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The association of basic numerical abilities and math achievement: The mediating role of visuospatial and arithmetical abilities

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Abstract

Basic numerical abilities such as number line estimation have been observed repeatedly to be associated with mathematical achievement. Recently, it was argued that the association between basic numerical abilities and mathematical achievement is fully mediated by visuospatial abilities. However, arithmetical abilities have not yet been considered as influencing this association, even though solution strategies in number line estimation as well as mathematical achievement often involve arithmetical procedures. Therefore, we investigated the mediating role of arithmetical *and* visuospatial abilities on the association between number line estimation and mathematical achievement in a sample of $n = 599$ German elementary school students. The results indicated that arithmetical abilities as well as visuospatial abilities mediated the association between number line estimation and mathematical achievement. However, neither visuospatial nor arithmetical abilities fully mediated the association between number line estimation and mathematical achievement when considered in isolation. This substantiates the relevance of the intertwined development of visuospatial and arithmetical abilities as well as basic numerical abilities such as number line estimation (i.e., the combination of domain-specific numerical and domain-general abilities) driving mathematical achievement.

Introduction

Mathematical competences are important for mastering everyday life as well as vocational success (e.g., Bruder et al., 2015; Grønmo et al., 2015; Organisation for Economic Co-operation and Development [OECD], 2004, 2014). Generally, high mathematical competences are accredited to someone who shows high mathematical achievement (e.g., Weinert, 2001). Thus, it is not surprising that many studies have investigated factors associated with mathematical achievement. Thereby, influences of domain-specific basic numerical skills (e.g., counting) as well as non-numerical domain-general factors (e.g., visuospatial abilities) to mathematical achievement have been observed (e.g., LeFevre et al., 2010).

Some studies primarily reported significant associations of domain-specific basic numerical abilities with mathematical achievement (e.g., Passolunghi & Lanfranchi, 2012), whereas others (also) identified non-numerical domain-general factors to significantly predict mathematical

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achievement (e.g., Deary et al., 2007; Friso-van den Bos et al., 2015). In a recent study by Sella and colleagues (2016), the authors investigated the relationship between all three variables: non-numerical domain-general factors, domain-specific basic numerical abilities, and mathematical achievement. For *adults*, they found evidence for a full mediation of the association between basic numerical abilities (operationalised by number line estimation) and mathematical achievement by visuospatial abilities, a non-numerical domain-general factor.

However, so far, most studies on the association between basic numerical abilities and mathematical achievement investigated (elementary school) *children* with the idea that basic numerical abilities are a building block for (later) mathematical development and achievement. From this, the question remains whether the association between basic numerical abilities (operationalised by number line estimation as in Sella and colleagues, 2016) and mathematical achievement is also fully mediated by visuospatial skills in children who still develop mathematical abilities. This question seems particularly relevant because mathematical achievement was found to depend on basic numerical abilities (e.g., Booth & Siegler, 2006; M. Schneider et al., 2018) as well as arithmetical abilities (e.g., Booth & Siegler, 2008; Link et al., 2014) for elementary school students (see also curricula for elementary school, e.g., National Council of Teachers of Mathematics [NCTM], 2000 or Kultusministerkonferenz [KMK], 2004).

In the present study, we examined the association of basic numerical abilities with mathematical achievement by considering the influence of both visuospatial and arithmetical abilities in a large sample of more than 600 elementary school students. In the following, we will first briefly summarise recent evidence on the relevance of domain-specific basic numerical and domain-general abilities for mathematical achievement before describing the details of the present study.

Association of basic numerical abilities and mathematical achievement

Domain-specific basic numerical abilities (e.g., counting, magnitude understanding, and number line estimation) were repeatedly observed to significantly predict mathematical achievement (e.g., Fuchs et al., 2010; Hirsch et al., 2018; Passolunghi & Lanfranchi, 2012). For instance, Hirsch and colleagues (2018) found significant positive effects of kindergartners' basic numerical abilities such as counting, seriation, and symbolic number knowledge on their mathematical achievement in Grade 6. One of the most prominent basic numerical abilities—observed to be associated with mathematical achievement—is the ability to estimate the spatial location of numbers on a number line (i.e., number line estimation, M. Schneider et al., 2009; Torbeyns et al., 2015; for a meta-analysis see M. Schneider et al., 2018). Performance in number line

estimation was argued to reflect children's understanding of number magnitude (e.g., M. Schneider et al., 2018). The mental representation of number magnitude is assumed to be "the most basic level of numerical cognition upon which all other (more complex) numerical and mathematical thinking builds" (Thompson et al., 2013, p. 325).

The results of numerous studies have revealed evidence for a positive association between elementary school children's ability to map numbers on a number line and mathematical achievement assessed by standardised achievement tests or grades (e.g., Booth & Siegler, 2006; Friso-van den Bos et al., 2015; M. Schneider et al., 2009; Torbeyns et al., 2015). Importantly, the association of number line estimation performance and mathematical achievement remained significant even after controlling for domain-general cognitive abilities (i.e., intelligence, working memory, e.g., M. Schneider et al., 2009).

