Data in Brief 57 (2024) 111145

Contents lists available at ScienceDirect

Data in Brief

journal homepage: www.elsevier.com/locate/dib

Data Article

Experimental datasets on synchronization in simplicial complexes

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ARTICLE INFO

Article history:
Received 16 July 2024
Revised 9 October 2024
Accepted 12 November 2024
Available online 16 November 2024

Dataset link: Experimental validation of simplicial complexes in multivariable coupled oscillators. Coupling: Lineal (Class II) vs NoLineal (Class II) (Original data)
Dataset link: Experimental validation of simplicial complexes in multivariable coupled oscillators. Coupling: Lineal (Class III) vs NoLineal (Class III) (Original data)
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ABSTRACT

Some real-world phenomena and human-made problems have been modeled as networks where the objects form pair-wise interactions. However, this is a limited approach when the existence of high-order interactions is inherent in a system, such as the brain, social networks and ecosystems. The way in which these high-order interactions affect the collective behavior of a complex system is still an open question. For this reason, it is necessary to analyze theoretically, numerically and experimentally the consequences of higher-order interactions in complex systems. Here, we provide experimental datasets of the dynamics of three nonlinear electronic oscillators, namely, Rössler oscillators, interacting into a simplicial complex whose connections rely on both linear (diffusive) and nonlinear (high-order) coupling. It is well-known that Rössler systems only achieve the synchronization when they are coupled by means of x or y variable. Considering this fact, we designed our experiment considering four

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https://doi.org/10.1016/j.dib.2024.111145

scenarios. The first one, when both linear and nonlinear coupling functions are introduced through the x variable. The second one, occurring when linear coupling is introduced through the x variable and the nonlinear coupling through the y variable. The third case happens when the linear coupling is introduced through the y variable whereas nonlinear coupling goes through the x variable. The last case, when both linear and nonlinear coupling are introduced through the y variable. For each scenario, we acquired 10000 times series when both the linear and nonlinear coupling strengths were modified. Each time series contained 30000 temporal points. These datasets are useful to corroborate the conditions to reach the synchronized state varying the linear/nonlinear coupling strengths and to test new metrics for better understanding the effects of higher-order interactions in complex networks.

Specifications Table

Subject	*Physics.*
Specific subject area	*Nonlinear dynamics, complex networks synchronization.*
Type of data	*Time series in text file*
Data collection	- We collected the data using a Data Acquisition Card (DAQ) from National Instruments, model USB-6363. The acquired voltages come from the x and y state variables of three electronic Rössler oscillators interacting in a simplicial complex. - Four different scenarios were implemented. The first one, when both, linear and nonlinear coupling functions are introduced by x variable. The second one occurs when linear coupling is introduced by x variable but nonlinear coupling by the y variable. The third case happens when linear coupling is introduced by y variable whereas nonlinear coupling by x variable. The fourth case, when both linear and nonlinear coupling are introduced by y variable. - For each scenario, the two coupling strengths (one for linear coupling and the other for nonlinear coupling) were varied 100 times, giving as a result a total of 10000 time series for each scenario. Each time series contains 30000 points.
Data source location	*Institution: Centro Universitario de los Lagos (Universidad de Guadalajara).* *City, Town, Region: Lagos de Moreno, Jalisco.* *Country: México.*
Data accessibility	Repository name: ZenodoDirect URL to data: • Dataset I: https://doi.org/10.5281/zenodo.10408252 • Dataset II: https://doi.org/10.5281/zenodo.10408257 • Dataset III: https://doi.org/10.5281/zenodo.10408265 • Dataset IV: https://doi.org/10.5281/zenodo.10392385
Related research article	V.P. Vera-Ávila, R.R. Rivera-Durón, Miguel S. Soriano-Garcia, R. Sevilla-Escoboza, Javier M. Buldú; **Electronic implementation of simplicial complexes.** (2024) *Chaos, Solitons & Fractals*, 183 art. no. 114915 DOI: https://doi.org/10.1016/j.chaos.2024.114915

