the water pressure in the main pipe network is quite low, many points are lower than 10 m. Therefore, the wrong elevation of 1-2 m gives a high error in the water pressure.

#### 3. MWA's main pipe network model

MWA's service area is large and supplies a high-density population, therefore the pipe network is large and complex. The main pipe network has a total length of 1823.663 km with 500–1800 mm diameters. The water supply system of MWA is supplied by 4 water treatment plants and 17 pumping stations. The main piping network consists of pipes, reservoirs, pumps and pressure reducing valves



Fig. 1. MWA main piping network model.

(PRV), etc. Bangkok is a densely populated area that has not been planned well in the past, so the main pipe network is connected like a spider's web (see Fig. 1). This complex network makes it difficult to manage the water supply. MWA has adopted EPANET to help manage the water distribution on the main pipe network. The most popular and widely used EPANET is freeware, and the open-source code can be developed further in the future [4]. Complete model preparation must be calibrated with field data. Without calibration, one can know how accurate the models are and they cannot predict the true performance of the system [5]. The current pipeline network model has been calibrated to 134 field pressure measurement points, beginning with U and UZ as shown in Fig. 2. The demand values used in EPANET are the true values at domestic metering areas which are measured by the water meter. The error of calibration results is quite high as shown in Fig. 3. The errors are caused by many reasons such as nodal elevation [6,7], valve opening, pipe roughness, head loss, leaks, etc.

#### 4. Revision of elevation of node

As shown in Fig. 3, a node's elevation error is a constant error. Simulation pressure and field measurement pressure have the same trend and are equally spaced from 6:00–23:00 h. The elevation of the node is a required parameter for the hydraulic calculation of EPANET. Sometimes it can be taken from a map or a drawing. Often, the elevation of the node is estimated using a topographic map. To be satisfactory, the map must show a contour line of ground elevation, with a resolution of 60 cm (2 ft) or more. We have revised nodal elevation in the Bangkok area using data from the mean elevation map along the roads in Bangkok, 2006–2007 of Land Survey and Map Division, Public Works Department, Bangkok Metropolitan Administration. This is a map showing elevations based on the mean sea level (MSL). The elevation of the roads varies between 0.09 and 1.90 m above the MSL (see Fig. 4). In Nonthaburi and Samut Prakarn elevations have been estimated using data from the Bangkok Metropolitan Region map of the Royal Thai Survey Department; the elevations vary between 0-3.5 m and 0–3 m respectively (see Fig. 5).

#### 5. Use of Hazen-Williams versus Darcy-Weisbach head loss formula

Todini and Pilati's algorithms were originally developed using the Hazen-Williams head loss formula. Now, MWA uses the Hazen-Williams head loss formula and sets the *C* value to 120. This value is recommended for brand-new steel pipes only. In fact, the MWA main pipe has been in use for a long time, and the pipe's inner surface roughness has changed [8]. Therefore, the C value will not be as above specified and the Hazen-Williams formula is less accurate than Darcy-Weisbach. Because the resistance in the Hazen-Williams formula does not vary with changes in fluid physical properties, such as flow rate and temperature, it is commonly used because it is easy to calculate; requiring a single roughness coefficient C for resistance factor (r = 10.676L/C1.852D4.871). If the physical properties of water are changed, the r value will not change in the old model. The more complicated Darcy-Wiesbach formula has better



Fig. 2. Field pressure measurement points on the main piping network.



Fig. 3. EPANET results from the current model and measured values for pressure. (a) at node UZ0309 (b) at node UZ0514 (c) at node UZ1210 (d) at node UZ1304.

accuracy when compared with the experimental data. The flow resistance is a function of the friction factor ( $r = f8L/g\pi^2D^5$ ), f is the friction factor related to the volume flow rate (Q), pipe roughness ( $\varepsilon$ ) and viscosity (v) varies with water temperature.

#### 6. Use of Hazen-Williams versus Darcy-Weisbach head loss formula

The pipe friction head loss formula in the form of an exponential can be used generally for the Hazen-Williams, Darcy-Weisbanch, and Chezy-Manning formulas [9–12].

$$h_f = rQ^n \tag{1}$$

where  $h_f$  = head loss (m).

r = resistance coefficient (unitless).

Q =flow rate (m<sup>3</sup>/s).

n = flow exponential (unitless).

The values for r and n depend on the section of the head loss formula.

#### 6.1. Hazen-Williams head loss formula

The Hazen-Williams formula is widely used because the calculation method is simple. For the Hazen-Williams equation the exponent is n = 1.852 so equation (1) becomes



Fig. 4. Bangkok roads elevation map.



Fig. 5. Ground elevation in the Bangkok metropolitan region map of the Royal Thai survey department.

$$h_f = rO^{1.852}$$

and the resistance coefficient r is

$$r = 10.676 \frac{L}{C^{1.852} D^{4.871}} \tag{3}$$

where C = Hazen Williams roughness coefficient (unitless).

 $\varepsilon =$ Darcy-Weisbach roughness coefficient (mm).

f = friction factor (unitless).

D = pipe diameter (m).

L = pipe length (m).

