

# *Article* **Analytical Model for Concentration (Pressure) Impedance of a Low-Pt PEM Fuel Cell Oxygen Electrode**

**Andrei Kulikovsky**



**Abstract:** In this study, a model for concentration/pressure impedance *ζ* of the cathode catalyst layer of a low-Pt PEM fuel cell is developed. The model is based on transient oxygen mass transport equations through the cathode catalyst layer modeled as a single pore with a thin Nafion film covering the pore surface. This structure is used to simulate oxygen transport through the catalyst layer depth and through the ionomer film covering Pt/C agglomerates in low-Pt cells. Analytical solution for zeta-impedance at high cell current is derived; this solution can be used for fast fitting of experimental zeta-spectra. Optimal conditions for measuring the zeta-spectra of a low-Pt cell are discussed. Zeta impedance is not affected by faradaic processes in the cell, which makes this technique a useful alternative to standard EIS.

**Keywords:** PEM fuel cell; low-Pt loaded cathode; concentration impedance; pressure impedance; modeling

# **1. Introduction**

Lowering of Pt loading is of large importance for the success of PEM fuel cells on the mass market. However, the first attempts to lower Pt loading on the cathode side have already revealed unexpected "overlinear" transport loss of the cell potential [\[1,](#page-9-0)[2\]](#page-9-1). This effect has been attributed to oxygen transport through a thin ionomer film covering Pt/C agglomerates in the cathode catalyst layer (CCL). Over the past decade, a lot of research has been done to measure the film transport resistivity  $\mathcal{R}_N$  in PEMFCs [\[2](#page-9-1)[–9\]](#page-9-2). Most of the works employed a limiting current method for measuring R*N*, moreover, in [\[9\]](#page-9-2) hydrogen instead of oxygen was used in the limiting current experiments to avoid undesired effects due to oxygen reduction reaction (ORR) in the electrode.

An alternative method, requiring standard cell, equipment, and procedures, is electrochemical impedance spectroscopy (EIS). A numerical model for low-Pt PEMFC impedance has been developed and fitted to the experimental impedance spectra [\[10\]](#page-9-3). Fitting returned the Nafion film thickness and oxygen diffusivity. Nonetheless, complexity of the impedance model [\[10\]](#page-9-3) stimulates searching for simpler ways for measuring  $\mathcal{R}_N$ . Generally, every transport process in a fuel cell is expected to form a separate peak in the distribution of relaxation times (DRT) spectrum [\[11\]](#page-9-4). The area under the DRT peak gives the contribution of the respective process to the total cell polarization resistivity. Thus, calculation of R*<sup>N</sup>* from experimental impedance spectra would be much simpler if the DRT peak due to oxygen transport in Nafion film were well separated from other peaks. However, a recent model [\[12\]](#page-9-5) shows that at low currents, the characteristic frequency of the film-transport peak is only 1.73 times less than the frequency of faradaic peak and the two peaks tend to merge. An impedance spectroscopy method insensitive to faradaic processes in the cell would be of great help.

The idea to perturb cell potential by harmonic variation of the oxygen concentration or pressure (EPIS) has been suggested in [\[13\]](#page-9-6) and developed further in [\[14](#page-9-7)[–18\]](#page-10-0). A review of pressure impedance works can be found in [\[16\]](#page-10-1). The great advantage of this technique



**Citation:** Kulikovsky, A. Analytical Model for Concentration (Pressure) Impedance of a Low-Pt PEM Fuel Cell Oxygen Electrode. *Membranes* **2022**, *12*, 356. [https://doi.org/](https://doi.org/10.3390/membranes12040356) [10.3390/membranes12040356](https://doi.org/10.3390/membranes12040356)

Academic Editor: Stefano Deabate

Received: 28 February 2022 Accepted: 22 March 2022 Published: 24 March 2022

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is that the zeta-spectrum of the cell is independent of the faradaic processes. So far, most of the reported EPIS models have been numerical. An analytical model for the PEMFC zeta-impedance taking into account oxygen transport in the gas diffusion layer (GDL) and CCL has been derived in [\[19\]](#page-10-2); however, this mean-field model ignores oxygen transport in ionomer film covering Pt/C agglomerates.

In this work, we report a model for the concentration impedance *ζ* of a low-Pt cell, with explicit account of oxygen transport through the ionomer film. We derive a formula for the *ζ*-impedance of a low-Pt cell suitable for fast fitting of experimental zeta-spectra. We show that measurements of the oxygen diffusion coefficient in the Nafion film using zeta-spectroscopy should be done at low oxygen concentration providing high rate of oxygen consumption in the CCL and at the cell current density close to the limiting current density due to Nafion film.