1. Value of the Data

- We provide the time series of N=3 electronic Rössler oscillators interacting in a simplicial complex and coupled by means of the x and y state variables. The datasets allow further in-

vestigation of the conditions to achieve the synchronization in simplicial complexes and to corroborate the theoretical findings in the master stability function for high-order interactions.
- Datasets can be used by researchers to analyze the synchronization phenomena with experimental data. Also, the data can be used to develop new metrics to quantify the level of synchronization between dynamical systems. Other classical metrics, such as the phase synchronization or the mutual information, can be extracted from the datasets.
- These datasets allow testing the robustness of the experimental synchronization since the oscillators in the experiment are not identical due to the tolerance of electronic components and the existence of noise.
- We considered these datasets are significant and they represent a good start-point for studying high-order interactions because out data considers the simplest simplicial complexes (triplets). The high-order interactions are so important since they reveal dynamics that cannot be understood by analyzing simple connections alone. Also, the high-order interactions can lead to emergent phenomena, such as it occurs in biological neural networks or in the propagation of information in social networks.

2. Background

In [1], we introduced the design and the implementation of an experimental simplicial complex, which consists of a complex network with higher-order interactions among nodes. Using a set of three Rössler-like electronic oscillators under a chaotic dynamical regime, we analyzed how the synchronization manifold is modified by introducing higher-order interactions between the triplet of nodes, as suggested in recent theoretical works [2,3]. The combination of pairwise (i.e., node-to-node) with high-order (i.e., triplet) coupling was studied by varying the corresponding coupling strengths. Publications dealing with experiments in simplicial complexes are scarce and the access to data obtained experimentally is not common. For this reason, we decided to provide all datasets generated in the laboratory, allowing other researchers to further investigate the properties of complex networks with high-order interactions and to inquire about the emergent phenomena that occur in this kind of structures.

3. Data Description

The datasets correspond to the experimental implementation of a simplicial complex consisting of a 3-node network as shown in Fig. 1. Each node is an electronic Rössler-like oscillator whose parameters were fixed to operate in a chaotic regime. In simplicial complexes, it is possible to model two types of interactions among nodes: pairwise interactions (i.e., linear) and high-order interactions (i.e., non-linear). The complete experiment consists of coupling simultaneously the simplicial complex by means of linear and non-linear interactions. The coupling can occur in state variables x (class III) and y (class II). In such a way, that the complete experiment considers four scenarios: the first one, when both, linear and nonlinear coupling functions

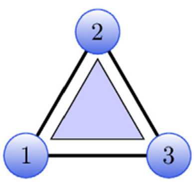

Fig. 1. Simplicial complex network used for the experimental setup to study the synchronization phenomenon.

are introduced through x variable; the second one occurs when linear coupling is introduced through x variable but nonlinear coupling through the y variable; the third case happens when linear coupling is introduced through y variable whereas nonlinear coupling through x variable; finally, the fourth case, when both linear and nonlinear coupling are introduced through y variable. Coupling though variable z is not considered, since the synchronization manifold is never reached using this variable. The datasets are organized in four files/folders, whose name explicitly indicates the variable used for the coupling. For example, “linearx-nonlineary” indicates that the linear coupling occurs in variable x (class III) whereas the non-linear coupling occurs in variable y (class II). Each folder contains 10000 files which come from varying the linear coupling and the nonlinear coupling 100 times. The file name is composed as follows: root name XX YY, where XX corresponds to the variation number in linear coupling, whereas YY corresponds to the variation number in non-linear coupling. Internally, we can find 6 columns and 30000 rows in each file. Each pair of columns corresponds to the x and y variables of each oscillator and the rows correspond to time-varying samples. The data were collected using an acquisition card USB-6363 of National Instruments. This device collects data from all channels in sequential order using a multiplexer. The setting time to achieve this task is 1 to 2 µs. This delay time is much less than the oscillation frequency of the Rössler system, which is around 1.5 kHz. The sequential order to collect data from the analog inputs in our setup is 3-4-2-5-1-0, which correspond to $y_2, x_3, x_2, y_3, y_1, x_1$ signals, respectively. The delay to acquire these 6 signals is around 10 µs. This point is important since the lag induced by the data collection equipment could hinder the identification of identical synchronization.