In particular, for arithmetical abilities (e.g., adding or subtracting) numerous studies have reported a positive association with children's accuracy in number line estimation (i.e., the percentage of absolute estimation error, e.g., Booth & Siegler, 2006, 2008; Link et al., 2014). For instance, Link and colleagues (2014) observed that children's performance in number line estimation was associated with performance in both addition and subtraction. Looking more closely at the number line estimation task, there are studies arguing that additional processes such as proportion judgement drive performance in number line estimation (Barth & Paladino, 2011; Slusser et al., 2013; see also Newcombe et al., 2019).

However, it is important to note that applying proportion judgement strategies depends on procedures such as, for instance, benchmark-based strategies (e.g., Barth & Paladino, 2011; Ebersbach et al., 2013). In a recent study, Peeters and colleagues (2016) reported that children's systematic use of benchmark strategies tended to correlate with their number line estimation performance. In addition, they reported a positive association of the use of benchmark strategies and mathematical achievement. This is not surprising, as applying benchmark -strategies is hypothesised to require arithmetical abilities (e.g., addition, see e.g., Link et al., 2014) which are also necessary to solve other numerical problems (see also M. Schneider et al., 2018).

When looking at how mathematical achievement is measured in elementary schools—independent of whether operationalising mathematical achievement by a standardised achievement test or grades in mathematics (cf. Haffner et al., 2005; KMK, 2004; NCTM, 2000)—being able to successfully solve as many numerical problems as possible mathematical achievement tends to be higher. In line with this argument, arithmetical abilities were observed to significantly predict mathematical achievement in several studies (e.g., Bailey et al., 2014; Cerda et al., 2015; Duncan et al., 2007; Watts et al., 2015, see also Rittle-Johnson &

Siegler, 1998; M. Schneider et al., 2011). For instance, Duncan and colleagues (2007) found that early arithmetic skills were the strongest longitudinal predictor of later more advanced mathematical competences even after controlling for important other variables (e.g., family background, socio-emotional skills, or general cognitive abilities). This is backed by the results of W. Schneider and colleagues (2016), who indicated that the association of arithmetic abilities and mathematical achievement seems to be stronger for children than for adults. As such, the role of arithmetical abilities should not be neglected when evaluating the association of number line estimation and mathematical achievement, with their accumulating evidence suggesting a close association of arithmetical abilities and mathematical achievement.

Association between domain-general cognitive abilities and mathematical achievement

Domain-specific basic numerical and arithmetical abilities are not the only determinants of mathematical achievement. In several studies, a complex interplay of non-numerical domain-general cognitive and domain-specific basic numerical abilities was observed to predict mathematical achievement. For instance, Passolunghi and Lanfranchi (2012) found a positive effect of kindergartners' domain-general cognitive abilities, such as working memory and processing speed on their mathematics achievement at the end of Grade 1. Overall, domain-general cognitive abilities seem to contribute to mathematical achievement (and therewith mathematical competences) over and above influences of domain-specific basic numerical abilities (e.g., Lee & Bull, 2016; Mazzocco & Kover, 2007; Skagerlund & Träff, 2016).

Interestingly, visuospatial abilities belong to the most relevant non-numeric domain-general factors for mathematical achievement (e.g., LeFevre et al., 2010, see also Mix & Cheng, 2012; Verdine et al., 2017c; for the association of visuospatial abilities to STEM-success in general see also Wai et al., 2009). In a sample of preschool children, Verdine and colleagues (2017a, 2017b, 2017c, 2017d) assessed the relationship between domain-general cognitive abilities, including visuospatial skills, domain-specific basic numerical abilities (e.g., counting), and mathematical achievement. Their results indicated visuospatial abilities (assessed at age 3) to be a significant predictor of mathematical achievement at age 5 (Verdine et al., 2017c). In contrast, domain-specific basic numerical abilities at age 3 (e.g., counting) were not predictive for mathematical achievement at age 5 (Verdine et al., 2017c).

Therewith, these results corroborate the notion that visuospatial abilities and mathematical achievement "share cognitive processes beginning early in development" (Cheng & Mix, 2014, p. 3). In their recent review, Newcombe et al. (2015) even argue that the development of quantitative skills depends on a generalised magnitude system, which is supposed to underlie development in both spatial and numerical cognition (see also Walsh, 2003 for a similar suggestion). The development of this generalised magnitude system is suggested to build on correlated dimensions (e.g., area, length) whose increasing precision with age stimulates the development of the discrete number system (Newcombe et al., 2015). Although the relative importance of the respective dimensions is still under scientific debate, Newcombe and colleagues (2015) suggested that the dimension of *space* seems to be the best candidate for building the basis upon other dimensions such as number project. Thus, visuospatial abilities are a good candidate for a domain-general ability—for more about the association of domain-general abilities and mathematical achievement see, for example, Passolunghi and Lanfranchi (2012), that systematically influences the development of both basic numerical abilities and mathematical achievement.

