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Research article

The improvement of 10th students' mathematical communication skills through learning ellipse topics

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ABSTRACT

Mathematical education in general, and mathematics education at high schools in particular, creates favorable conditions for students to develop essential and core competencies and assists students in improving their mathematical competence as a foundation for good learning. It also promotes necessary skills for society, in which mathematical communication skills are essential skills. The purpose of the research is to enhance students' mathematical communication skills while studying ellipse topics. In this study, 87 students in the tenth grade were tested to see if the teaching process for accelerating the development of mathematical communication skills related to ellipse topics was effective and possible. Students in the experimental class were instructed using a four-step learning model with the activities regarding the ACODESA method and mind map, whereas students in the control class were guided using the conventional way of instruction. A study design involving a pre-test, a treatment, and a post-test was used to determine the effectiveness of this type of teaching approach. Quantitative and qualitative analyses of the gathered data assessed the students' progress in mathematical language activities. The findings indicated that most students' mathematical communication skills regarding ellipse topics had improved significantly. The learners towards the above process also showed a positive attitude. In addition, the research findings have important implications and recommendations.

1. Introduction

In mathematics classes, a variety of different types of communication can take place between students. Involvement in this activity can take the form of interaction with the teacher, small group work, or standing in front of the classroom to make a presentation in order to clarify a discovered idea. The teacher can set up the classroom so that students face one another to encourage more discussion and participation in front of the whole class. For students who are hesitant to speak up in front of their peers, this is especially beneficial. A critical component of mathematics and mathematics education is the ability to communicate effectively with one another. To communicate, one must first share ideas with others, then be willing to discuss those ideas with others, and then be capable of reflecting on those ideas quickly after they have occurred. The students can benefit from doing mathematics communication so that they can comprehend mathematics in depth.

The teacher must be the sole channel through which information is transferred when it comes to teaching students. Teachers use various methods to convey information to students during a lecture, including language, speech, writing, and other audiovisual tools. As the intended recipients of messages, students have an easier time hearing and deducing information from them. Based on their knowledge and experience, they can also evaluate and make changes to data. Each student will have a reaction or feedback after hearing the message (surprised, confused, or disagree). It is undeniable that communication is dependent on feedback; it enables the source to identify more accurately, correct errors, or revise the message to reflect the current situation better. The author also stressed that the communication process is constantly dynamic, unstable, and changing; communication flow factors are always interconnected. Communication in the classroom is broken down into steps, making it more transparent for all parties concerned.

Future life will necessitate the development of a mathematical mindset as well as communication skills. Mathematics is a unique language made up of words, tables, and illustrations such as graphs and symbols to improve students' communication skills. It is more stable and effective for students to use their mathematical knowledge when they are asked to investigate and explain a mathematical problem, write or speak

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about the results, and argue. Throughout that time, we discovered two things: first, students use communication to assist them in learning math, and second, students use communication to assist them in learning math. When evaluating a student's ability to solve problems, educators must consider various factors and provide opportunities for students to practice their mathematical communication skills.

1.1. The concept of mathematical communication

It has been demonstrated that mathematical representation, explanation, argument, and presentation are all related to mathematical communication. Students can express their opinions and ideas in various ways, including mathematical representations, written words, and other forms of writing. It is possible for students to better understand their ideas and points of view in this situation by engaging in discussions with other students. When teachers reason with students, they are putting their critical thinking skills to the test. The final step involves students presenting evidence to ensure that others understand the problem clearly and accurately. Presentations, explanations, and arguments are basic mathematical communication methods that encourage students to share, exchange, and reflect (Wood, 2012). During group work, students are faced with various questions as they explain, argue, and debate their findings. According to Smieskova (2017), mathematical communication skills are useful tools for encouraging students' creativity and motivation in their mathematics studies.

It is possible to communicate with others more easily when using mathematical representation to describe relationships between objects and symbols. When discussing issues, students will demonstrate their understanding of math concepts related to problem-solving (Sari and Darhim, 2020), the explanation and discussion stages. Throughout the course, students will practice and learn various methods for demonstrating their understanding and determining whether or not the statement is accurate. A common misconception is that an argument is a collection of arguments that resolve a problem. Particularly noteworthy is the fact that one can argue without regard to whether the conclusion desired by the listener is correct. Students can either demonstrate or disprove counter-examples, regardless of whether they are correct or incorrect in their reasoning (Salsabila, 2019). Because of this, it follows that the argument is about discovering mathematical proofs in general rather than about discovering specific proofs. In order to assist others in understanding a problem, it is necessary to prove theorems or the correctness of specific judgments, which can be done either in writing or verbally. Mathematical communication is divided into four categories by teachers: verbal communication (including speaking and listening), listening (including verbal communication via reading), and written communication (including writing assignments) (Utomo and Syarifah, 2021; Wilson, 2009; Utari et al., 2020).

Certain mathematical communication standards must be followed, and this is extremely important to understand. Students begin organizing and formulating ideas by using various visual and verbal representations to get their ideas off the ground. Second, they can communicate their ideas (point of view) to their peers and teachers coherently and clearly. Third, students can assist their peers by examining, evaluating, thinking about, and even solving the mathematical problems that their peers are encountering and experiencing. The final step is for students to accurately utilize their mathematical abilities to express their points of view through written communication.

1.2. Student's difficulties in mathematical communication

Some difficulties are likely to arise for students during their math communication. In mathematics classrooms, various forms of mathematical communication address math problems, which aids students in comprehending and applying mathematical knowledge and skills. During this time, students must focus, debate, explain, and prove math problems logically. Students must first and foremost possess strong mathematical abilities in order to accomplish this goal. Because students lack sufficient mathematical knowledge, they cannot effectively communicate mathematical ideas. However, a mathematical vocabulary is necessary for mathematical communication; without it, the process fails. Mathematical vocabulary limitations should not impede communication in mathematics, especially given the demands placed on writing skills. Correspondingly, in general, the mathematical communication abilities of junior high school students are still severely lacking (Rustam and Ramlan, 2017).

A further barrier to effective mathematical communication is students' lack of self-confidence in their abilities. According to the study, these students are often afraid to speak up or ask questions of their teachers and disagree with other students about the content of the lessons. Critical thinking helps teachers develop appropriate questions, evaluate potential answers, and assess the reliability of information sources. Assume students' critical thinking skills are still lacking when participating in mathematical communication. In that case, they are unlikely to question their teachers' information accuracy or the problems' simplicity. It will be impossible to achieve high efficiency in mathematical communication between teachers and students if the problem is not understood and explained properly.