### **2. Model**

### *2.1. Basic Transient Equations*

It is assumed that the following conditions hold

- Oxygen transport loss in the gas-diffusion layer is small;
- Proton transport in the CCL is fast;
- Oxygen consumption in the CCL is large.

The limiting current density due to oxygen transport in the GDL is typically much larger, than the limiting current due to Nafion film [\[20\]](#page-10-3) and to a first approximation oxygen transport loss in the GDL can be neglected. The second and third assumptions are discussed in Section [3.](#page-5-0)

The low–Pt CCL is modeled by a single cylindrical pore penetrating through the whole CCL depth. The pore volume is separated from the coaxial  $Pt/C$  tube by a thin Nafion film (Figure [1\)](#page-1-0). Oxygen is transported along the pore and in the radial direction through the ionomer to Pt surface, where the ORR occurs. The static version of this model has been reported in [\[20\]](#page-10-3); here we briefly repeat the basic equations necessary for understanding the impedance model.

<span id="page-1-0"></span>

**Figure 1.** Schematic of a single-pore model for impedance of the low-Pt cathode catalyst layer.

Oxygen transport along the pore is described by

<span id="page-1-1"></span>
$$
\frac{\partial c}{\partial t} - D_p \frac{\partial^2 c}{\partial x^2} = \frac{2N_{N,p}}{R_p}, \quad \frac{\partial c}{\partial x}\Big|_{x=0} = 0, \quad c(l_t) = c_1 \tag{1}
$$

where  $R_p$  is the pore radius,  $D_p$  is the oxygen diffusion coefficient in the pore, *c* is the oxygen concentration in the pore,  $c_1$  is the oxygen concentration at the pore/GDL interface and

$$
N_{N,p} = D_N \frac{\partial c_N}{\partial r} \bigg|_{r=R_p} \tag{2}
$$

is the radial oxygen flux in the Nafion film at the pore/film interface.

Radial oxygen transport through the film is described by the diffusion equation

$$
\frac{\partial c_N}{\partial t} - \frac{D_N}{r} \frac{\partial}{\partial r} \left( r \frac{\partial c_N}{\partial r} \right) = 0, \quad c_N(R_p) = K_H c(x),
$$
\n
$$
D_N \frac{\partial c_N}{\partial r} \bigg|_{r=R_m} = -\frac{R_p i_*}{2(4F)} \left( \frac{c_{N,m}}{c_{ref}} \right) \exp\left( \frac{\eta}{b} \right) \quad (3)
$$

where  $c_N$  is the oxygen concentration in the Nafion film,  $c_{N,m} \equiv c_N(R_m)$ ,  $D_N$  is the oxygen diffusion coefficient in the film,  $i_*$  is the ORR exchange current density,  $\eta$  is the positive by convention ORR overpotential, and *b* is the ORR Tafel slope. The left boundary condition for Equation [\(3\)](#page-2-0) is Henry's law for oxygen dissolution in Nafion. The right boundary condition to Equation [\(3\)](#page-2-0) describes consumption of dissolved oxygen in the ORR; the factor *Rp*/2 provides correct transition to the standard oxygen mass conservation equation in the CCL for the limiting case of zero Nafion film thickness.

To simplify the calculations we introduce dimensionless variables

$$
\tilde{x} = \frac{x}{l_t}, \quad \tilde{r} = \frac{r}{l_t}, \quad \tilde{t} = \frac{ti_*}{4Fc_{ref}}, \quad \tilde{\eta} = \frac{\eta}{b}, \quad \tilde{j} = \frac{j}{i_*l_t},
$$
\n
$$
\tilde{D} = \frac{4FDc_{ref}}{i_*l_t^2}, \quad \tilde{\omega} = \frac{\omega 4Fc_{ref}}{i_*}, \quad \tilde{\zeta} = \frac{\zeta c_{ref}}{b} \quad (4)
$$

where *l<sup>t</sup>* is the pore length (CCL thickness), *ω* is the angular frequency of AC signal, and *ζ* is the concentration impedance (see below).