Link between basic numerical abilities and visuospatial abilities

Based on the assumption that basic numerical and visuospatial abilities are associated with mathematical achievement, the question is whether there is also a link between basic numerical skills such as number line estimation and visuospatial abilities. Thompson and colleagues (2013) found that mental rotation ability was significantly correlated with number line estimation performance in university students. The authors supposed that higher mental rotation abilities led to a more sophisticated mental number line representation, providing the "mental organization and framework within which information about the cognitive concept of numbers is stored" (Thompson et al., 2013, p. 325). In another study, LeFevre and colleagues (2013) reported significant associations between performance in number line estimation and visuospatial abilities. Visuospatial abilities predicted the increase in children's number line estimation performance over time. In addition, the authors observed associations between number line estimation and mathematical achievement. Based on this, they argued that both visuospatial and basic numerical abilities influence mathematical achievement (LeFevre et al., 2013).

Recently, Sella and colleagues (2016) went so far as to argue that the association between basic numerical abilities (which they operationalised as number line estimation performance) and mathematical achievement is fully mediated by visuospatial skills. They assessed the performance in number line estimation and visuospatial abilities of both mathematicians and non-mathematicians. Their results indicated that the relation between number line estimation and mathematics achievement (i.e., being a professional mathematician or not) was fully mediated by visuospatial abilities. The authors concluded that this

result indicates that the "relation between basic, even specific, numerical abilities and advanced mathematical achievement can be artefactual" (Sella et al., 2016, p. 1458).

The present study

Previous studies have consistently reported an association of number line estimation performance and mathematical achievement (i.e., performance on a [subscale of a] math achievement test or school grades; for a meta-analysis see M. Schneider et al., 2018). Moreover, there are studies investigating the influence of visuospatial abilities on number line estimation performance (e.g., Gunderson et al., 2012), but also mathematical achievement (e.g., LeFevre et al., 2010; Sella et al., 2016; for the association of visuospatial abilities with STEM-success in general see also Wai et al., 2009). In addition, an association between number line estimation performance and arithmetical abilities has been reported frequently (e.g., Booth & Siegler, 2008; Link et al., 2014). Taken together, this seems to indicate a close relationship between number line estimation performance as well as visuospatial abilities and mathematical achievement.

For instance, in a longitudinal study, Gunderson and colleagues (2012) reported that the association of visuospatial abilities (as assessed by mental rotation) and arithmetical abilities (measured by approximate symbolic calculation) was mediated by number line estimation performance. Furthermore, Cheng and Mix (2014) found that visuospatial training (i.e., mental rotation training) led to an improvement in arithmetical abilities (i.e., basic arithmetic operations). In sum, this provides evidence suggesting that the association of number line estimation performance and mathematical achievement may be confounded by influences of both visuospatial and arithmetical abilities.

Therefore, we evaluated the association of number line estimation and mathematical achievement (i.e., grades in mathematics) in a sample of $n=599$ German elementary school children while considering both visuospatial and arithmetical abilities. Following the analytic approach of Sella and colleagues (2016), we investigated the association of number line estimation and mathematical achievement in a series of mediation analyses to evaluate the potentially mediating role of arithmetical and visuospatial abilities.

Based on the above described previous findings, our hypotheses were as follows. First, we expected to replicate the positive association between number line estimation performance and mathematical achievement as observed repeatedly in previous studies (e.g., M. Schneider et al., 2009). Second, we hypothesised according to the results of Sella and colleagues (2016) that including visuospatial abilities as a mediator should change the result patterns in

a way that the direct path between number line estimation performance and mathematical achievement should no longer be significant. This would reflect the influence of visuospatial abilities as confounding the association of number line estimation performance and mathematical achievement. Third, we expected that considering arithmetical abilities in the same way should also diminish the direct path between number line estimation performance and mathematical achievement—again arguing for a confounding effect on the latter association. Finally, combining the results regarding the influence of arithmetical and visuospatial abilities on the association between number line estimation performance and mathematical achievement, we expected that considering visuospatial and arithmetic abilities should reveal both to eliminate the direct path between number line estimation performance and mathematical achievement. In turn, this reflects that the association of number line estimation and mathematical achievement is confounded by the interplay of arithmetical and visuospatial abilities.

Method

Sample, participants, and procedure

Data were collected from 24 different elementary schools in the south of Germany. In sum, $N=1,048$ elementary school children (47% male, age [year;months]: $M=9;6$ years, $SD=.0;8$, $min=7;6$, $max=12;2$) from 78 classes (43, 3rd Grade, 35, 4th Grade classes) participated in the study. All student measures were assessed in paper-pencil-format and administered by trained researches and research assistants. To assess the demographic background and students' school grades in mathematics and German, parents' questionnaires were handed to students, which had to be sent back by the parents ($n=678$, return rate of 64.7%). Because mathematical achievement was operationalised by school grades in mathematics, we only analysed data of the respective subsample of $n=678$ children (44.7% male, age: $M=9;5$, $SD=0;8$, $n_{3rd\ Grade}=420$, $n_{4th\ Grade}=247$). In addition, we excluded those children who had missing values on one of the variables of interest (i.e., mathematical achievement, number line estimation performance, arithmetical abilities, paper folding, or mental rotation). Therewith the final analyses were conducted on data of $n=599$ students (45.0% male, age: $M=9;5$, $SD=0;9$, $n_{3rd\ Grade}=366$, $n_{4th\ Grade}=230$; for further information see Table 1). Local school authorities as well as the local ethics committee approved the study. Parents' informed written consent was obtained prior to data collection.