1.3. The ways of developing students' mathematical communication skills in some specific topics

Math communication skills can be improved by teachers using lesson plans based on RME (Andriani and Fauzan, 2019; Hutapea et al., 2019; Palinussa, 2013; Palinussa et al., 2021; Rahman et al., 2012; Supriyanto et al., 2020). Aside from that, the authors have pointed out differences in the effects of problem-based learning (Hidayati et al., 2020; Sudia and Muhammad, 2020); and another study is based on project-based learning models without scaffolding (Paruntu et al., 2018). Meanwhile, some authors hope to improve students' mathematical communication skills through learning models such as digital teaching module (Setiyani et al., 2020), cooperative learning (Salsabil et al., 2017), ASSURE learning (Sundayana et al., 2017), learning devices based on constructivism approach (Zulaika and Syarifuddin, 2018), problem-posing and think-pair-share learning models (Rohim and Umam, 2019), school location (Juliarta and Landong, 2020), self-regulated learning (Sudia and Muhammad, 2020), quantum learning (Utari et al., 2020). Reikerås (2020) says that in toddlers, play abilities and mathematical abilities are linked, including the capability for mathematical communication. The results from Yang et al. (2016) indicate that pupils utilize computer-supported reciprocal peer tutoring to improve their ability to communicate in mathematics. This is similar to the findings of Lestari et al. (2019) on GeoGebra-assisted reciprocal peer tutoring strategy for students' mathematical communication skills, which were derived from gender differences in students. Following that, Fuchs et al. (2020) implemented peer-assisted learning strategies (PALS) to help students improve their reading and mathematics performance.

Recently, there have been several studies on mathematical communication skills related to a wide range of mathematical topics, including algebraic factorization (Disasmitowati and Utami, 2017), set material (Salsabil et al., 2017), circle material (Alfarisyi and Sutiarso, 2020), relation and function (Setiyani et al., 2020) and absolute maximum value of a function (Sudia and Muhammad, 2020).

1.4. Some studies focused on mathematical communication skills

According to the Treffinger model and the typical learning style, it was created based on the previous mathematical knowledge (high, medium, and low) (Alhaddad et al., 2015). The participants in this study were all Faculty of Mathematics Pedagogy students who had studied discrete mathematics at a university in Ternate City and participated in an empirical study. The study found no interactive effects between learning and prior math knowledge in the Treffinger and common learning frameworks for improving students' mathematical communication skills (Alfarisyi and Sutiarso, 2020).

There is a great deal of study done on math instructional skills in mathematics classes. For example, they described students' written mathematical communication skills in open math problems based on their mathematical abilities. The educators were able to elaborate on the situation and make recommendations due to conducting descriptive qualitative research. Math proficiency tests, written math communication skills tests on open-ended problems (Zayyadi and Saleh, 2020), and interview guides are some of the tools that have been used. There were three indicators of written communication skills in subjects with high and average mathematical abilities. That was the ability to express mathematical ideas in writing and through performance in a variety of settings. Writing situation models requires understanding, interpreting, and evaluating written mathematical ideas and other visual forms. This task necessitates a working knowledge of the written mathematical symbols used in constructing written situation models. Palinussa et al. (2021) investigated the impact of rural mathematics education (RME) on mathematical reasoning and communication abilities in rural settings. The instrument was designed to collect mathematical reasoning and communication skills problem statements. It is shown that RME influences students' abilities in mathematical reasoning and communication. Thus, RME is suitable for helping students reason mathematically and communicate better in the rural island context.

As with problem-posing, real-world math education is more effective than direct instruction and problem-posing models in promoting defensive cognitive style (Kamid et al., 2020). It is important to note that each cognitive style and problem-posing learning model in Indonesian real-world math education is different. When it comes to learning, instructional learning models tend to favor fields that are more field-dependent. Alfarisyi and Sutiarso (2020) found a significant difference between male and female students' mathematical communication skills and educational backgrounds. Female students outperform male students when conveying ideas to explain each indicator of the mathematical communication ability test. Yaniawati et al. (2019) used the CORE learning model in another study to improve students' mathematical communication abilities. Instead of expository learning, students' mathematical communication and connections were deemed more effective (Minarti and Wahyudin, 2019; Sari and Darhim, 2020).

2. Theoretical framework

2.1. Mathematical communication skills in curriculum and mathematics textbooks in Vietnam

Vietnam's Ministry of Education and Training's 2018 General Education Curriculum states that mathematics is taught to develop ten core competencies (Vietnamese Ministry of Education and Training, 2018). Specifically, there are three general competencies – autonomy and self-study – and seven professional competencies that follow the subject system in each grade level. Preparing students for high school mathematics requires mathematical modeling, problem-solving, communication, and tool use skills. Ability to effectively use mathematical language (letters, symbols, diagrams and graphs) in conjunction with a common language; demonstrated through mathematical texts, asking questions and answering reasoning questions when proving the correctness of propositions; demonstrated through solving math problems and solving mathematical problems.

As a result, mathematics communication skills require knowledge, skills, and attitude in addition to mathematical knowledge. It is well known that students must have a solid foundation in mathematics. The result is that they understand mathematical language (words, terms) and can communicate their ideas more effectively. As a final note, students must approach mathematics with a spirit of cooperation, sharing, and positivity. The following are elements of mathematical communication skills.

(1) Recognizing and remembering necessary mathematical information presented in mathematical text or by others, reading comprehension, and taking notes on what was heard or written.
(2) Communicating mathematical contents, ideas, and solutions to others through oral or written expression (speaking or writing) (with appropriate completeness and accuracy).
(3) Effectively combining mathematical language (numbers, letters, symbols, charts, graphs, logical connections) with a common language or physical movements when presenting, solving similar problems, and evaluating math ideas in interactions (discussing, debating) with others is a requirement.
(4) Demonstrating self-assurance when presenting, expressing, asking questions, discussing, and debating mathematical concepts.

As a result, this program defines mathematical communication skills as the ability to represent and communicate information. The presence of these two essential elements ensures that the other is supported and bolstered by the other. To summarize, mathematics communication skills are the ability to express and explain ideas so that others can comprehend. The following are the specific manifestations of mathematical communication skills and requirements that must be met by junior high school students in this program. First and foremost, students must pay attention to comprehension, read thoroughly, and take notes (summarizing) the necessary math information, which is the text's primary focus (in the form of written or spoken text). As a result, students examine the text, select key points, and extract the mathematical information they require (in the form of written or spoken text). They then interact with others to perform mathematical contents and ideas and express, question and discuss mathematical contents, ideas, and solutions (at a relatively complete and accurate level). They also use mathematical language to express mathematical ideas and to show evidence, methods, and argument outcomes in conjunction with a common language. They then show confidence in presenting, expressing, discussing, arguing, and explaining mathematical content in simple situations like classroom presentations.

2.2. Ellipse topics in curriculum and mathematics textbooks in Vietnam

By analyzing the ellipse topic's content in the Geometry 10 textbook program and the types of tasks (Hao et al., 2015), we found that the knowledge related to the ellipse appeared explicitly, mostly acknowledged. The main activities of students are reading comprehension, summarizing basic mathematical information with the aim of mastering definitions, writing elliptical canonical equations, determining factors and solving basic math problems involving ellipse.