With Equation [\(4\)](#page-2-1), Equations [\(1\)](#page-1-1) and [\(3\)](#page-2-0) transform to

<span id="page-2-3"></span><span id="page-2-2"></span><span id="page-2-1"></span><span id="page-2-0"></span>
$$
\frac{\partial \tilde{c}}{\partial \tilde{t}} - \tilde{D}_p \frac{\partial^2 \tilde{c}}{\partial \tilde{x}^2} = \frac{2\tilde{N}_{N,p}}{\tilde{R}_p} \tag{5}
$$

$$
\frac{\partial \tilde{c}_N}{\partial \tilde{t}} - \frac{\tilde{D}_N}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial \tilde{c}_N}{\partial \tilde{r}} \right) = 0, \quad \tilde{c}_N(\tilde{R}_p) = K_H \tilde{c}(\tilde{x}),
$$

$$
\tilde{D}_N \frac{\partial \tilde{c}_N}{\partial \tilde{r}} \bigg|_{\tilde{r} = \tilde{R}_m} = -\frac{\tilde{R}_p}{2} \tilde{c}_{N,m} \exp(\tilde{\eta}) \quad (6)
$$

Systems [\(5\)](#page-2-2) and [\(6\)](#page-2-3) form the basis for the concentration impedance model.

#### *2.2. Equations for Perturbation Amplitudes*

<span id="page-2-4"></span>Now we apply small-amplitude perturbations of the form

$$
\tilde{c}(\tilde{x}, \tilde{t}) = \tilde{c}^{0}(\tilde{x}) + \tilde{c}^{1}(\tilde{x}, \tilde{\omega}) \exp(i\tilde{\omega}\tilde{t}) \n\tilde{c}_{N}(\tilde{x}, \tilde{t}) = \tilde{c}_{N}^{0}(\tilde{x}) + \tilde{c}_{N}^{1}(\tilde{x}, \tilde{\omega}) \exp(i\tilde{\omega}\tilde{t}) \n\tilde{\eta}(\tilde{t}) = \tilde{\eta}^{0} + \tilde{\eta}^{1}(\tilde{\omega}) \exp(i\tilde{\omega}\tilde{t}).
$$
\n(7)

where the superscripts 0 and 1 mark the static functions and the perturbation amplitudes, respectively. Note that fast proton transport means that the static and perturbed ORR overpotentials are independent of  $\tilde{x}$ .

Substituting Equation [\(7\)](#page-2-4) into Equations [\(5\)](#page-2-2) and [\(6\)](#page-2-3) and performing standard procedure of linearization, we come to the system of linear equations for  $\tilde{c}^1$  and  $\tilde{c}^1_N$ :

<span id="page-2-5"></span>
$$
\tilde{D}_p \frac{\partial^2 \tilde{c}^1}{\partial \tilde{x}^2} = -\frac{2\tilde{N}_{N,p}^1}{\tilde{R}_p} + i\tilde{\omega}\tilde{c}^1, \quad \frac{\partial \tilde{c}^1}{\partial \tilde{x}}\bigg|_{\tilde{x}=0} = 0, \quad \tilde{c}^1(1) = \tilde{c}_1^1 \tag{8}
$$

$$
\frac{\tilde{D}_N}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial \tilde{c}_N^1}{\partial \tilde{r}} \right) = \mathrm{i} \tilde{\omega} \tilde{c}_N^1, \quad \tilde{c}_N^1(\tilde{R}_p) = K_H \tilde{c}^1(\tilde{x}),
$$
\n
$$
\tilde{D}_N \frac{\partial \tilde{c}_N^1}{\partial \tilde{r}} \bigg|_{\tilde{r} = \tilde{R}_m} = -\frac{\tilde{R}_p}{2} e^{\tilde{\eta}^0} \left( \tilde{c}_N^1(\tilde{R}_m) + \tilde{c}_{N,m}^0 \tilde{\eta}^1 \right), \quad (9)
$$

where  $\tilde{c}_{N,m}^0 = \tilde{c}_N^0(\tilde{R}_m)$ ,  $\tilde{c}_1^1$  is the oxygen perturbation at the pore/GDL interface, and

<span id="page-3-0"></span>
$$
\tilde{N}_{N,p}^1 = \tilde{D}_N \frac{\partial \tilde{c}_N^1}{\partial \tilde{r}} \bigg|_{\tilde{r} = \tilde{R}_p} \tag{10}
$$

is the perturbed oxygen flux in the Nafion film at the pore interface. Note that due to assumption of fast oxygen transport in the GDL we have

$$
\tilde{c}_1^1 = \tilde{c}_{h'}^1 \tag{11}
$$

i.e.,  $\tilde{c}_1^1$  is equal to the applied oxygen concentration perturbation in the channel  $\tilde{c}_h^1$ .