Measures

Basic numerical abilities. In line with Sella and colleagues (2016), we used a bounded number line estimation task to

Table 1. Descriptive statistics.

	Missing rate	M	SD	Min	Max	α	Number of items
Sample							
Age (years)	.01	9;5	0;9	7;6	12;2		
3rd Grade (n = 266)							
Age (years)	.01	9;1	0;6	7;6	12;0		
4th Grade (n = 260)							
Age (years)	.00	10;1	0;6	8;1	12;2		
Measures							
Number line estimation	.00	9.01	5.39	2.30	56.39	.90	15
Mathematical achievement	.00	2.08	0.71	1.00	4.75		1
Arithmetical abilities	.00	63.07	13.73	27.00	110.00	.79	120
Mental rotation	.00	12.32	3.13	1.00	19.00	.63	24
Paper folding	.00	10.41	3.20	1.00	16.00	.74	18

n = 599, α = Cronbach's alpha.

assess student's number line estimation performance (e.g., Ludewig et al., 2019; Siegler & Opfer, 2003). On a 21-cm long 0- to 1,000 number line with specified starting- and end-point, students had to estimate the position of a given target number without counting (see, for example, Booth & Siegler, 2006; Link et al., 2014). One practice item (i.e., 500) was used to exemplify the task followed by 20 items targeting 10 numbers smaller than 500 and 10 numbers bigger than 500.[1] Target numbers were presented in the same order for each participant: two items were presented per page with a horizontal jitter in starting points of 3 cm to avoid the use of external reference points. The mean percentage of absolute estimation errors across all 20 items was used as dependent variable. Thereto, the absolute difference of the estimated position and the correct position of the target number for each item was divided by the length of the number line (and multiplied with 100). Accordingly, small values of the dependent variable reflect better performance (i.e., more accurate estimation) than larger values.

Arithmetical abilities. To assess arithmetical abilities, we used three subscales of a German standardised arithmetic test (Heidelberger Rechentest; Haffner et al., 2005): (a) addition (1- to 3-digit numbers; e.g., $1 + 6$ or $26 + 13$); (b) multiplication (1- and 2-digit numbers; e.g., 3×1 or 11×2); and (c) problem completion (children had to fill in the missing number to correctly solve the problem; e.g., $6 + \_ = 7$, $13 - 12 = 9 - \_$). Each subscale consisted of 40 items presented in ascending difficulty. Students were asked to solve as many items as possible within a time limit of 2 min on each scale. The sum score of correctly solved problems across all three subscales was considered as the dependent variable.

Visuospatial abilities. Two different tasks reflecting the sub-component of object transformation within visuospatial abilities (Hegarty & Waller, 2004; Kozhevnikov &

Hegarty, 2001) but covering two different aspects of visuospatial skills as categorised by Linn and Petersen (1985, i.e., mental rotation and spatial visualisation) were used to assess visuospatial abilities.

Mental rotation (Peters et al., 1995; Vandenberg & Kuse, 1978) was measured by 24 items. Each item illustrated two three-dimensional (3D) figures built of cubes printed on paper and students had to decide whether the two figures were the same or different. The sum score of correctly solved items was used for further analyses.

For spatial visualisation, we used a paper folding test (subscale of a German cognitive ability test, namely KFT 4-12 + R, Heller & Perleth, 2000). In total, 15 items were presented and students were asked to identify the one of five perforated sheets of paper that resulted when a blank sheet of paper was folded as indicted by arrows. The sum score of correctly solved items was used in further analyses.[2]

Mathematical achievement. School grades in mathematics were used as an indicator for students' mathematical achievement. Parents were asked about their children's grade in mathematics in the last certificate on a scale ranging from 1 = *very good* to 6 = *insufficient*. Thus, the smaller the values the better the achievement in mathematics was.

Statistical analyses

To evaluate our four hypotheses, a total of four different models were specified examining the association of number line estimation performance and mathematical achievement step by step. First, a simple linear regression of mathematical achievement on number line estimation performance was computed (Model 1, revealing the total effect *c*, cf. Baron & Kenny, 1986). In a second step, the confounding roles of visuospatial and arithmetical abilities to the association of number line estimation performance and mathematical achievement were examined separately, by conducting two full mediation models: first considering visuospatial abilities as a mediator

Table 2. Bivariate correlations.

		Number line Estimation	(1)	(2)	(3)
Mathematical achievement	r	.30***			
Mental rotation	r	−.34***	−.38***		
Paper folding	r	−.25***	−.24***	.33***	
Arithmetical abilities	r	−.39***	−.51***	.32***	.24**

Note. Two-tailed significance levels are reported. $n = 599$.
*$p < .05$, ***$p < .001$.