The most common types of solving-problem tasks are finding the components of an ellipse and writing the ellipse equation. Nevertheless, real-world problems have not appeared much, and the briefly mentioned phenomena in life have not been clearly explained. The activities in the textbooks only stop the students from responding according to their senses; the mathematical debate activities take place relatively little, not to mention the training for students' communication ability and the development of mathematical communication capacity. Some of the developed components are listening comprehension, reading comprehension and writing, summarizing basic and central mathematical information in spoken or written texts and presenting fully, accurately and logically the contents and math ideas.

2.3. The process of designing lesson plans in the direction of developing mathematical communication skills for students

The process of developing students' mathematical communication abilities is a lengthy one. Teaching and developing this student capacity requires teachers to design and organize the teaching process to carry out learning activities suitable to their abilities, stimulating their interest in learning. Designing math lesson plans is an important job of teachers before organizing student learning activities in class. It is not merely a copy of a math textbook but vividly shows the organic relationship between the goals, contents, methods and conditions of teaching. Designing math lesson plans should consider both objectives, such as behavioral indicators that can be observed, measured, counted, and methods that entail organizing activities. Lessons are thought to be most effective when problems are used that are integrated, practical, and important. Figure 1 depicts a three-step process for creating a lesson plan.

Step 1. Determine the lesson objectives.

The teacher's goal is to achieve certain outcomes after completing the lesson that is referred to as its objectives. The objectives serve as the foundation for developing and implementing educational programs, subject programs, lessons, and learning activities. The objectives guide the development of the remaining elements of the instructional process. Defining goals in developing mathematical communication competence includes knowledge, skills, attitudes, and ways and paths from knowledge, skills, and attitudes to those results: competence. The goal of developing mathematical communication competence must be what students can do with the knowledge they have learned. The lesson's objectives serve as a framework for the activities that are planned during the lesson. Hence, it is essential to ensure the consistency between lesson objectives and activities: Lesson objectives regulate activity goals, activity objectives serve lesson objectives. For example, through the analysis of textbook content, statistics of task types. We define the goal of teaching the ellipse, such as understanding the definition of an ellipse, finding the canonical equation of the ellipse, identifying the elements of an ellipse, solving elliptic problems with practical applications, and explaining some phenomena in life-related to ellipses.

Step 2. Identify and select lesson content.

Content of teaching is based on social values and the knowledge and experiences accumulated by previous generations in morality, intelligence, labor, physical and aesthetics. Pedagogical and professional considerations are meticulously considered to ensure that the educational goal is met, the learning abilities of students, and the performance conditions of students. The educational program specifies the content that will be covered in the classroom. The lesson's content is derived from the program's content, which helps to make it more concrete. Determining the specific details of the lesson content is dependent on the lesson objectives. Put another way, once a goal has been established, the teacher must use the goal to determine and select the appropriate content for the students. A successful lesson can therefore be achieved if the content corresponds to the goal. Determining the content of the lesson to develop mathematical communication competence for students in teaching ellipse needs:

1) Based on the math program: By analyzing the content of textbooks and types of tasks, determining the knowledge to be taught.
2) Based on the lesson objectives: The content must be appropriate and help students achieve the set lesson objectives.
3) Associated with practice: The ellipse is a familiar and common line in practice. There are numerous applications in the context of solving and explaining real-world problems. Thus, we create phenomena and situations that students may encounter in their everyday lives to help them learn. Practical knowledge helps students better understand the nature of mathematical knowledge, see the meaning of mathematical knowledge, make students more interested in learning mathematics, and a necessary condition for developing mathematical communication capacity and developing other competencies for students.

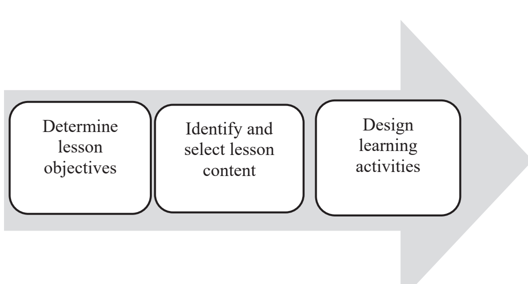

Figure 1. Lesson plan design process.

Step 3. Design learning activities.

Teachers must plan learning activities by the objectives and content of the lesson. Students will have opportunities to experience, interact, exchange, learn from, and apply what they have learned through well-designed learning activities tailored to their abilities and needs. Each activity is designed to help students achieve one or more of the lesson's objectives in most cases. Scheduling activities at specific times and arranging them logically according to how far the lesson is important. For instance, learning activities in elliptical topics include warm-up, knowledge-forming, problem-solving, and application and expansion. Each activity we build has the following components: operational goals, content, methods and forms, a product of activity. Thereby, in each component, we apply appropriate measures to develop mathematical communication competence for students through elliptical teaching activities.

2.4. Purpose of the study and research questions

This study aims to promote the mathematical communication skills of 10th-grade students when they study the topic ellipse. The following questions will be answered in the research:

1. What do students take away from learning about ellipse topics?
2. How will students' mathematical communication skills improve from learning about ellipse topics in a four-step teaching process?
3. What kind of attitude do students have when learning with the teaching process described above?

3. Method

The quantitative approach applied in this research is the conclusion of the data from the research results, which is illustrated through statistical formulas. The goal of quantitative quasi-experimental research is to determine the effectiveness of treatment. The researchers (Baye et al., 2021; Darmawijoyo and Hartono, 2020; Palinussa, 2013) most commonly use the pre-test, post-test control group design. This study design is included in the methods used for behavioral sciences research, as mentioned by Gravetter and Forzano (2018). As illustrated in Table 1, the empirical sequence of pedagogy was followed.

An independent t-test was employed in this investigation to see any difference in the mean scores between the experimental and control groups. Additionally, this t-test allowed applying the mean of the two pre-tests for the above two groups to determine the equivalence between the groups. It is stated that the level of the effect represents the magnitude of the impact. When it comes to an impact, the degree of influence is a good indicator of the extent of the effect. The t-test showed a significant difference in the mean; the magnitude of the effect then told us how big the effect was. Researchers used the Pearson correlation coefficient to

Table 1. The study design.

Group	Pre-test	Treatment	Post-test
Class 10A1 (experimental group, 44 students)	test	X	test
Class 10A2 (control group, 43 students)	test	—	test

Note. Experimental group: EG; Control group: CG.
X: The teaching process is oriented to developing students' mathematical communication skills.

determine how closely the two tests in the experimental class were related.

3.1. Participants

The experiment was carried out on 87 students from classes 10A1 and 10A2 at An Nhon Tay High School in the Cu Chi district of Ho Chi Minh City, Vietnam. Because this is quasi-experiment research, the participants were selected from the classes of a public school. It was decided which participants would take part in the learning process based on their availability and willingness. Furthermore, this research has revealed no student prejudice or disregard and no negative consequences for the students. In addition, the research team conducted a training course for 25 teachers to help them have the professional competence to develop students' ability to communicate mathematically. However, only one teacher was chosen to conduct instruction for the students in the experimental class because this teacher met the standards of mastery of knowledge, skills, and competence in developing mathematical skills in students set by the researchers.