The system of Equations [\(8\)](#page-2-5) and [\(9\)](#page-3-0) with the coefficient functions given by Equations [\(13\)](#page-3-1) and [\(14\)](#page-3-2) in the next section determine the CCL concentration impedance  $\zeta$ 

$$
\tilde{\zeta} = \frac{\tilde{\eta}^1}{\tilde{c}_h^1}.\tag{12}
$$

Here,  $\tilde{c}_h^1$  is the oxygen concentration perturbation applied in the gas channel, and  $\tilde{\eta}^1$  is the measured perturbation of the cell potential.

## *2.3. Static Equations and Solutions*

Static equations for  $\tilde{c}^0$  and  $\tilde{c}^0$  are obtained from Equations [\(5\)](#page-2-2) and [\(6\)](#page-2-3) simply by chalking out the time derivatives:

<span id="page-3-1"></span>
$$
- \tilde{D}_p \frac{\partial^2 \tilde{c}^0}{\partial \tilde{x}^2} = \frac{2}{\tilde{R}_p} \tilde{D}_N \frac{\partial \tilde{c}_N^0}{\partial \tilde{r}} \bigg|_{\tilde{r} = \tilde{R}_p}, \quad \frac{\partial \tilde{c}^0}{\partial \tilde{x}} \bigg|_{\tilde{x} = 0} = 0, \quad \tilde{c}^0(1) = \tilde{c}_1 \tag{13}
$$

<span id="page-3-2"></span>
$$
\frac{\tilde{D}_N}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial \tilde{c}_N^0}{\partial \tilde{r}} \right) = 0, \qquad \tilde{c}_N^0(\tilde{R}_p) = K_H \tilde{c}^0(\tilde{x}), \quad \tilde{D}_N \frac{\partial \tilde{c}_N^0}{\partial \tilde{r}} \bigg|_{\tilde{r} = \tilde{R}_m} = -\frac{\tilde{R}_p}{2} \tilde{c}_{N,m}^0 \exp \tilde{\eta}^0 \quad (14)
$$

The solution to Equation [\(14\)](#page-3-2) is

<span id="page-3-3"></span>
$$
\tilde{c}_N^0(\tilde{r}) = \left(\frac{\tilde{R}_p \tilde{R}_m \ln(\tilde{R}_m/\tilde{r}) e^{\tilde{\eta}^0} + 2\tilde{D}_N}{\tilde{R}_p \tilde{R}_m \ln(\tilde{R}_m/\tilde{R}_p) e^{\tilde{\eta}^0} + 2\tilde{D}_N}\right) K_H \tilde{c}^0(\tilde{x})
$$
\n(15)

Equation [\(15\)](#page-3-3) allows us to calculate the flux  $\tilde{D}_N \partial \tilde{c}^0_N / \partial \tilde{r}|_{\tilde{r} = \tilde{R}_p}$ :

<span id="page-3-4"></span>
$$
\tilde{D}_N \frac{\partial \tilde{c}_N^0}{\partial \tilde{r}}\bigg|_{\tilde{r} = \tilde{R}_p} = -\frac{\tilde{R}_m \tilde{D}_N e^{\tilde{\eta}^0} K_H \tilde{c}^0(\tilde{x})}{\tilde{R}_p \tilde{R}_m \ln(\tilde{R}_m / \tilde{R}_p) e^{\tilde{\eta}^0} + 2\tilde{D}_N}
$$
(16)

With Equation [\(16\)](#page-3-4), Equation [\(13\)](#page-3-1) takes the form

<span id="page-3-5"></span>
$$
\tilde{D}_p \frac{\partial^2 \tilde{c}^0}{\partial \tilde{x}^2} = \rho \tilde{c}^0, \quad \frac{\partial \tilde{c}^0}{\partial \tilde{x}} \bigg|_{\tilde{x}=0} = 0, \quad \tilde{c}^0(1) = \tilde{c}_1^0 \tag{17}
$$

where

<span id="page-3-6"></span>
$$
\rho = \frac{2\tilde{R}_m \tilde{D}_N e^{\tilde{\eta}^0} K_H}{\tilde{R}_p \left(\tilde{R}_p \tilde{R}_m \ln(\tilde{R}_m/\tilde{R}_p)e^{\tilde{\eta}^0} + 2\tilde{D}_N\right)}
$$
(18)