(Model 2) and arithmetical abilities (Model 3) in a second step. In such mediation models, it is evaluated whether the effect of an independent variable (X) on a dependent variable (Y) is mediated by one or more mediator(s) (Ms). Thereby, the path of (X) on (Y) is referred to as the direct effect (c') and is equivalent to the beta regression coefficient of a linear regression of (Y) on (X) when considering the mediator(s) as a covariate. Association(s) between the independent variable (X) and the mediator(s) is/are named as a (a_1, a_2, . . .) path and also correspondents to the beta regression coefficient of a linear regression of (Ms) on (X). The third path, from the mediator(s) to the dependent variable (Y) is called as b (b_1, b_2, . . .) path and is equivalent to the beta regression coefficient of a linear regression of (Y) on (Ms) while controlling for the independent variable (X). To investigate the indirect effect of (X) on (Y) via the mediator(s), the product of a and b needs to be calculated. Independent of the significance of the individual a and b paths, the difference form zero for $a \cdot b$ has to be checked, indicating that also the indirect effect is statistically significant (see, for example, Hayes, 2009; Mackinnon et al., 2004; Preacher et al., 2007).

In a final step, a mediation model was run considering both visuospatial abilities and arithmetical abilities as mediators for the association between number line estimation performance and mathematical achievement (Model 4).

All models were run in R (R Core Team, 2015) using the R package lavaan (Rosseel, 2012). In all analyses, we used the robust maximum likelihood estimator, which corrects the standard errors for potential non-normality of variables. To prevent differences in sample sizes between the different mediation models, we excluded all participants with at least one missing value on one of the five variables of interest listwise (i.e., mathematical achievement, number line estimation performance, mental rotation, paper folding, and arithmetical abilities, R Core Team, 2015; Rosseel, 2012), meaning that data of $n = 599$ participants was considered in the analyses. To test whether effects in mediation analyses were different form zero, bias-corrected bootstrapping was suggested (e.g., Hayes, 2017; for small to moderate sample sizes see also Shrout & Bolger, 2002). As such, in addition to standardised regression weights obtained using the robust maximum likelihood estimator, we also report bootstrapped 95% confidence intervals for the indirect effect[3] (1,000 samples). All reported

coefficients are standardised ones and the reported p values are two-tailed.

Results

Descriptive statistics as well as bivariate correlations between all variables are displayed in Tables 1 and 2. The results of our models are summarised in Figure 1 (for further details see Appendix A). Statistical significance of the estimated coefficients, especially those for the indirect effects ab, did not differ using p values or the bootstrapped 95% confidence intervals as an indicator (see Appendix A).

In the following, results of the respective models are reported step by step. For each model, we first describe the direct effect (path c, respectively, c' for the mediation models) of performance in number line estimation on mathematical achievement. For the mediation models, effects of performance in number line estimation on the mediator (path a) and the effect of the mediator on mathematical achievement controlling for number line performance (path b) are reported. For all models we report standardised coefficients. According to Baron and Kenny (1986) a mediation model reflects full mediation when inclusion of the mediator(s) reveals significant paths a and b (i.e., influences of number line estimation on the mediator and the mediator on mathematical achievement controlling for number line estimation) and path c' is no longer significant (compared to c). When path c' (i.e., the association of performance in number line estimation and mathematical achievement) is not equal to zero (i.e., still statistically significant different from zero) partial mediation is in place (Baron & Kenny, 1986).

Model 1 revealed a significant positive effect (c) of performance in number line estimation on mathematical achievement, indicating that better number line estimation performance (i.e., smaller estimation error) was associated with better school grades in mathematics ($\beta = 0.30, p < .001$, see Figure 1).