4. Experimental Design, Materials and Methods

In the simplicial complex of Fig. 1, pairwise interactions are represented by single links, whereas high-order interactions are triplets. From a mathematical point of view, the dynamic of each node in the simplicial complex is given by:

$$\begin{aligned} \dot{x_i} = f(x_i) + \sigma_1 \sum_{j_1=1}^{N} a_{ij_1}^{(1)} g^{(1)}\left(x_i, x_{j_1}\right) + \sigma_2 \sum_{j_1=1}^{N} \sum_{j_2=1}^{N} a_{ij_1j_2}^{(2)} g^{(2)}\left(x_i, x_{j_1}, x_{j_2}\right) \\ + \cdots + \sigma_D \sum_{j_1=1}^{N} \cdots \sum_{j_D=1}^{N} a_{ij_1\cdots j_D}^{(D)} g^{(D)}\left(x_i, x_{j_1}, \cdots, x_{j_D}\right) \end{aligned} \tag{1}$$

where $\mathrm{x_i}$ is a vector of dimension m governing the dynamic of the node i; $\sigma_1, , \sigma_\mathrm{D}$ are the coupling strengths of each D-dimensional simplex and the functions f are coupling functions describing the interactions at different orders.

Our experimental validation consists of two stages: first, the design and implementation of the electronic oscillators, which represent the nodes in the simplicial complex; and the second, the complete experimental setup that includes the programming and control of the laboratory equipment to efficiently acquire the time series.

Regarding the electronic implementation of the nodes, each node is a Rössler oscillator constructed with analog devices (operational amplifiers), passive elements (resistors, variable resistors, capacitors), and some specific purpose integrated circuits (multipliers) as is shown in the schematic diagram of Fig. 2. To ensure that the chaotic oscillators are as identical as possible, we use 1% precision resistors and adjust the variable resistors to guarantee the oscillators display a nearly similar attractor shape, which was verified employing the frequency spectrum. The complete list of values for voltages, resistors and capacitors is given in Table 1. The dynamics of isolated electronic Rössler system is governed by:

$$\dot{x_i} = -\frac{1}{R_1C_1}\left(x_i + \frac{R_1}{R_2}y_i + \frac{R_1}{R_4}z_i\right)$$