The experimental class consisted of students who studied through the teaching process outlined below, whereas the students in the control class were guided by another untrained teacher who followed the conventional teaching model. In the control group, participants were subjected to traditional didactic lectures. In other words, compared to the experimental class, they did not benefit from a teaching process geared toward improving mathematical communication competence. This group was not made aware of the subject matter that would be covered. In addition, the lecture was not separated into smaller sub-topics, and students were neither encouraged nor discouraged from asking questions during the course.

3.2. The teaching process

The teaching process is associated with four learning activities of students to develop mathematical communication competence, usually including warm-up activities, knowledge-forming activities, practice activities, and activities of application and expansion.

(1) Warm-up activity.

Content: Psychological warm-up (ready, happy, positive mood) or thinking warm-up (makes students brainstorm, think, raise questions, want to learn and solve).

Purpose: Stimulate students' curiosity, arouse students' interest in the lesson they will learn or mobilize the knowledge, skills, and experiences already in learning and practice related to the knowledge they are about to learn cognitive contradictions.

How to do: Show learners clearly the goal, meaning and necessity of the lesson; ask questions, quizzes, tell stories, set a situation, organize games about issues related to the content of knowledge in the lesson.

For example: In teaching ellipse, teachers give situations close to reality that play an attractive role, motivating students to participate in activities. Students will be forced to think due to the explanation, which will lead to perceptional conflicts and a heightened interest in the lesson they will be learning.

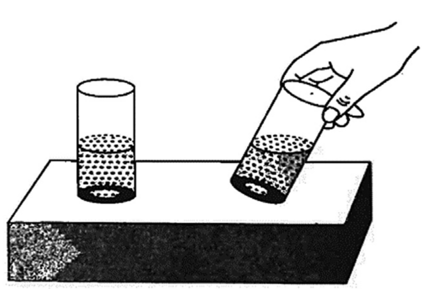

Figure 2. Illustration for the teacher's Question 1.

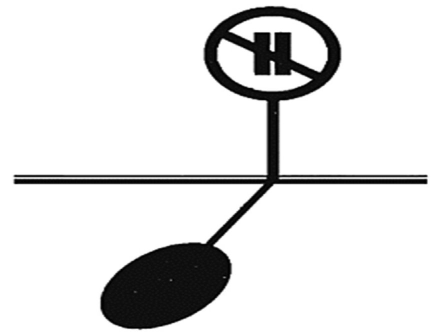

Figure 3. Illustration for the teacher's Question 2.

Question 1: (With illustrations) According to you, when standing upright in a cylindrical cup (bottle) containing water, what is the line between the surface of the water and the wall of the cup (bottle)? If we tilt the cup (bottle) and you see if it is still a circle? Explain? (Figure 2).

Then the teacher asked Question 2: Tell if the shadow of a circle on a plane is a circle or not? (Figure 3).

(2) Knowledge forming activities

Content: An activity in which students acquire knowledge by themselves through the organization and guidance of teachers. Thereby, capacity is formed and developed.

Purpose: Help students learn new content of the lesson, provide students with the scientific basis of the new knowledge mentioned in the lesson.

How to do it: Use prompting questions/requests, examples, and metaphors for students to find information, discover, discover and absorb new knowledge; Students self-study individually, in pairs or groups or the whole class through the organization and guidance of teachers to gain new knowledge of the lesson.

For example: In teaching ellipse, teachers can ask students to perform the following tasks to form how to draw ellipses by group activities of 4 people.

Requirements: Try to draw an ellipse as accurately as possible on the board. How will a piece of rope and two friends help you do this?

The teacher asked some questions: 1) How do you know it is an ellipse? 2) So, where are the two points and located on the figure?

(3) Practice activities.

Content: Students directly apply knowledge and skills to solve situations/problems in learning or practice.

Purpose: To help students consolidate and perfect their knowledge and skills acquired in knowledge-forming activities; Create conditions for students to properly express knowledge or describe skills learned in the language in their way.

How to do it: Students use the knowledge they have just learned to resolve basic tasks and exercises of the lesson; Students self-study individually, in pairs or groups or the whole class through the organization and guidance of teachers to practice. For example: In teaching the ellipse, the teacher can ask students to use the system table of knowledge related to the ellipse and apply it to the following exercises:

Requirements: Determine the lengths of the axes, coordinates of the focal points, coordinates of the vertices of the ellipse with equation

$$\frac{x^2}{9}+\frac{y^2}{4}=1 \quad (1)$$

(4) Activities of application and expansion

The principles of the Realistic Mathematics Education approach are applied in the development of the learning activity. According to the perspective influenced by Freudenthal (1974), mathematics should be regarded as a human activity that should be valued. The contents require students to apply knowledge and skills to solve new situations/problems, which are not similar to those instructed or give reasonable responses to a situation/new problems in study or life (Kamid et al., 2020).

Purpose: To help students have the ability to apply the lesson knowledge to certain situations and specific conditions in learning practice, school life, community and family, thereby supplementing and expanding the knowledge learned from practice.

How to do: Develop or suggest students build exercises in the direction of capacity development; Guide students to connect and arrange learned knowledge and skills into successfully solving exercises/situations/problems in real life or learning; Guide students to have the practical experience to complete the task; Activities are research and creative, so it is necessary to help students get close to their families and localities, taking advantage of the guidance of families and localities to complete their learning tasks. For example: In teaching elliptical content, teachers can ask students to perform tasks:

Requirements: Using the system table of knowledge related to the ellipse, complete the personal worksheet.

ELLIPSE Equation (Section 1) WORKSHEET NUMBER.

Orbital problem.

Given an object moving in an elliptical orbit around the moon, the closest point of the object to the center of the moon is called the perihelion, and the farthest point is called the aphelion; these points are called the vertex of the Moon Orbit, the center of the moon is a focal point of the orbit. The Apollo 11 spacecraft was launched into lunar orbit 68 miles (1 mile = 1.6 km) from the moon's surface and 195 miles at perihelion. Assume that the moon has a radius of 1075 miles. (Set the coordinate axis so that the origin is at the center of the orbit and the focal point is on the axis) (Stewart et al., 2015, p. 798). (Figure 4)

a) Draw a graph and determine the components of the Apollo 11 orbit.
b) Find the equation of the orbit of the ship.

With the lecture series, students will learn about elliptic equations and engage in activities that will help them improve their mathematical communication skills. Instruction necessitates that students learn the mathematical language (terms, symbols, graphs), comprehend communication-based math issues, and successfully communicate mathematical material and concepts through math language. Mathematical linguistically speaking, students are better prepared to understand and comprehend the canonical equation, the major axis and minor axes, the focal point and focal length, the graph, and equations comprising base rectangle edges. Concerning understanding and identifying mathematical problems through communication, learners must summarize mathematical problems presented in oral or written form and use the correct mathematical language to solve problems of writing canonical equations, finding the components of the ellipse. Regarding the effective use of mathematical language in presenting and expressing mathematical contents and ideas, learners are required to present and express their thoughts and ideas in addition to those of others. They make connections or convert everyday language into mathematical language and vice versa to accurately represent mathematical objects, relationships, or solutions to mathematical problems in various contexts. As a result, they exude self-assurance throughout their presentation, improving their ability to express mathematics effectively.