The solution of Equation [\(17\)](#page-3-5) reads

<span id="page-4-0"></span>
$$
\tilde{c}^{0}(\tilde{x}) = \frac{\tilde{c}_{1}^{0} \cosh\left(\tilde{x}\sqrt{\rho/\tilde{D}_{p}}\right)}{\cosh\left(\sqrt{\rho/\tilde{D}_{p}}\right)}.
$$
\n(19)

Using Equation [\(19\)](#page-4-0) in Equation [\(15\)](#page-3-3) and setting  $\tilde{r} = \tilde{R}_m$ , we get  $\tilde{c}_{N,m}^0$ , which appears in Equation [\(9\)](#page-3-0):

<span id="page-4-1"></span>
$$
\tilde{c}_{N,m}^0 = \frac{\rho \tilde{c}_1^0 \cosh\left(\tilde{x}\sqrt{\rho/\tilde{D}_p}\right)}{e^{\tilde{\eta}^0} \cosh\left(\sqrt{\rho/\tilde{D}_p}\right)}.
$$
\n(20)

For further calculations we need the polarization curve of the CCL. The steady-state proton current conservation equation in the CCL is

$$
\frac{\partial \tilde{j}^0}{\partial \tilde{x}} = -\tilde{c}_{N,m}^0 \exp \tilde{\eta}^0
$$
 (21)

where  $\tilde{j}^0$  is the local proton current density in the film. Integrating this equation over  $\tilde{x}$ from 0 to 1 with Equation [\(20\)](#page-4-1) and the boundary conditions  $\tilde{j}^0(0) = \tilde{j}_0$ ,  $\tilde{j}^0(1) = 0$ , we find

<span id="page-4-2"></span>
$$
\tilde{j}_0 = \frac{\tilde{c}_1^0 \rho \tanh \sqrt{\rho / \tilde{D}_p}}{\sqrt{\rho / \tilde{D}_p}}.
$$
\n(22)

With  $\rho$  given by Equation [\(18\)](#page-3-6), this equation provides the explicit dependence of  $j_0$  vs. the ORR overpotential  $\tilde{\eta}^0$ , i.e., the static polarization curve of the system.

In two limiting cases, Equation [\(22\)](#page-4-2) can be simplified. If the argument of the tanhfunction is small, we may approximate tanh  $x \approx x$  and Equation [\(22\)](#page-4-2) simplifies to

$$
\tilde{j}_0 = \tilde{c}_1^0 \rho. \tag{23}
$$

With  $\rho$ , Equation [\(18\)](#page-3-6), we get expression of  $\tilde{\eta}^0$  through  $\tilde{j}_0$ :

<span id="page-4-3"></span>
$$
e^{\tilde{\eta}^0} = \frac{2\tilde{D}_N \tilde{R}_p \tilde{j}_0}{\tilde{R}_m \left(2\tilde{D}_N K_H \tilde{c}_1^0 - \tilde{R}_p^2 \ln(\tilde{R}_m/\tilde{R}_p) \tilde{j}_0\right)}, \quad \text{when } \sqrt{\rho/\tilde{D}_p} \ll 1 \tag{24}
$$

Equating the zero denominator of Equation [\(24\)](#page-4-3) we get a limiting current density due to the oxygen transport in the Nafion film [\[20\]](#page-10-3)

<span id="page-4-4"></span>
$$
\tilde{j}_N^{\text{lim}} = \frac{2\tilde{D}_N K_H \tilde{c}_1^0}{\tilde{R}_p^2 \ln(\tilde{R}_m/\tilde{R}_p)}.
$$
\n(25)

Equations [\(24\)](#page-4-3) and [\(25\)](#page-4-4) do not contain  $\tilde{D}_p$ , meaning that this case corresponds to fast oxygen transport in the void pore.

In the limit of  $\sqrt{\rho/\tilde{D}_p} \geq 2$ , we may replace the tanh-function in Equation [\(22\)](#page-4-2) by unity, which leads to

$$
\tilde{j}_0 = \frac{\tilde{c}_1^0 \rho}{\sqrt{\rho/\tilde{D}_p}}.\tag{26}
$$

With Equation [\(18\)](#page-3-6) we find

<span id="page-4-5"></span>
$$
e^{\tilde{\eta}^0} = \frac{2\tilde{D}_N \tilde{R}_p \tilde{J}_0^2}{\tilde{R}_m \left(2\tilde{D}_p \tilde{D}_N K_H (\tilde{c}_1^0)^2 - \tilde{R}_p^2 \ln(\tilde{R}_m/\tilde{R}_p) \tilde{J}_0^2\right)}, \quad \text{when } \sqrt{\rho/\tilde{D}_p} \ge 2 \tag{27}
$$