The coefficient C depends only on the pipe material, independent of flow rate.

#### 6.2. Darcy – weisbach head loss formula

For the Darcy – Weisbach formula the exponent is n = 2 so equation (1) becomes

 $h_f = rQ^2$ 

and the pipe resistance coefficient r is

$$r = f \frac{8L}{\pi^2 g D^5} \tag{5}$$

where g = gravitational acceleration constant, friction factor f depends on flow rate and the ratio of roughness and pipe diameter ( $\epsilon/D$ ). You can find the value of f by using the Moody diagram with the relation of Re and  $\epsilon/D$  or using the appropriate formula as shown in Table (1) according to the flow range (Reynolds Numbers) [13,14].

Finding value of f by using Moody Diagram and Equation in Table 1 is inconvenient in computer programming. Estimating the value of f in computer programming and also in EPANET depends on three flow ranges as the following [15].

Lamina Flow (Re  $\leq$  2100) applied Hagen-Poiseuille formula (Bhave 1991).

$$f = 64/\mathrm{Re} \tag{6}$$

where  $\text{Re} = 4|Q|/v\pi D$  the friction factor f in terms of Q is

$$f = \frac{16\pi\nu\epsilon D}{|Q|} \tag{7}$$

Where  $\nu =$  kinematic viscosity.

Transitional flow (2100 < Re < 4000) Dunlop's cubic interpolation from Moody diagram (Bhave, 1991).

$$f = \sum_{k=0}^{3} (\alpha_k + \beta_k / \theta) \eta^k \tag{8}$$

**Turbulent flow** ( $Re \ge 4000$ ) applied Swamee and Jain approximation to the Colebrook-White equation (Bhave, 1991).

$$f = \frac{0.25}{\left[\log(\varepsilon/3.7D + 5.74/\text{Re}^{0.9})\right]^2} = \frac{\ln^2 10}{4\ln^2 \theta}$$
(9)

Thus, friction factors *f* for the 3 difference flow regimes are

Table 1			
Darcy – W	eisbach	friction	factor.

Title 1	Title 2	Title 3
Laminar	$f = 64/\mathrm{Re}$	Re < 2000
Smooth pipe	$1/\sqrt{f} = 2 \log_{10}(\text{Re}/f) - 0.8$	${ m Re} > 4000$ & $\varepsilon/D \ge 0$
Transitional Colebrook-White Eq.	$1/\sqrt{f} = 1.14 - 2\log_{10}\left(rac{arepsilon}{D} + rac{9.35}{ ext{Ref}} ight)$	Re > 4000
Wholly Rough	$1/\sqrt{f} = 1.14 - 2 \log_{10} ig(rac{arepsilon}{D}ig)$	$\operatorname{Re} > 4000$

(2)

(4)

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$$f = \begin{cases} 64/\text{Re} \\ \sum_{k=0}^{3} (\alpha_k + \beta_k/\theta) \eta^k \\ \ln^2 10/4 \ln^2 \theta \end{cases}; \text{transitional flow} \\ \text{; turbulent flow} \end{cases}$$

where  $\alpha_k$  and  $\beta_k$  are defined in Table 2.

and the three variables are defined as:

$$\eta = \operatorname{Re} \left/ 2,000, \theta = \frac{\varepsilon}{3.7D} + \frac{5.74}{\operatorname{Re}^{9/10}}, \widehat{\theta} = \frac{\varepsilon}{3.7D} + \frac{5.74}{4000^{9/10}}, \tau = 0.00514215 \text{ and } \xi = -0.86859 \right)$$

So, flow resistance coefficients of the 3 different flow ranges are

$$r = \begin{cases} \frac{16\pi\nu D}{|Q|} \frac{8L}{g\pi^2 D^5} \\ \left(\sum_{k=0}^{3} (\alpha_k + \beta_k/\theta)\eta^k\right) \frac{8L}{g\pi^2 D^5} \\ \vdots \text{ transitional} \\ \frac{\ln^2 10}{4\ln^2 \theta} \frac{8L}{g\pi^2 D^5} \\ \vdots \text{ turbulent} \end{cases}$$
(11)

The Darcy-Weisbach head loss formular in each regime is summarized as follows.

$$h_{f} = \begin{cases} \frac{128VL}{g\pi QD^{4}} Q^{2} & ; \text{laminar} \\ \left(\sum_{k=0}^{3} (\alpha_{k} + \beta_{k}/\theta)\eta^{k}\right) \frac{8L}{g\pi^{2}D^{5}}Q^{2} & ; \text{transitional} \\ \frac{\ln^{2}10}{4\ln^{2}\theta} \frac{8L}{g\pi^{2}D^{5}}Q^{2} & ; \text{turbulent} \end{cases}$$
(12)

The Darcy-Weisbach head loss formula has a higher accuracy than the Hazen-Williams as it varies with the flow rate. The pipe network model using Hazen-William set the value of *C* equal to 120, which is the value for the new steel pipe only [16,17]. The network model parameters using the Darcy-Weisbach formula are as follows;  $\varepsilon = 4$  mm,  $\nu = 0.802 \times 10^{-6}$  m<sup>2</sup>/s at 30 °C and stopping criteria  $<10^{-6}$ .