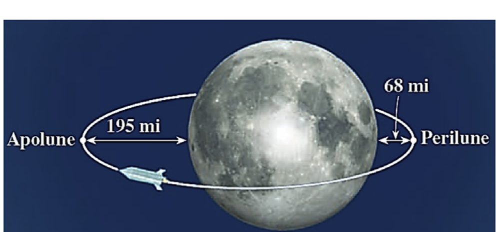

Figure 4. Illustration of the Orbital problem.

The learning activities in the experimental class are organized by the ACODESA method, which integrates both the individual and the sociocultural settings to assist the students in developing their knowledge and understanding (Hitt and González-Martín, 2015). Regarding peer-assisted learning strategies, the instructional method is regarded in the Fuchs et al. (2020) study. This procedure can be broken down into the five steps listed below.

(1) Individual work: Students are assigned an unfamiliar task by the teacher; the student must think and perform that task.
(2) Teamwork: Students work in groups with the same task in phase 1. In this phase, students choose answers through discussion among group members. Students' work can be displayed as posters for teachers to see and comment on.
(3) Debate: The teacher chooses a poster of the group (usually the group with the wrong answer) for the whole class to discuss. In this phase, teachers create conditions for students to develop reasoning abilities.
(4) Self-reflection: The teacher assigns students learning tasks for them to do at home. This stage aids students in consolidating their knowledge.
(5) Institutionalization process: The teacher comments and explains the problem based on the students' group work results.

Furthermore, mind maps associated with elliptical topics systematize knowledge by utilizing images and symbols to represent concepts. As a result of summarizing and visualizing mathematical symbols and language in this manner, students are given more opportunities to develop mathematical communication skills (Armstrong, 2020; Yorulmaz and Uysal, 2021).

3.3. Assessing the level of mathematical communication skills regarding ellipse topics

The skills to communicate mathematically are measured using the scale shown in Table 2.

The tests have eight ideas to assess mathematical communication competence; each is scored on a 10-point scale. The maximum total score is 80 points, coded according to five levels of behavior and ranked respectively in Table 3.

3.4. Data collection and analysis

To determine whether or not the teaching process is feasible, the research team observes students and collects data to determine whether or not it is effective. The data was gathered through observations in the classroom, the pre-test and post-test, and a teacher interview. Before learning about ellipses, the students in two classes took a pre-test that consisted of six questions about circles that they were instructed to study before moving on to the next topic. In this study, the post-test instrument was a description containing six items related to elliptic equations. Particularly challenging were items 4 and 5, which dealt with real-world carpentry and architectural design problems, respectively. When dealing with these math problems, it was expected that students would need adequate mathematical communication skills. Each of the two classes was given test instruments at the start and end of the lesson, respectively. Also, the researchers interviewed teachers to learn about the methods they used to assist students in developing their mathematical communication skills and their attitudes toward learning better to understand the current state of elliptical topics education.

Table 2. Scale to assess the level of mathematical communication skills regarding ellipse topics.

Component Skills	Skills and criteria for each skill	The expression levels of each criterion				
		Level 1	Level 2	Level 3	Level 4	Level 5
Ability to understand and identify mathematical problems through communication	Understand, identify, and solve mathematical problems in written form.	Unable to understand or identify math problems.	Understand and identify the part of math problems.	Understand and identify mathematical problems and summarize a part of basic, central mathematical information.	Understand, identify, and summarize key mathematical information in a text.	Understand, identify and correctly use mathematical language in summarizing mathematical problems.
	Understand and identify mathematical problems in the form of mathematical language (digits, symbols, graphs).	Cannot understand, cannot identify mathematical information.	Identify some core mathematical information.	Know how to analyze, select and extract some necessary mathematical information.	Know how to connect, link and synthesize mathematical information.	Know how to connect, link, synthesize and select mathematical information using mathematical language.
Ability to use mathematical language in presenting, expressing mathematical contents and ideas	The ability to effectively use mathematical and mathematical language to present, explain, reason, and prove mathematics accurately, logically, and clarify mathematical ideas in a specific context.	Lack of basis when speaking and writing math. Students are not yet able to express their understanding of mathematical language.	The first step is to present and explain mathematical contents in familiar situations with single and discrete sentences. When speaking or writing a math problem, it is not logical, strict, and concise.	Understand and use mathematical language in familiar symbols and symbols to summarize and present mathematical ideas and solutions in a relatively accurate and appropriate manner.	Ability to write about mathematical ideas and solutions concisely and clearly; Analyze, evaluate, and respond to math problems logically and accurately.	Coherent presentation, close argument, proper use of mathematical language convincingly and effectively. Make connections or convert natural language to mathematical language and vice versa to accurately represent mathematical objects, relationships or solutions to mathematical problems in specific contexts.

Table 3. Ranking of test results before and after the experiment.

Levels	Score range	Classification
Level 1	[0; 16]	Poor
Level 2	(16; 32]	Weak
Level 3	(32; 48]	Medium
Level 4	(48; 64]	Good
Level 5	(64; 80]	Very good

First, validation and testing were required before running the experiment to determine if it would be successful. To increase the accuracy of the research, the team went about designing high-quality and advanced instruments. Two well-known specialists in mathematics education believed that the assessments were credible; Salsabila's research (2019) used a similar approach, which was also successful. After the evaluation and adjustments, the instruments and research were implemented. The instruments' integrity was proven, as each individual asserted that the instrument was suitable. In the end, they agreed to conduct the tests since they viewed the research subject as important. In addition, the researcher team assessed the degree of coverage of academic content and communication skills about mathematics. According to Thao et al. (2020), one way to determine a test's reliability is how well it covers academic content and skills; this method was used to design the test in this research. A more specific answer is that the tests in this research were designed to assess a student's mathematical communication ability in a variety of forms, including the use of mathematical language, mathematical representation, and presentation of ideas and concepts to resolve a certain math problem regarding the topics covered by elliptic curves.

Also, using the same grading process they used for the Vietnamese national exam, the research team could replicate their findings. It is then necessary to analyze and process the experimental data to evaluate the findings, which is accomplished using the statistical method for evaluating students' mathematical communication skills. Because of the use of the SPSS 20 software, the quantitative aspects of the data were examined. When analyzing the data, the researchers used two sets of independent t-tests with two tails, the Pearson correlation coefficient and the Cohen influence level criteria table, to determine the correlation between data from the same group. A qualitative evaluation of the effectiveness of the teaching process was also decided to assess students' perceptions of knowledge, skills, and attitudes. Students' worksheets were evaluated quantitatively and qualitatively throughout this period, and the worksheets that they had in their possession were evaluated in both ways at the same time. As an additional measure of students' mathematical communication skills, these worksheets were graded on a four-level scale throughout the instructional process, as shown in Tables 2 and 3. Also, students' learning attitudes were assessed through classroom observations, participation in learning activities, and feedback from teachers, among other methods.