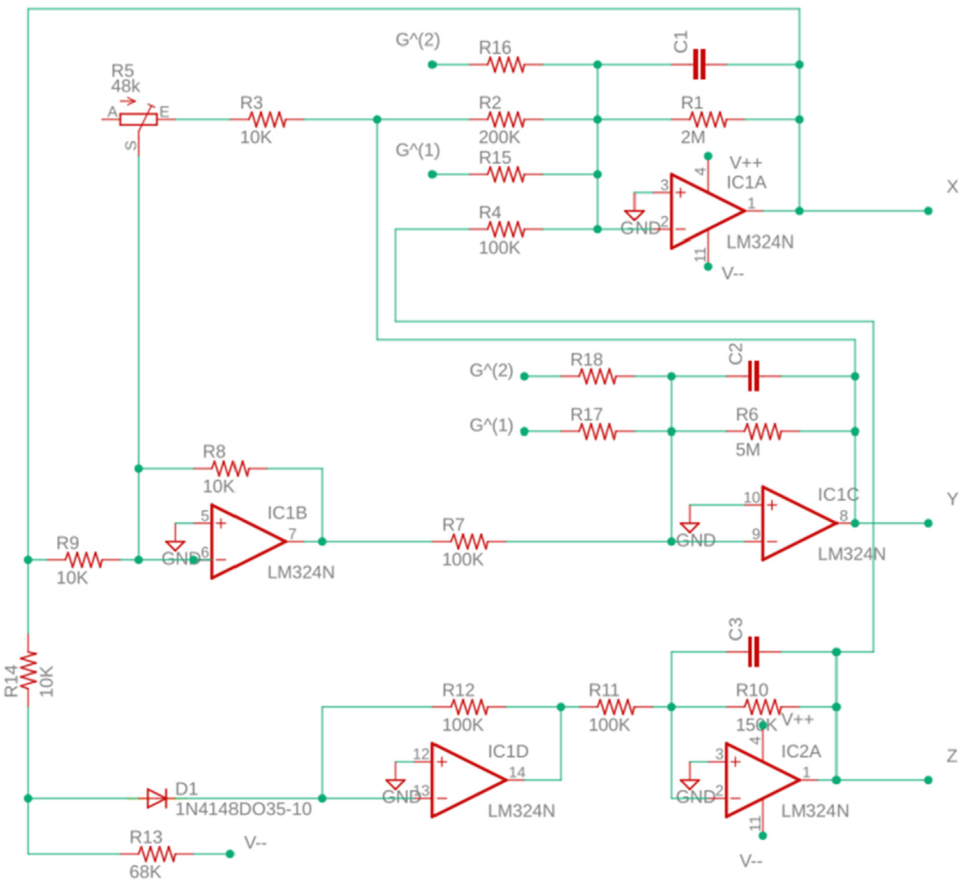

Fig. 2. Schematic diagram of the experimental electronic Rössler oscillator. The circuit is constructed using only analog devices as operational amplifiers (LM324N), resistors, capacitors, and diodes.

Table 1
Values of the electronic components for the schematic diagram shown in Fig. 2. Using these values the oscillator displays chaotic behavior.

$C_{1,2,3}$ = 1 nF	V_d = 0.7 V	V_{ee} = 9 V	R_C = R_3+R_5
R_1 = 2 MΩ	R_2 = 200 kΩ	R_3 = 10 kΩ	R_4= 100 kΩ
R_5 = 48 kΩ	R_6 = 5 MΩ	R_7 = 100 kΩ	R_8= 10 kΩ
R_9 = 10 kΩ	R_{10} = 100 kΩ	R_{11} = 100 kΩ	R_{12}= 150 kΩ
R_{13} = 68 kΩ	R_{14} = 10 kΩ	R_{15} = 300 kΩ	R_{16}= 33.2 kΩ
R_{17} = 10 kΩ	R_{18} = 51 kΩ	$\sigma_1 = [0-2]$ Vcc	$\sigma_2 = [0-0.02]$ Vcc

$$\dot{y}_i = -\frac{1}{R_6C_2}\left(-\frac{R_6R_8}{R_9R_7}x_i + \left[1 - \frac{R_6R_8}{R_CR_7}\right]y_i\right)$$

$$\dot{z}_i = -\frac{1}{R_{10}C_3}\left(-\frac{R_{10}}{R_{11}}G_{x_i} + z_i\right) \tag{2}$$

where x, y and z represent the voltages at the outputs of the three operational amplifiers (denoted as IC1A, IC1C, and IC2A in Fig. 2) and G_{x_i} is a nonlinear function given by:

$$G_{x_i} = \left\{ 0,\ if\ x_i \leq\ V_d + V_d\frac{R_{14}}{R_{15}} + V_{ee}\frac{R_{14}}{R_{15}}\ \frac{R_{12}}{R_{14}}x_i -\ V_{ee}\frac{R_{12}}{R_{13}} - V_d\left(\frac{R_{12}}{R_{14}} + \frac{R_{12}}{R_{13}}\right),\ if\ x_i >\ V_d + V_d\frac{R_{14}}{R_{15}} + V_{ee}\frac{R_{14}}{R_{15}} \right\} \tag{3}$$