When including visuospatial abilities as a mediator in Model 2, the direct effect (c') of number line estimation performance on mathematical achievement was still statistically significant ($\beta = 0.18, p = .002$). The mediation analysis further revealed that the effects of number line estimation performance on visuospatial abilities ($a_{paper folding}$ and $a_{mental rotation}$) were also significant (paper folding: $\beta = -0.34, p < .001$, mental rotation: $\beta = -0.25, p < .001$), indicating that better
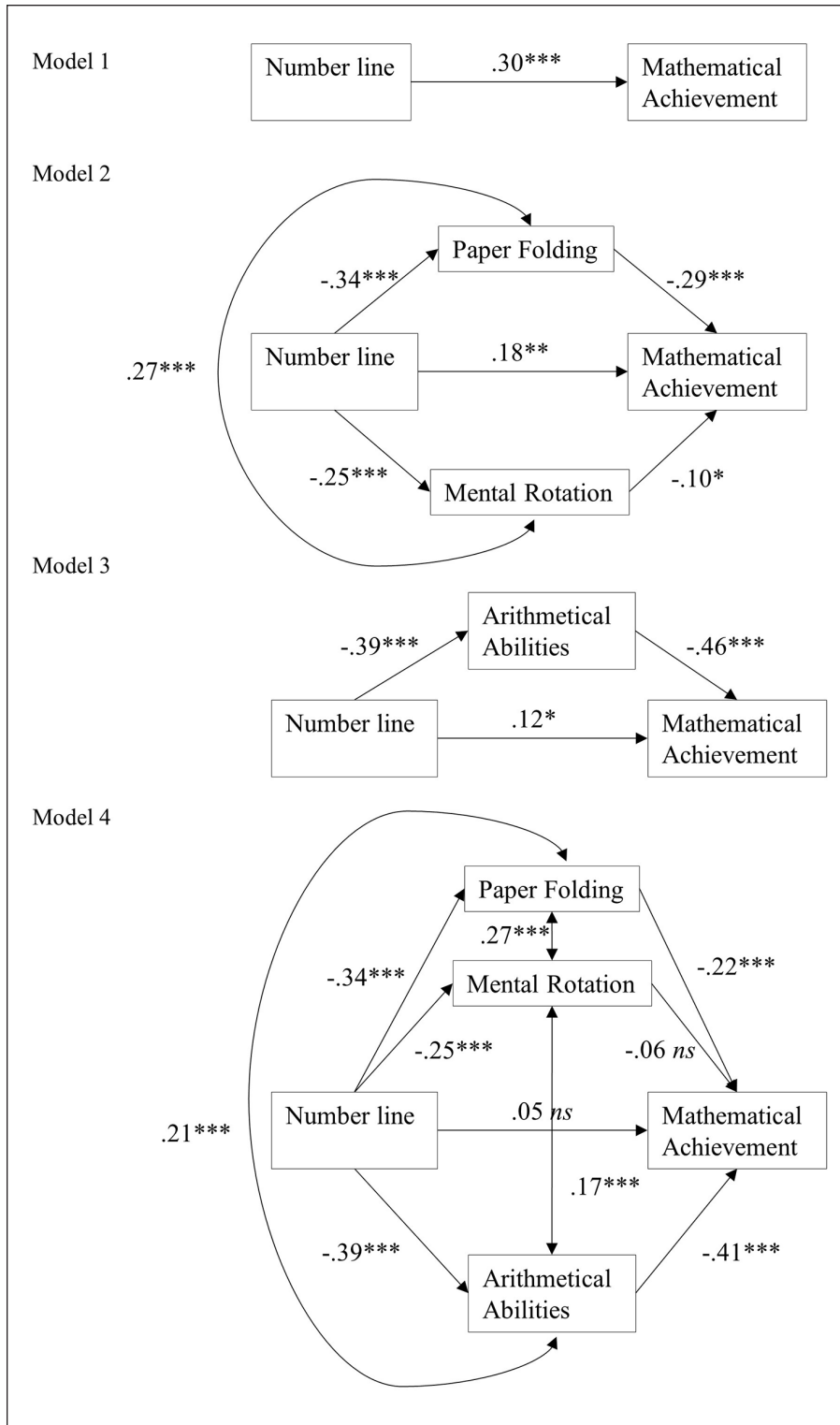
Figure 1. Mediation analyses between performance in bounded number line estimation and mathematical achievement.
Note. Models 1 to 4 are described in detail in the "Method" section. Standardised coefficients are reported. All participants with at least one missing value were excluded for analyses. The sample size for all mediation models was $n = 599$.
*$p < .05$, **$p < .01$, ***$p < .001$.

performance in number line estimation was associated with better visuospatial abilities. Controlling for performance in number line estimation, the effect of visuospatial abilities on mathematical achievement ($b_{paper\ folding}$ and $b_{mental\ rotation}$) was also significant (paper folding: $\beta = -0.29$, $p < .001$, mental rotation: $\beta = -0.10$, $p = .009$). Thus, the results indicated that

better visuospatial abilities were associated with better mathematical achievement (at constant number line estimation performance). Based on the criteria for mediation analyses suggested by Baron and Kenny (1986), visuospatial abilities partially mediated the association of number line estimation performance and mathematical achievement, indirect effect via paper folding ($a_{paper folding} \cdot b_{paper folding}$): $b=0.013$, 95% CI = [0.008, 0.019], $\beta=0.097$, $p<.001$, indirect effect via mental rotation ($a_{mental rotation} \cdot b_{mental rotation}$): $b=0.003$, 95% CI = [0.001, 0.06], $\beta=0.025$, $p=.019$.

By adding arithmetical abilities instead of visuospatial abilities as a mediator in Model 3, the direct effect (c′) of number line estimation performance on mathematical achievement was still significant ($\beta=0.12$, $p=.019$, Model 3, Figure 1). Furthermore, the results of Model 3 revealed a significant effect of number line estimation performance on arithmetical abilities, indicating that better number line estimation performance was associated with better arithmetical abilities ($\beta=-0.38$, $p<.001$, Model 3, Figure 1). Controlling for number line estimation performance, arithmetical abilities were also a significant predictor of mathematical achievement ($\beta=-0.47$, $p<.001$). As such, arithmetical abilities also partially mediated the association between number line estimation and mathematical achievement (cf. Baron & Kenny, 1986) (indirect effect: $b=0.023$, 95% CI = [0.017, 0.031], $\beta=0.179$, $p<.001$).

When considering both visuospatial abilities and arithmetical abilities as mediators (Model 4), we observed full mediation (cf. Baron & Kenny, 1986): The direct effect (c′) of number line estimation performance on mathematical achievement was no longer significant when controlling for influences of visuospatial abilities and arithmetical abilities (i.e., the mediators paper folding, mental rotation and arithmetical abilities, $\beta=0.06$, $p=.230$).