4. Results

4.1. Quantitative results of pre-test and post-test

The data processing results through SPSS show that the pre-experimental test scores of the two classes have a normal distribution. The independent t-test method is used to test the hypothesis that the pre-test scores of the experimental and control classes are not significantly different. Tables 4 and 5 indicate the descriptive statistics and t-test of the average score in Math of the experimental and control classes through SPSS.

To determine whether or not the mean difference between the experimental and control groups was statistically significant, an independent t-test was carried out on both groups. Accordingly, with

Table 4. Descriptive statistics of pre-test results.

	N	Minimum	Maximum	Mean	Std. Deviation
Pretest- EG	44	12	68	36.50	14.539
Pretest- CG	43	14	70	38.23	15.462

Table 5. Independent t-test of pre-test results.

Independent degree: df	t Stat	Sig. (2-tailed)	Mean Difference
85	0.539	0.592	1.733

significance level α=0.05 and degrees of freedom df=85, the critical value (Sig.) equal to 0.529 is greater than 0.05. Accordingly, the difference in mean scores between the experimental class and the control class is not significant. In other words, the test findings indicate that the levels of the experimental class and the control class are equivalent.

The Independent t-test method is used to test the hypothesis that the mean score of the post-experiment test of the experimental class is higher than that of the control class. Tables 6 and 7 reveal the descriptive statistics and independent t-test of post-test scores of the experimental and control classes through SPSS 20 software.

The calculated influence level value is 0.93 in the range (0.08–1.00). For this reason, it can be drawn that the magnitude of the experimental effects is quite large. In order to see if the experimental and control groups had different means, an independent t-test was utilized. Accordingly, with the significance level $\alpha = 0.05$ and degrees of freedom $df = 85$, the critical value (Sig.) equal to 0.000 is less than 0.05, so the difference in mean score value between the experimental the control classes is statistically significant. Therefore, the null hypothesis is ruled out, allowing us to accept the alternative hypothesis. Therefore, the test results show that the post-test results of the experimental class and the control class have significant differences. Specifically, the test results of the experimental class were higher than that of the control class.

Further, according to the two classes' means, it appears that the students in the experimental class perform better academically than the students in the control class. The standard mean deviation has been calculated as 0.93. According to Cohen's criteria, this value is in the range of 0.8–1.0, so it can be concluded that the magnitude of the effect is large.

Furthermore, the correlation between the results of the two tests administered to the experimental class was addressed. It was also possible to examine the relationship between the post-test and the pre-test.

The results of the correlation test from Table 8 show that, with the Sig significance level. (2-tailed) less than 0.05 experimental class scores in the two tests before and after the experiment are correlated. Accordingly, the Pearson correlation coefficient is equal to 0.856, showing that the correlation is very large. Furthermore, based on Figure 5, the majority of the above scores are dispersed around the line, indicating that students in the experimental class who achieved high results in the pre-test would similarly achieve high results in the post-test.

Statistical results from Figure 6 show that most students in the experimental class scored 40 or more points (34/44 students), and no student scored less than 20 points. Meanwhile, most students in the control class scored from 20-39 points, accounting for more than 50%, only 16/43 students got over 40 points, and no student got over 70 points. Thus, between the two experimental and control classes, there is a significant difference in score differentiation. Specifically, the experimental class has a uniform score distribution, concentrated at relatively high scores. Meanwhile, the scores in the control class are distributed at different levels of high and low, and the differences between the levels are significant, especially the number of students who achieve high scores (over 50 points) is relatively small.

Table 9 shows that the rate of Poor grades in the experimental class (accounting for 0%) is much lower than that in the control class

Table 6. Descriptive statistics of post-test results.

	N	Minimum	Maximum	Mean	Std. Deviation
Pretest- EG	44	22	78	51.18	14.699
Pretest- CG	43	12	68	37.77	14.443

Table 7. Independent t-test of post-test results.

Independent degree: df	t Stat	Sig. (2-tailed)	Mean Difference
85	4.293	0.000	13.414

Table 8. Correlation.

		PretestEG	PosttestEG
PretestEG	Pearson Correlation	1	0.856**
	Sig. (2-tailed)		0.000
	N	44	44
PosttestEG	Pearson Correlation	0.856**	1
	Sig. (2-tailed)	0.000	
	N	44	44

**. A significant correlation is found with a p-value of 0.01 (2-tailed).

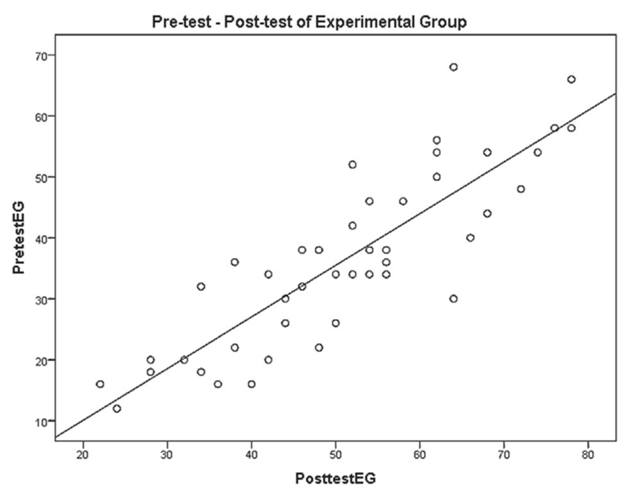

Figure 5. Scatter chart of experimental group data.

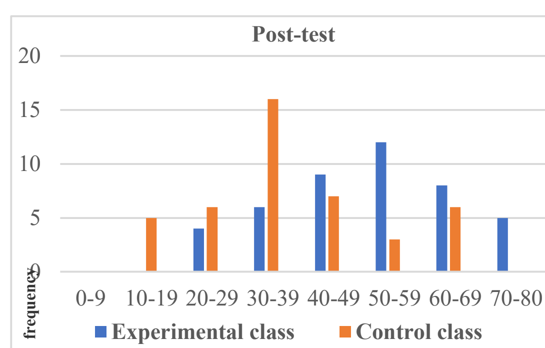

Figure 6. Diagram of score differentiation after the experiment teaching.

(accounting for 7%). Besides, in the control class, the percentage of assignments graded as Weak and Medium was quite high (about 72.1%), nearly two times higher than that of the experimental class (accounting for 40.8%) and accounted for the majority of the class. A total number of assignments. For the Good and Very good grades, 59.2% of the assignments in the experimental class were superior to those of the control class (accounting for 20.9%). From here, we have grounds to believe that the learning results of equations of the ellipse in the experimental class are significantly better than those in the control class.

Table 9. Results of grading test scores after the experiment.

Classification	Poor	Weak	Medium	Good	Very good
Levels of mathematical communication	Level 1	Level 2	Level 3	Level 4	Level 5
	[0; 16]	(16; 32]	(32; 48]	(48; 64]	(64; 80]
Experimental group	0	5	13	18	8
	0%	11.3%	29.5%	40.9%	18.3%
Control group	3	14	17	7	2
	7.0%	32.6%	39.5%	16.3%	4.6%

4.2. Assessment of mathematical communication skills

In order to ensure objectivity, it is necessary to compare the ability to develop mathematical communication abilities of students in the experimental group and the control group. Before the experiment, we looked at how students used mathematical language in speaking, writing, thinking, discussing or presenting their mathematical ideas in interaction with others. In general, both experimental and control students showed the following symptoms.