After the implementation of the electronic oscillators, the next step is to integrate them into the simplicial complex, using a combination of the abovementioned couplings. In this way, the dynamics of the nodes considering the couplings are:

$$\dot{x_i} = -\frac{1}{R_1C_1}\left(x_i + \frac{R_1}{R_2}y_i + \frac{R_1}{R_4}z_i - \sigma_1\frac{R_1}{R_{15}}\sum_{j=1}^{N} g_{ij}^{(1)}\left(x_j - x_i\right)\right) - \frac{1}{R_1C_1}\left(-\sigma_2\frac{R_1}{R_{16}}\sum_{j=1}^{N}\sum_{k=1}^{N} g_{ijk}^{(2)}\left(x_j^2x_k - x_i^3\right)\right)$$

$$\dot{y_i} = -\frac{1}{R_6C_2}\left(-\frac{R_6R_8}{R_9R_7}x_i + \left[1 - \frac{R_6R_8}{R_CR_7}\right]y_i - \sigma_1\frac{R_6R_8}{R_{17}R_7}\sum_{j=1}^{N} g_{ij}^{(1)}\left(y_j - y_i\right)\right) - \frac{1}{R_6C_2}\left(-\sigma_2\frac{R_6R_8}{R_{18}R_7}\sum_{j=1}^{N}\sum_{k=1}^{N} g_{ijk}^{(2)}\left(y_j^2y_k - y_i^3\right)\right)$$

$$\dot{z_i} = -\frac{1}{R_{10}C_3}\left(-\frac{R_{10}}{R_{11}}G_{x_i} + z_i\right) \tag{4}$$

Fig. 2 shows an overview of the complete experimental setup. It is important to notice that a crucial point of the experiment is to control, at the same time, the coupling for simplexes of dimension 1 and 2. This implies that we need to vary two coupling strengths, the former for the 1-simplex (i.e., single links) and the latter for the 2-simplex (i.e., triplets). With this aim, we used a two-channel programmable power supply (B), whose two output voltages directly control the values of the coupling strengths (σ_1 and σ_2). Next, to got the time series, each coupling strength was modified 100 times through a Labview code (A) that controls the programmable power supply (B) in such a way that when each scenario in the experiment was finished, we obtained 100×100 time series for each of the three nodes of the simplicial complex. The values of σ_1, σ_2, and the two state variables of the three nodes (x_i and y_i) were recorded simultaneously by an analog input module of the embedded system CompaqRIO (C). In real-time, these input signals were processed by another Labview program (D) that calculates the coupling functions $g^{(1)}$ and $g^{(2)}$. After the terms of the linear diffusive coupling function $g^{(1)}$ and the high-order coupling function $g^{(2)}$ have been calculated, they were fed back to the electronic Rössler oscillators (E) through the analog output module of the CompaqRIO (C). Finally, a data acquisition card NI-6363 (F) captures the temporal series of the electronic Rössler and sends them to a Labview program (A) that stores them in the hard disk to be processed later. The results of this last stage of the experiment are the datasets which are supplied in this paper.

At this point, let us clarify two facts about the acquisition equipment we utilized. The former, we used the CompaqRIO as a FPGA configuration because it is more flexible, customizable compared with scan interface mode. This configuration allows us performing all the high-order functions. The second, regarding to the data acquisition card USB-6363, which does not collect data simultaneously, it collects data in a multiplexed way, to achieve this task the setting time is around 1 to 2 µs. This delay time is practically despicable because it is much less than the oscillation frequency of the Rössler systems, which is around 1.5 kHz. The acquisition card collects data from $y_2, x_3, x_2, y_3, y_1, x_1$ signals in that sequential order. The USB-6363 requires a time of 10 µs to acquire these six signals. We mentioned this fact because is important since when the synchronization is analyzed, the lag induced by the data acquisition equipment could complicate the identification of identical synchronization. This issue can be solved if we know the temporal skew on each channel, then we could fit a function to interpolate our data and provide them as they were sampled in simultaneously way Fig. 3.