Furthermore, performance in number line estimation had significant negative effects on both measures of visuospatial abilities ($\beta_{paper folding}=-0.34$, $p<.001$, $\beta_{mental rotation}=-0.25$, $p=.001$), and arithmetical abilities ($\beta=-0.38$, $p<.001$, for more details see Figure 1). This reflected that better number line estimation performance was associated with better visuospatial abilities and arithmetical abilities. Controlling for performance in number line estimation, there were significant negative effects of visuospatial abilities, but only for paper folding ($\beta_{paper folding}=-0.22$, $p<.001$, $\beta_{mental rotation}=-0.06$, $p=.104$), and arithmetical abilities ($\beta=-0.41$, $p<.001$) on mathematical achievement. This indicated that better visuospatial abilities and arithmetical abilities were associated with better mathematical achievement even when controlling for number line estimation performance (indirect effect via paper folding: $b=0.009$, 95% CI = [0.006, 0.014], $\beta=0.072$, $p<.001$, indirect effect via mental rotation: $b=0.002$, 95% CI = [−0.001, 0.004], $\beta=0.014$, $p=.138$, indirect effect via arithmetical abilities: $b=0.021$, 95% CI = [0.015, 0.027], $\beta=0.157$, $p<.001$).

Discussion

In this study, we aimed at evaluating two potential mediators (i.e., visuospatial and arithmetical abilities) of the association of basic numerical abilities (reflected by number line estimation performance) and mathematical achievement (i.e., grades in mathematics) for elementary school students. Recently, Sella and colleagues (2016) reported the association of number line estimation performance (used as a proxy for basic numerical abilities) and mathematical achievement (i.e., being a mathematician or not) to be fully mediated by visuospatial abilities. However, number line estimation was found to be associated with arithmetical abilities repeatedly (e.g., Link et al., 2014), which in turn seem associated with mathematical achievement (e.g., W. Schneider et al., 2016). Thus, we were interested in the mediating role of both visuospatial abilities and arithmetical abilities to the association of number line estimation performance and mathematical achievement.

In line with the results of previous studies, we observed a positive association of number line estimation performance and mathematical achievement (e.g., M. Schneider et al., 2009; Torbeyns et al., 2015). Importantly, however, we did not observe that this association was fully, but only partially, mediated by visuospatial abilities as found by Sella and colleagues (2016, cf. Model 2). Importantly, we also found the association between number line estimation performance and mathematical achievement to be partially mediated by arithmetical abilities (Model 3). Only when considering both factors as mediators, full mediation of the association between number line estimation performance and mathematical achievement was observed (Model 4).

Taken together, our mediation analyses indicated that for elementary school children arithmetical as well as visuospatial abilities seem to matter for the association of basic numerical abilities and mathematical achievement. In particular, the interplay of the two mediators seems to drive the association of basic numerical abilities and mathematical achievement. Therewith, these results are in line with our expectations and suggest that the association of number line estimation performance (as a proxy for basic numerical abilities) and mathematical achievement cannot be reduced to be fully mediated by visuospatial abilities for elementary school children as argued by Sella and colleagues (2016) for adults.

Furthermore, arithmetical abilities should not be neglected as underlying the association of number line estimation performance and mathematics achievement in elementary school children. In fact, it seemed the interplay of both visuospatial and arithmetical abilities drives the association of basic numerical abilities and mathematical achievement. As such, the interplay of both domain-specific numerical (i.e., arithmetical) and domain-general abilities (i.e., visuospatial abilities) was observed to underlie the association of number line estimation performance and mathematical achievement.

This finding perfectly fits the more general debate on whether mathematical achievement is predicted by domain-specific numerical or/and domain-general abilities (cf. Alcock et al., 2016; Fuchs et al., 2010; Passolunghi & Lanfranchi, 2012) and provides further evidence for the proposition of an intertwined development of domain-general visuospatial and domain-specific numerical abilities. In their recent review, Newcombe and colleagues (2015) argued that the development of quantitative skills relies on a generalised magnitude system, which is supposed to underlie the development of both spatial and numerical cognition (see also Walsh, 2003 for a similar suggestion). Our results corroborate this claim, while indicating that there is more than just domain-general visuospatial abilities to drive the association of domain-specific number line estimation performance and mathematics achievement. In particular, our results argue for an important role of arithmetical abilities in this association with a full mediation only observed when considering visuospatial as well as arithmetical abilities as mediators.

Moreover, the present results also suggest that apart from obvious visuospatial aspects of the number line estimation task, it seems that arithmetical abilities may also be crucial for solving it successfully (cf. Link et al., 2014; Petitto, 1990). Thereby, our results are also informative with respect to the ongoing debate on the role of specific solution strategies in bounded number line estimation and their influence on the association of number line estimation and mathematical achievement. In particular, the mediating role of arithmetical abilities is hard to reconcile with the notion that performance in number line estimation reflects a direct measure of numerical estimation or the mental number line representation and—in turn—the idea that a more accurate mental number line underlies better mathematical achievement (e.g., Siegler & Opfer, 2003). Instead, these results add to accumulating evidence (e.g., Link et al., 2014), suggesting that number line estimation is solved applying specific solution strategies such as proportion-judgements (e.g., Barth & Paladino, 2011), which require both visuospatial processes and arithmetical abilities to be carried out successfully. Thereby, these data are theoretically important, as they provide converging evidence for the above described debate from a new perspective; this means not evaluating number line estimation performance in itself to derive information on how the task is solved (see Dackermann et al., 2015 for an overview), but by evaluating factors considering the context of task performance. One might even speculate that the association of domain-specific numerical abilities as reflected by number line estimation and mathematical achievement is an artefact of the influences of visuospatial and arithmetical abilities which were repeatedly observed to be associated with both domain-specific numerical abilities (e.g., Link et al., 2014; Thompson et al., 2013; Torbeyns et al., 2015) and mathematical achievement (e.g., LeFevre et al., 2010; M. Schneider et al., 2011).