The students were successful in comprehending the concept of ellipse equations and thus arrived at the answer. Students were able to utilize their skills by using their knowledge to deal with mathematical problems such as writing an equation for an ellipse's axis, finding lengths of its axes, focus coordinates, and vertex coordinates, and applying this knowledge to real-world problems regarding ellipse concepts to expand their understanding.

These surveys assessed the ability of the experimental class students to express mathematics objectively and scientifically, with initial observations on students' abilities to communicate mathematics.

Nonetheless, through general observations, some limitations are found, such as students have a habit of long expressions and often do not have enough ideas, so teachers often have to explain and describe to help students.

Many students can understand math problems but have difficulty expressing them in mathematical language. Students have almost no habits and have many difficulties in expressing their thoughts and sharing their understanding. Teachers need to help students overcome psychological obstacles such as fear of making mistakes, being laughed at or expressing their views in front of teachers, and actively participating in the communication.

In organizing mathematical communication activities for students, teachers are often used to explaining to students when they find it difficult to express but rarely give suggestions for students to express or present better. Sometimes, the questions are unnecessary, and many are true-false, yes-no, few questions require students to explain.

4.3. Results of a survey of student attitudes

Observing experimental teaching and interviewing students after the experiment demonstrated that most students in the experimental group agreed that teaching and developing mathematical communication competence helped them be confident and self-confident. They have more mastery when participating in learning activities, overcoming timidity and psychological obstacles, and promoting positive, proactive and creative learning, thereby understanding mathematics. The knowledge of students is reinforced more profoundly. At the same time, students also found that thanks to the strengthening of activities to practice skills in using mathematical language (including terms, symbols, graphs, mathematical symbols) in close relationship with the language of mathematics they study, have explanatory questions, open problems, solving practical problems and debate in group activities. The students were helped to talk about math, think coherently, express clearly and have the ability to remember more deeply and better apply mathematical knowledge in specific contexts.

Through observing during class time and discussing with the teachers of the control group, we found that the change of students in the control class was not much; many students lacked flexibility and initiative in learning. Communicating or memorizing lessons is not much, not effective. They also show timidity, lack of confidence, shyness to communicate, do not know how to ask questions and are often passive in answering questions from teachers. It is observed that effective mathematical communication helps students be bold, express their mathematical ideas coherently and clearly, create friendliness and openness, and make the learning process active. For 10th graders, this is particularly important as they prepare to approach the foundational knowledge of high school mathematics at a higher level. These individuals displayed an above-average aptitude for learning during this time and an excessive amount of enthusiasm and excitement when they discovered the solution. While they were confident in completing the exercises correctly, they also expressed high confidence in completing more advanced manipulation activities. Student participation in exploratory learning activities is depicted in Figure 7 below.

5. Discussion

It has been shown that the results of the teaching process, which included learning activities based on the ACODESA method and mind mapping, have improved students' mathematical communication abilities. This study produced results that corroborate the findings of a great deal of the previous work in mathematics education (Hitt and González-Martín, 2015; Setiyani et al., 2020; Yaniawati et al., 2019; Rohim and Umam, 2019). After a period of experimental teaching, the shortcomings mentioned above are gradually overcome. Students understand the core content that needs to be recorded. Know how to read and understand to summarize the main idea and ask questions and answers to find the direction of proof. In particular, students can read pictures, read graphs, recognize mathematical relationships expressed in mathematical language, and use mathematical language effectively such as digits, symbols, graphs, relationships, logical connections in combination with mathematical language or physical movements when presenting, explaining and evaluating mathematical ideas in interaction with teachers and students in the classroom (Nuraina and Mursalin, 2018; Lestari et al., 2019; Fuchs et al., 2020). This finding supports the views of the students who participated in the research of Mukuka et al. (2021), who believe that mathematics is a subject that is best learned through interaction between teachers and students as well as between students.

Due to enhanced communication and encouragement to express ideas, students in the experimental group were more confident in learning, boldly giving ideas, willing to share their understanding and know-how to accept new ideas. Different opinions, solutions of classmates, know how to contribute ideas and skills to convince listeners. Nevertheless, language is sensitive in teaching, especially the

Figure 7. Illustration of student activity.

mathematical language used by students in the learning process, so the teachers need to make timely adjustments with practical and effective effects (Sudia and Muhammad, 2020).

Discovery teaching techniques similar to those used in the research of Juliarta and Landong (2020) were also employed in the students' learning; these techniques stimulated students' math skills, particularly their communication skills. Students were allowed to continue practicing once they had finished the classroom activities, after which they were given an additional support test to help them get ready for the upcoming test. Several students' lessons focused on summarizing the experiment results were employed, and the results of those experiments served as a foundation for ensuing ideas and discussions (Kosko and Wilkins, 2012). Similarly, student success in the planned lesson will be determined by their ability to communicate effectively and collaborate with others (Smieskova, 2017). After the research, it was speculated that the group could reach a consensus on the suggested teaching method, which could be considered a legitimate teaching approach for the group. It was discovered in this study that inquiry teaching strategies enabled students to develop their skills in mathematical communication (Supiyanto et al., 2020).

In contrast, students exhibited a high level of classifications within the communication system, including comprehension questions and requests, comprehension of the questions' system, and the use of the questions to generate answers. While there were a variety of additional capabilities the student developed, including public speaking and the breadth of math topics and concepts they explored through group interactions and communication with their peers and class representatives, they also developed the capacity to facilitate effective class meetings, something essential for leaders of successful classes. This confirms that a deep understanding of mathematics was applied in dealing with the problems described in Utomo and Syarifah's research (2021). While visuals and representations like logos and metaphors are also valuable in developing good problem-solvers, problem-solving processes such as researching topics and formulating conclusions are also conducted using visual and symbolic representation (Utomo and Syarifah, 2021). This research further demonstrates that they have greater efficacy because they apply their findings to real-world issues while utilizing natural language and mathematical jargon in their research (Hutapea et al., 2019; Palinussa, 2013). The researchers also discovered something else interesting: students with stronger communication skills in mathematics were better at verifying a mathematical theorem, which matched the conclusions of the study of Salsabila (2019). This finding corroborates the ideas of Zhou et al. (2021), who suggested that incorporating good communication skills into the context of a mathematical argument increases students' overall ability to perform well. Most students are conscious of learning, listening attentively to lectures, and being serious in performing the tasks required by the teacher, such as answering questions, making presentations, performing group activities (Paruntu et al., 2018; Salsabil et al., 2017), participating in lesson development. In the exercise part, students are quite economical in presenting and explaining but only focus on the results of the problem, thereby having difficulty in solving problems related to reality or explaining a problem; this result is also similar to the study of Sudia and Muhammad (2020). Additionally, most students still have difficulty presenting their ideas and solutions orally, have confusion, lack confidence, and choose inappropriate expressions (Salsabila, 2019).