Equation [\(27\)](#page-4-5) determines the limiting current density

<span id="page-5-5"></span>
$$
\tilde{j}_N^{\text{lim}} = \sqrt{\frac{2\tilde{D}_p \tilde{D}_N K_H (\tilde{c}_1^0)^2}{\tilde{R}_p^2 \ln(\tilde{R}_m/\tilde{R}_p)}}
$$
(28)

Equation [\(27\)](#page-4-5) correlates with the assumption of large oxygen transport loss in the CCL and below, this equation will be used in numerical calculations.

### <span id="page-5-0"></span>**3. Results and Discussion**

Static solutions of the previous section allow us to calculate the concentration impedance. The solution to Equation [\(9\)](#page-3-0) is a rather cumbersome expression containing Bessel functions. This solution is only needed for calculation of the perturbed oxygen flux  $\tilde{N}_{N,p}^1$  appearing in Equation [\(8\)](#page-2-5). Rather tedious algebra leads to

<span id="page-5-1"></span>
$$
\tilde{N}_{N,p}^1 = \frac{\tilde{D}_N}{Q} \left( P_c K_H \tilde{c}^1(\tilde{x}) + P_\eta \tilde{c}_{N,m}^0(\tilde{x}) e^{\tilde{\eta}^0} \tilde{\eta}^1 \right)
$$
\n(29)

where the independent of  $\tilde{x}$  coefficients  $P_c$ ,  $P_\eta$  and  $Q$  are given in Appendix [A.](#page-9-8) Equation [\(29\)](#page-5-1) with  $(20)$  allow us to solve the key Equation  $(8)$ :

$$
\tilde{c}^{1}(\tilde{x}) = \frac{2\rho P_{\eta}\tilde{D}_{N}\tilde{c}_{1}^{0}\tilde{\eta}^{1}}{\tilde{R}_{p}Q\cos(\phi)(\tilde{D}_{p}\phi^{2}+\rho)\cosh\sqrt{\rho/\tilde{D}_{p}}}\times \left(\cos(\phi\tilde{x})\cosh\sqrt{\frac{\rho}{\tilde{D}_{p}}}-\cos(\phi)\cosh\left(\tilde{x}\sqrt{\frac{\rho}{\tilde{D}_{p}}}\right)\right) + \frac{\tilde{c}_{h}^{1}\cos(\phi\tilde{x})}{\cos\phi} \quad (30)
$$

where

<span id="page-5-6"></span><span id="page-5-2"></span>
$$
\phi = \sqrt{\frac{2P_c\tilde{D}_N K_H}{\tilde{R}_p\tilde{D}_p Q} - \frac{i\tilde{\omega}}{\tilde{D}_p}}.
$$
\n(31)

As discussed above, the model is valid in the limit of strong oxygen transport loss through the CCL depth. In this regime, the perturbation of oxygen concentration at the membrane surface is nearly zero:  $\tilde{c}^1(0)\simeq 0$ . Setting in Equation [\(30\)](#page-5-2)  $\tilde{x}=0$  ,  $\tilde{c}^1(0)=0$  and dividing the resulting equation by  $\tilde{c}^1_{h'}$ , we get an algebraic equation for zeta-impedance

<span id="page-5-3"></span>
$$
0 = \frac{2\rho P_{\eta} \tilde{D}_{N} \tilde{c}_{1}^{0} \tilde{\zeta} \left( \cosh \sqrt{\rho / \tilde{D}_{p} - \cos \phi} \right)}{\tilde{R}_{p} Q (\tilde{D}_{p} \phi^{2} + \rho) \cosh \sqrt{\rho / \tilde{D}_{p}}} + 1
$$
\n(32)

Solving Equation [\(32\)](#page-5-3) for  $\tilde{\zeta}$ , we finally find

<span id="page-5-4"></span>
$$
\tilde{\zeta} = \frac{\tilde{R}_p Q (\tilde{D}_p \phi^2 + \rho) \cosh \sqrt{\rho / \tilde{D}_p}}{2\rho P_\eta \tilde{D}_N \tilde{c}_1^0 \left(\cos \phi - \cosh \sqrt{\rho / \tilde{D}_p}\right)}.
$$
\n(33)

Equation [\(33\)](#page-5-4) is the main result of this work.