When interpreting the results of the current study, some limitations may need to be considered. First, the measurements implemented in our study were in some parts different from those implemented in other studies: For example, visuospatial abilities are a multidimensional construct (see, for example, Linn & Petersen, 1985) and we did not assess all dimensions, but only *mental rotation* and *spatial visualisation* (i.e., paper folding). In this context, it also needs to be noted that the reliability of our mental rotation measure was not optimal. This should be kept in mind when interpreting the present results regarding visuospatial abilities.

In addition, when interpreting the results of the current study it should also be considered that there were specific assumptions regarding the operationalisation of the dependent as well as the independent variable. First, the dependent variable mathematical achievement was only assessed by asking parents for their children's math grades, which may not be the most reliable measure of mathematical achievement (see, for instance, Bos and colleagues [2004], who reported a correlation of math grade and performance in a mathematical achievement test of about −.55,[4] or also Lintorf, [2012]). Some previous studies referred to above (e.g., Booth & Siegler, 2006, 2008; Passolunghi & Lanfranchi, 2012; M. Schneider et al., 2009) used other measures to assess mathematical achievement (e.g., [sub-]scales from standardised tests like the Woodcock-Johnson III Test of Achievement, Woodcock et al., 2001); see Torbeyns and colleagues (2015) as well as M. Schneider and colleagues (2009) for the use of math grades. Second, the independent variable number line estimation performance was assessed by bounded number line estimation tasks only for which there is an ongoing scientific debate on what exactly it assesses (e.g., Barth & Paladino, 2011). Nevertheless, bounded number line estimation performance was broadly used as a measure of basic or domain-specific numerical abilities in numerous studies (e.g., Ludewig et al., 2019; Sasanguie et al., 2012; Siegler & Opfer, 2003; Thompson et al., 2013). However, further studies using different measures would be necessary to substantiate the results of the present study.

Furthermore, the fact that our results did diverge from those of Sella and colleagues (2016), who reported visuospatial abilities to fully mediate the association of basic numerical abilities and mathematical achievement, needs to be interpreted against the background that we did not compare adult mathematicians with non-mathematicians. Instead, we evaluated the association of visuospatial abilities (as well as arithmetical abilities) and mathematical achievement in elementary school children to learn about their interplay in children's cognitive development. As such, future studies evaluating the present tripartite association in adults would be desirable.

Moreover, our results indicated that both visuospatial and arithmetical abilities were important mediators of the association between basic numerical abilities and mathematical achievement. However, it is important to note that

this was not a longitudinal study. As such, it would be interesting to evaluate the differential prediction of these two factors for future mathematical achievement. Of course, there are studies that already indicated the influence of visuospatial (e.g., Sella et al., 2016; Thompson et al., 2013) and arithmetical abilities (e.g., W. Schneider et al., 2016) for later mathematical achievement (i.e., arithmetical, respectively, mathematical competences). However, these studies did not consider both variables in combination to evaluate their differential contribution.

Conclusion

Extending the results of Sella and colleagues (2016), the current study provided evidence that the association of basic numerical abilities (as reflected by number line estimation) with mathematical achievement in elementary school children was not fully mediated by visuospatial abilities alone. Instead, our results suggest mediating roles of the combination of visuospatial and arithmetical abilities. In particular, this indicates that in addition to the obvious visuospatial demands of number line estimation, arithmetical abilities necessary to apply specific solution strategies (e.g., proportion judgements) mediate the association of basic numerical abilities (i.e., number line estimation) and mathematical achievement. These findings provide additional evidence for an intertwined development of visuospatial and numerical skills driving mathematical achievement.

Declaration of conflicting interests

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Data accessibility statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Supplementary material

The supplementary material is available at qjep.sagepub.com.

Notes

1. We also assessed unbounded number line estimation but we did not consider it in the analyses due to largely missing associations with the other variables of interest.
2. Although data of the present study indicated a small- to medium-sized bivariate correlation of mental rotation and paper folding, $r(667) = .340$, $p < .005$, we kept the two measures separate in line with previous research. Nevertheless, we also conducted our analyses with pooled sum scores for both visuospatial tasks. This did not change our results substantially.
3. For the indirect effects we report the estimation coefficient b and its 95% CI, as well as the standardised regression coefficient β and the respective p value.
4. Please note that the analyses are based on the German coding system for grades which range from "1" (*reflecting best performance*) to "6" (*representing fail*) and thus the negative correlation indicates that better math grades were associated with better performance in a math test.

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