In particular, most students are still afraid of mathematical communication, often using mathematical language, so the expression is quite lengthy, inaccurate and incoherent (Rustam and Ramlan, 2017).

6. Conclusions and limitations

The research findings help to answer the questions raised earlier in the research. In the first place, the students in the experimental class were able to show they understood the subject matter of ellipse concepts clearly. In particular, they could also translate their general know-how and specific knowledge and skills into math problem-solving skills required for the study area. Next, concerning the ability to communicate mathematical information using math text effectively, students listened, read, understood, and recorded important mathematical information found in math text or delivered verbally or in writing by others. Another factor is that the learners engaged in discussions, explanations, and demonstrations of mathematical ideas and concepts while maintaining an appropriate level of completeness and accuracy. In their presentations, explanations, and evaluations, the students in the experimental group used mathematical language and non-verbal techniques like hand gestures or movement. When presented, expressed, asked questions, discussed, and debated mathematical content and ideas increased their competence. Another striking feature of their thinking was that they were cognitively engaged, which allowed them to predict and uncover new information and gain more quickly than the students in the control class. Even if students were not formally taught any math communication skills, they were trained in independent thinking, problem-solving, and teamwork using authentic teaching situations.

In addition, students built their confidence and authority in their learning by conducting personal investigations and finding ways to solve problems. We have demonstrated that a reciprocal, two-way relationship exists between student attitudes and academic performance through this research. The use of real-world mathematical solving activities created a significant learning motive for the students, who participated in activities under the teaching model because they wanted to improve their communication skills with mathematics.

According to research, mathematical language in children's thinking is critical for their mathematical development. To this day, mathematical thinking is characterized by sign language, which is a formalized language. Students go through an expansion process in every maths lesson, during which they must first express ideas in an expressive language and then demonstrate practical fluency in mathematics. Initial research indicates that students continue to have significant difficulties communicating and presenting mathematical content on their own. Despite recent improvements, students' ability to speak and write mathematics remains limited. Students are perplexed when presented with visual representation problems because they have only been exposed to arithmetic representations during their mathematics training, and as a result, they have difficulty finding mathematical solutions in practice. According to the implication, the ability to express mathematical ideas effectively in writing impacts the students' ability to construct valid proofs and may even pose difficulties in some cases.

Assume that teachers do not have effective strategies to motivate students to participate in learning activities in general and math communication activities in particular. Consequently, it is observed that students lack initiative rather than confidence, are in a stressful or uncomfortable learning environment and lack the motivation to participate in educational activities. When students are presented with real-world problems, they are unable to use mathematics to handle them. Providing students with opportunities to practice mathematical communication will help to stimulate the activities listed above. Other benefits include a clearer focus on innovation in teaching based on the learners' mathematical abilities and increasing students' accountability and proactiveness in building mathematics. While renovating the high school mathematics curriculum, building measures to foster mathematical communication skills for students in math teaching becomes increasingly necessary for the formation and development of competencies for students.

Furthermore, substantial implications of this type of experiment have also emerged. Furthermore, the teacher developed classroom management and management skills, including lesson design and implementation, and enhanced students' mathematics communication skills with his/her overall efforts. They also examined learners' capacity to speak mathematically and designed a lesson plan to improve this capability. The students also practiced more specialized communication skills, such as problem-solving complex situations, making comparisons, and providing logical explanations of mathematical concepts. Additionally, an instructional strategy that intentionally focuses on students' ability to convey mathematical concepts through oral presentation in an experimental class devoted to ellipse concepts can also teach other mathematics topics. Once the teacher has reached this point, they can begin instructing students to compare the characteristics of three different conic sections.

There are some good suggestions for teachers looking for useful materials to teach students about math communication skills. To assist their students in improving their mathematical language skills, teachers can implement several different strategies. Teachers create speaking or writing tasks when students learn mathematical concepts, theorems, rules, and methods. Students are then guided through various mathematical communication activities that they can use to cope with math problems. Following that, some interactive learning activities (group, pair, or in-class discussion) will take place require students to perform a variety of learning tasks in terms of solutions, with practical components, and with a variety of appropriate representations in order to increase their awareness, practice, memorization, and mathematical communication. Additionally, mathematics teachers should incorporate contextual learning into their classroom instruction to significantly improve their students' mathematics communication skills.

Moreover, students benefit from visual representations when expressing mathematical concepts. By providing an ideal platform for individuals to demonstrate their abilities, an event that places a high premium on performance will foster the development of new mathematical knowledge. Students can learn math more effectively when they see visual representations, courtesy of the enhanced learning environment. There is compelling evidence that utilizing alternative representations enables students to comprehend and approach the nature of the problem, ultimately leading to the development of a solution. The classroom's overall layout can help promote mathematical communication if it emphasizes teamwork in lessons with conflicting knowledge, either teamwork in class or outdated knowledge. They express themselves verbally and in writing when working in groups. When they describe that concept, they will use the visual symbols that they commonly use, such as diagrams, drawings, letters, and mathematical symbols.

Additionally, learners need to be aware of the importance of strong mathematical communication skills to ensure that students succeed in mathematics. Because having good mathematical communication capabilities is advantageous for all aspects of learning, it helps students become more logical thinkers and better communicators. This approach contributes to and encourages the development and strengthening of the capacity mentioned above, which focuses on learners and helps build their confidence. As students advance in their math learning, the greatest value is finding the answers to problems and asking for their teachers' assistance. This happens, for example, when students are working on problems and thus must come up with several ways to reach a solution.

Students who are unable to understand mathematics apply more information to help them better understand the problem, while at the same time, they put in additional effort to assimilate new mathematics concepts by using them in their context. A similar approach to the one described here should be applied to other mathematics subjects in the future. Our enthusiasm for enhancing students' understanding of how to communicate mathematically comes from other researchers who support learning kits to aid in that task. Some research should be carried out to determine whether or not it is related to students' abilities in communicating mathematically and in problem-solving. Furthermore, as information technology advances, it is becoming increasingly vital for teachers to incorporate it into their lesson plans. Due to the widespread usage of computers in schools, the results of this study suggest ways to create digital modules for the classroom.

Since this was a quasi-experimental study, only 44 students in grade 10 were involved in it. It was discovered that the capacity and willingness of subjects to learn differed significantly between genders and that they could be classified according to their learning styles. A further limitation of the research was that it only focused on exploiting and using the mathematics language, which included mathematical symbols and terms, to improve students' mathematical communication skills while learning ellipse topics in class 10.

Declarations

Author contribution statement

Duong Huu Tong: Conceived and designed the experiments; Wrote the paper.

Bui Phuong Uyen: Contributed reagents, materials, analysis tools or data.

Ngo Van Anh Quoc: Performed the experiments; Analyzed and interpreted the data.

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Data availability statement

Data included in article/supplementary material/referenced in article.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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