The spectra of Equation [\(33\)](#page-5-4) in the dimension form  $\zeta = \tilde{\zeta}b/c_{ref}$  for the two oxygen diffusion coefficients  $D_N$  in the Nafion film are shown in Figure [2.](#page-6-0) The base-case set of parameters used in the calculations is collected in Table [1.](#page-6-1) As can be seen, for these parameters, the zeta-spectrum is quite sensitive to the value of *D<sup>N</sup>* (Figure [2a](#page-6-0)), which makes zeta-spectroscopy a good candidate for measuring film transport properties. Note that the variation of  $D_N$  does not change the characteristic frequency of the curve in Figure [2b](#page-6-0).

Tafel slope <i>b</i> , V	0.03
Exchange current density $i_{*}$ , A cm <sup>-3</sup>	$10^{-3}$
Double layer capacitance $C_{d,l}$ , F cm <sup>-3</sup>	20
Oxygen diffusion coefficient in	
the Nation film, $D_N$ , cm <sup>2</sup> s <sup>-1</sup>	$\approx$ 1 $\times$ 10 <sup>-6</sup>
Dimensionless Henry's constant for	
O <sub>2</sub> solubility in water at 80 °C, $K_H$	$6.76 \times 10^{-3}$
Catalyst layer thickness $l_t$ , cm	$3 \times 10^{-4}$ (3 µm)
Nafion film thickness $l_N$ , cm	$10 \times 10^{-7}$ (10 nm)
Pore radius $R_p$	$30 \times 10^{-7}$ (30 nm)
Cell current density $j_0$ , A cm <sup>-2</sup>	1.5
Pressure	Standard
Cell temperature $T$ , K	$273 + 80$

<span id="page-6-1"></span>**Table 1.** The base-case cell parameters used in calculations.

<span id="page-6-0"></span>

**Figure 2.** (**a**) The Nyquist spectra and (**b**) the frequency dependence of imaginary part of *ζ*-impedance, Equation [\(33\)](#page-5-4), for the indicated values of oxygen diffusion coefficient in the Nafion film  $D_N$ , cm<sup>2</sup> s<sup>-1</sup>. The other parameters for calculations are listed in Table [1.](#page-6-1)

Variation of the zeta-spectrum with pore diffusivity  $D_p$  is illustrated in Figure [3:](#page-6-2) the growth of  $D_p$  increases the static "resistivity" of the spectrum and shifts the peak of  $-\text{Im}(\zeta)$ to higher frequency (Figure [3b](#page-6-2)). The dependence of −Im(*ζ*) peak frequency on *D<sup>p</sup>* is close to linear: twice higher *D<sup>p</sup>* shifts the peak to twice larger frequency (Figure [3b](#page-6-2)). The characteristic frequency of peaks in Figure [3b](#page-6-2) is between 1 and 10 kHz, far above the characteristic frequency of faradaic processes in the cell, which typically does not exceed 100 Hz. It is interesting to note that variation of  $D_N$  and  $D_p$  lead to the opposite trends in the spectrum diameter: the curl increases with the decrease in  $D<sub>N</sub>$  and with the growth of *D<sup>p</sup>* (cf. Figures [2a](#page-6-0) and [3a](#page-6-2)).

<span id="page-6-2"></span>

**Figure 3.** (**a**) The Nyquist spectra and (**b**) the frequency dependence of imaginary part of *ζ*-impedance, Equation [\(33\)](#page-5-4), for the indicated values of oxygen diffusion coefficient in the pore  $D_p$ , cm<sup>2</sup> s<sup>-1</sup>. The other parameters for calculations are listed in Table [1.](#page-6-1)

Of particular interest is the static value  $\tilde{\zeta}^0$ , corresponding to the rightmost point of the spectra in Figures [2](#page-6-0) and [3.](#page-6-2) Unfortunately, Maple<sup>®</sup> fails to calculate  $\zeta^0 = \lim_{\tilde{\omega} \to 0} \tilde{\zeta}$ . Numerically calculated *ζ* <sup>0</sup> as a function of the Nafion film diffusivity is shown in Figure [4.](#page-7-0)

For  $D_N$  below 1.0  $\times$  10 $^{-6}$  cm<sup>2</sup> s $^{-1}$  the curve is very steep due to effect of limiting current: with the decrease in  $D_N$ , the right side of the dimension version of Equation [\(28\)](#page-5-5) tends to the current of 1.5 A cm<sup>-2</sup> fixed in the calculations and the zeta-impedance rapidly increases. From Figure [4](#page-7-0) it follows that measurements of  $D<sub>N</sub>$  should be best done close to the limiting current  $j_N^{\text{lim}}$  due to oxygen transport in the film. In this range of currents, the zeta-impedance is most sensitive to Nafion film transport parameters.