#### 7. Results and discussion

Table 2

120.1

After adjusting the nodal elevations in the pipe network model, the current and new models were compared with 134 fieldmeasured pressure points. It was found that the new model was more accurate than the current model. The new pipe network model has increased accuracy to 87.63% from the existing model 83.68%; that is 3.95%. For example, at measurement points UZ0309, UZ0514, UZ1210 and UZ1304 the accuracy of the new mean was better than the existing model (see Fig. 6). The errors of these 4 measurement points were reduced by 12.34%, 16.28%, 10.35% and 23.92%, respectively.

Although overall this first new model is quite good, there are still some points that have high errors, such as at point numbers UZ0309, UZ0514, UZ1210 and UZ1304 (see Fig. 6). It can be seen that the results almost coincide with actual measurements at 6:00-23:00 h, but are quite different at 0:00-5:00 h period of night time with low water consumption (night flow). One cause of the remaining errors is the head loss formula used [18]. The research of Liou on the impact of the use of the head loss formula [19]. Reported that if the Hazen-Williams formula with only one resistance coefficient C is used outside of the boundary, it can cause up to 40%, error and Fabian's research (2003) [20] was in the same direction. The Hazen-Williams formula is often used in the design of large pipelines, without regard to its limitations. Using the Hazen-Williams head loss formula is limited to the flow rate range and is accurate only for transitional flow, smooth flow and turbulent flow. Christensen suggested using Hazen-Williams within the flow range  $10^5 < \text{Re} < 10^8$ . In actual work, the flow of water is usually outside of this limitation. Fabian A. (2003) recommends using the

The values of $a_k$	and $\beta_k$ .	
k	$\alpha_k$	$\beta_k$
0	$5/(\xi^2 \ln^2 \widehat{ heta})$	$\tau/(\xi^3\ln^3\widehat{\theta})$
1	$0.128 - 12/(\xi^2 \ln^2 \widehat{ heta})$	$- 5 au/(2\xi^3 \ln^3 \widehat{ heta})$
2	$=0.128+9/(\xi^2{ m ln}^2\widehat{ heta})$	$2 au/(\xi^3 \ln^3 \widehat{ heta})$
3	$0.032-2/(\xi^2{ m ln}^2\widehat{ heta})$	$- \ \tau/(2\xi^3 \ {\rm ln}^3 \widehat{\theta})$

(10)



Fig. 6. Computed results from the current model, the new model and measured values for pressure. (a) at node UZ0309 (b) at node UZ0514 (c) at node UZ1210 (d) at node UZ1304.

Darcy-Weisbach formula to cover all flow ranges and various physical characteristics of water and pipe. These studies were consistent with the current study (Fig. 5). It was found that during high water consumption (6:00–23:00), the simulation results were close to the actual field measurement pressures. But, during low water consumption (0:00 a.m.–5.00 a.m.), the simulation results from EPANET were different from the actual field measurement results. Therefore, the Darcy-Weisbach head loss formula should be used instead of the Hazen-Williams formula, but further implementation can be made by refining the Jacobian matrix.

After revision of the elevation of nodes and using the Darcy-Weisbach instead of Hazen-Williams, the simulation results of the two head loss models were compared with field measured pressure data. It was found that the Darcy-Weisbach model was again more accurate than the Hazen-Williams model. The Darcy-Weisbach model has increased accuracy to 92.33% from the existing model at 83.68% which is accuracy up 8.65%. Now the effect of the head loss model can be illustrated. For example, at measurement points UZ0309, UZ0514, UZ1210 and UZ1304 the accuracy of the new mean was better than the existing model (see Fig. 7) where it can be seen that the results almost coincide with actual measurement pressures at all times. The accuracies of these 4 measurement points were increased by 15.36%, 22.68%, 19.04%, and 29.60% respectively.

#### 8. Conclusions

This study estimates the effect of inputting nodes elevation on the accuracy of EPANET final calculation. The new pipe network model has increased accuracy to 87.63% from the existing model 83.68%; that is of 3.95%. The coincidence of EPANET calculation and actual measurements indicates acceptable results. In addition, the errors of 4 arbitrary measurement points were reduced between 12 and 23% which sounds good. from EPANET. The maximum deviation can be seen in the period of night time which relates to low water



Fig. 7. Computed results from the Hazen-Williams model, the Darcy - Weisbach model and measured values for node pressure. (a) at node UZ0309 (b) at node UZ0514 (c) at node UZ1210 (d) at node UZ1304.

consumption. By using the Darcy-Weisbach instead of Hazen-Williams, the software results were compared with field-measured pressure data. It was seen that the Darcy-Weisbach model was more accurate than the Hazen-Williams model.

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#### Data availability statement

Data associated with this study will be made available on request.

#### CRediT authorship contribution statement

**Rangsan Wannapop:** Investigation, Methodology, Writing – original draft. **Thira Jearsiripongkul:** Conceptualization, Investigation, Supervision, Writing – review & editing. **Krit Jiamjiroch:** Investigation, Supervision.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to

influence the work reported in this paper.

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