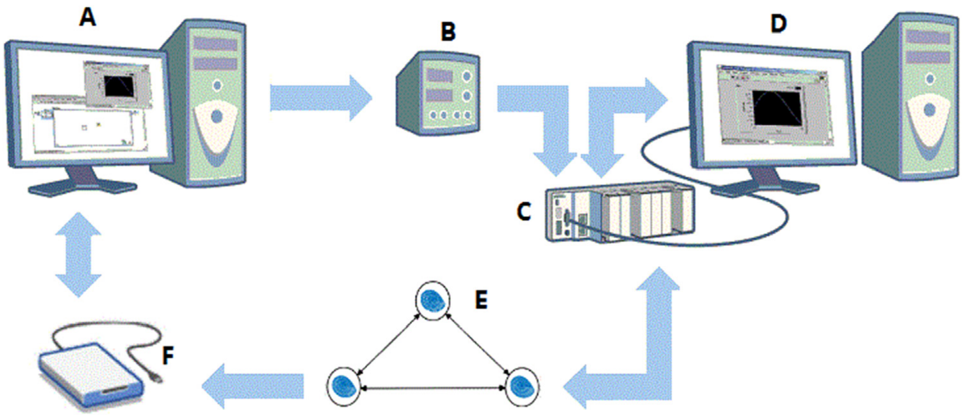

Fig. 3. Schematic representation of the experimental setup. (A) Labview virtual instrument controlling the coupling strengths σ_1, σ_2 and storing the outputs of the simplicial complex. (B) Two-channel programmable power supply. (C) Embedded system CompaqRIO reading the state variables using the analog inputs module and introducing the coupling functions through the analog outputs module. (D) Labview virtual instrument calculating the coupling functions. (E) Simplicial complex composed of 3 electronic Rössler oscillators. (F) Data acquisition card capturing the outputs of the simplicial complex.

Limitations

'Not applicable'.

Ethics Statement

The authors have read and followed the ethical requirements for publication in Data in Brief and confirm that the current work does not involve human subjects, animal experiments, or any data collected from social media platforms.

CRediT Author Statement

V. P. Vera-Ávila: Methodology, Validation, Investigation, Writing (original draft). **R. R. Rivera-Durón:** Methodology, Validation, Investigation, Writing (original draft). **Onofre Orozco-López:** Methodology, Validation, Writing (review and editing). **J. Ricardo Sevilla-Escoboza:** Methodology, Validation, Writing (review and editing), Supervision, Funding. **M. S. Soriano-García:** Methodology, Validation, Writing (review and editing). **Javier M. Buldú:** Methodology, Supervision, Writing (review and editing), Funding.

Data Availability

Experimental validation of simplicial complexes in multivariable coupled oscillators. Coupling: Lineal (Class II) vs NoLineal (Class II) (Original data) (Zenodo).

Experimental validation of simplicial complexes in multivariable coupled oscillators. Coupling: Lineal (Class III) vs NoLineal (Class III) (Original data) (Zenodo).

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Acknowledgements

J.M.B. acknowledges the support by Ministerio de Ciencia e Innovación, Spain under grant PID2020-113737GB-I00 and grant PID2023-147827NB-I00. R.S.E acknowledges support from the Consejo Nacional de Ciencia y Tecnología, Mexico call SEP-CONACYT/CB-2016–01, Grant No. 285909. V.P.V.A. acknowledges support from the Secretaria de Investigación y Posgrado (SIP) del Instituto Politécnico Nacional (IPN) Programa especial de consolidación de investigadores 2023, No. 20231415. R.R.R.D. acknowledges to Consejo Nacional de Humanidades, Ciencia y Tecnología, Programa de Estancias Posdoctorales por México, Grant I1200/331/2023.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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