<span id="page-7-0"></span>

**Figure 4.** The dependence of static concentration impedance *ζ* <sup>0</sup> on the oxygen diffusion coefficient in the Nafion film  $D_N$ . The spectra in Figure [2](#page-6-0) are plotted at the points indicated by red dots.

The model above is developed assuming fast proton transport in the CCL. This means that the cell current density must be much less than the characteristic current for proton transport in the CCL:

<span id="page-7-3"></span>
$$
j_0 \ll j_* = \frac{\sigma_N b}{l_t} \tag{34}
$$

where  $\sigma_N$  is the CCL proton conductivity. In a working PEMFC,  $\sigma_N \simeq 0.02$  S cm<sup>-1</sup>, hence with  $b = 0.03$  V and  $l_t = 3 \times 10^{-4}$  cm we get  $j_* = 2$  A cm<sup>-2</sup>.

On the other hand, the condition  $\tilde{c}_1^1 \simeq 0$  holds for the cell current density satisfying to

<span id="page-7-1"></span>
$$
j_0 \gtrsim \frac{4FD_p c_1^0}{l_t} \tag{35}
$$

while the largest sensitivity of the zeta-spectrum to the Nafion film oxygen diffusivity is achieved for the cell currents satisfying to

<span id="page-7-2"></span>
$$
j_0 \simeq 4Fc_1^0 \sqrt{\frac{2D_p D_N K_H}{R_p^2 \ln(R_m/R_p)}}
$$
(36)

which is the dimension version of Equation [\(28\)](#page-5-5). Equations [\(35\)](#page-7-1) and [\(36\)](#page-7-2) allow one to select optimal  $c_1^0$  and  $j_0$  for experiments.

## **4. Conclusions**

In this study, a model for concentration (zeta-) impedance of the low-Pt cathode catalyst layer in a PEM fuel cell is developed. The model is based on the transient oxygen mass transport equations in the cylindrical pore surrounded by a thin Nafion film separating the pore volume from Pt/C surface. An analytical solution for zeta-impedance is obtained, which can be used for fast fitting experimental zeta-spectra. It is shown that the zetaspectrum is very sensitive to the value of oxygen diffusion coefficient in the Nafion film, provided that the cell operates at a high rate of oxygen consumption in the CCL and close to the limiting current density due to oxygen transport in the film.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Data is contained within the article.

**Conflicts of Interest:** The authors declare no conflict of interest.

# **Nomenclature**



- *m* Pt/C (metal) surface *N* Nation film
- *N* Nafion film
- *p* Pore/Nafion film interface

## **Superscripts:**





#### <span id="page-9-8"></span>**Appendix A. Coefficients in Equations** [\(29\)](#page-5-1)**–**[\(33\)](#page-5-4)

$$
P_c = \gamma q^2 \tilde{D}_N (K_1(-q\tilde{R}_m)I_1(q\tilde{R}_p) - K_1(-q\tilde{R}_p)I_1(q\tilde{R}_m)) + q e^{\tilde{\eta}^0} (K_0(-q\tilde{R}_m)I_1(q\tilde{R}_p) - K_1(-q\tilde{R}_p)I_0(q\tilde{R}_m))
$$
 (A1)

$$
P_{\eta} = q(K_0(-q\tilde{R}_p)I_1(q\tilde{R}_p) - K_1(-q\tilde{R}_p)I_0(q\tilde{R}_p))
$$
\n(A2)

$$
Q = \gamma q \tilde{D}_N (K_1(-q\tilde{R}_m)I_0(q\tilde{R}_p) - K_0(-q\tilde{R}_p)I_1(q\tilde{R}_m)) + e^{\tilde{\eta}^0} (K_0(-q\tilde{R}_m)I_0(q\tilde{R}_p) - K_0(-q\tilde{R}_p)I_0(q\tilde{R}_m))
$$
 (A3)

<span id="page-9-9"></span>
$$
q = \sqrt{\frac{i\tilde{\omega}}{\tilde{D}_N}}, \quad \gamma = \frac{2}{\tilde{R}_p}
$$
 (A4)

Here, *I*0, *I*<sup>1</sup> are the Bessel functions of the second kind, *K*0, *K*<sup>1</sup> are the modified Bessel functions of the second kind.

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