Research article

Study on *Spirulina platensis* growth employing non-linear analysis of biomass kinetic models

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ABSTRACT

Spirulina platensis has been considered a promising source of food supplement to combat malnutrition worldwide. Numerous investigations have stated its immune activity, ability to absorb CO_2 during the growth period, and antioxidant potential. Well-known theoretical biomass kinetic model sheds are capable of qualitative analysis of the fast microalgae growth. In this regard, we considered eight popular biomass models: Monod, Haldane, Andrews & Noack, Teissier, Hinshelwood, Yano & Koga, Webb, and, Aiba model comprising analytical investigation within the numerical simulation. Besides, in this study, we establish a new mathematical biomass growth model by merging the well-known Hinshelwood and Yano & Koga models. We explored the most suitable *Spirulina* growth model to minimize the overstated and understated growth trends in the assorted eight biomass kinetic models. Our findings show microalgae biomass growth and substrate diminishes along with time, and these results were compared with available experimental data. Results present a high value of R^2(0.9862), a low value of RSS (0.0813), AIC (-9.7277), and BIC (-8.2148) implied significantly fitted with the investigated data for the growth of *Spirulina platensis* compared with popular eight studied models.

1. Introduction

Algae are photosynthetic organisms that can convert carbon dioxide (CO_2) to foods and are used for fuel, potential drugs, high-value bio-actives, as well as numerous remarkable commercial goods (Aikawa et al., 2018; Bilal et al., 2017; Bleakley and Hayes 2017; Liu and Chen 2014; Wells et al., 2017; Yan et al., 2016). Microalgae culture has multiple promising advantages; for example, it uses abandoned land or damped canal for culturing, so it does not compete with fertile land, which means food security is not vulnerable. Algae cultivation contributes to CO_2 fixation and provides higher oil production compared to crop food per acre, and it helps to delete adverse materials from wastewater. Furthermore, some studies revealed that it boosts immunity and is active against heart disease, obesity, and other diseases (De Morais and Costa 2007; Metting and Pyne 1986; Metting 1996; Metzger and Largeau 2005; Ramanan et al., 2010; Singh et al., 2005; Walker et al., 2005). Recently, microalgae are getting biotechnological attention due to the

nutraceutical compounds available in them (Miranda et al., 1998; Odjadjare et al., 2017). For example, algae and seaweed are excellent sources of protein comparable with that of soybean, egg, meat, and milk (Gouveia et al., 2008). Furthermore, *Spirulina* has been widely used to produce biofuel, biogas, capsule, nutrition drinks and bars, noodles, fairness cream, powder, beauty soap, skincare products, lotion, shampoo, moisturizing gel, mask cream, eye zone repair gel, medicine, and other therapeutical products (Borges et al., 2013; Yan et al., 2016; Wells et al., 2017).

There is no need for fertile land or freshwater to cultivate marine algae/seaweed. But, the cultivation of microalgae and isolation process is not viable economically (Miranda et al., 1998). The filamentous cyanobacterium *Spirulina platensis* grows in enormous volume in brackish water, freshwater, as well as the marine environment (Converti et al., 2006). The strains of *Spirulina platensis*, initially isolated from Chad-an African lake, have seen extensive commercial production (Ciferri 1983). It has been grown all over the world owing to its highest

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concentration of protein (approximately 65–70%), fatty acid (linoleic acid, gamma-linolenic acid, oleic acid, etc.), some essential amino acids, vitamins, minerals, coloring agents, and enzymes (Vonshak and Richmond 1988).

Understanding the physiology, ecology, and biotechnology of biomass growth is crucially essential in which biomass kinetics theoretical framework is used as a popular tool to represent the structure of growth formation. In 1913, Michaelis and Menten proposed a mathematical model describing the kinetics of microbial growth and the lessening in substrate concentration in a specific period. Following, Monod, 1949; Tang et al., (1997); Roos et al., (2004) established a new non-linear model for substrate concentration and specific growth rate that is considered as a primitive model on biomass growth kinetics. Huppert et al. (2005) showed that Monod functional form enhances the uniform phase evolution with chaotic amplitudes in a model. Moreover, Feng et al. (2021) used Monod kinetics to describe the symbiotic bacterial biofilm growth. Meanwhile, Teissier improved the Monod model by showing cell concentration through the consumption of substrate concentration. Yurt et al. (2002) successfully showed that combining both Monod and Teissier kinetics models for pyruvate and oxygen, respectively, exhibits a suitable correlation with investigated data for Leptothrix discophora SP-6. Beyenal et al. (2003) and McHenry and Werker (2002) verified that Tessier growth kinetics was in well-agreement with experimental data of Pseudomonas aeruginosa based on the dual-substrate model and biomass amounts, respectively. Notably, the Monod model has high accuracy for simple substrates and pure cultures (Contois 1959).

To explore the specific growth rate of the biomass as a function of substrate, Haldane (1930) introduced a model known as the Haldane kinetics model in the literature. Morozov (2016) stated that Haldane (1930) is the pioneer of the mathematical modeling of biological evolution. Many researchers widely accept this model for its inhibitive substance (Polizzi et al., 2017), and the specific growth rate is a non-linear function. According to Sathya et al. (2015) Haldane kinetic model was analyzed using the homotopy perturbation method for phenol biodegradation. Furthermore, Webb (1963) introduced the modified Haldane model, but it did not improve the situation significantly.

Besides the models mentioned earlier, Aiba et al. (1968) investigated the kinetic growth model by substrate inhibition effects with pragmatic correlation and simulated data from experiments. Yano et al. (1966) stated a model that is based upon a theoretical analysis of the kinematic behavior of the growth inhibition at a high concentration of the substance. Schroder et al. (1997) used steady states continuous culture of Pseudomonas cepacia G4 and observed that Yano & Koga model fitted for various inhibition models better than any other known models. The model of Andrews (1968) described a model for continuous and batch cultures of biomass with substrate inhibition parameters (Beltrán-Prieto and Son Nguyen 2018). The main feature of this model is to establish a relation among substrate concentration to an inhibition parameter, and specific growth rate. Hinshelwood (1947) developed a model to illustrate the influence of alcohol in biomass cultivation. Ethanol inhibition affected cell growth, product yield, and specific growth rate, according to the study of Amenaghawon et al. (2012).

The objective of the research is to compare several popular kinetic models with existing experimental data in the literature and to propose a model blended from Yano & Koga and Hinshelwood models. The eight models of Monod, Teissier, Haldane, Andrews & Noack, Yano & Koga, Aiba, Webb, and Hinshelwood were used to explain the maximum specific growth rate, yield coefficient, substrate inhibition, and half-saturation parameters effect on biomass growth. Unlike previous works, this study mainly focuses on eight models with extensive analytical and numerical analysis with experimental values for four parametric effects. We studied the different growth parameters concerning biomass-growth concentration and substrate-diminish concentration for the *Spirulina platensis*. The proposed model illustrates the effect of substrate inhibition term on the *Spirulina* growth. We have simulated the validity of the model with experimental data using the statistical quantity, such as root mean square deviation (RMSD), correlation coefficient (R^2), Akaike's Information Criteria (AIC), and Bayesian Information Criteria (BIC).

2. Mathematical model and methods

2.1. Biomass growth model

The microbial biomass growth velocity is proportional to the present population,

$$x^{'}(t) = \mu(t)x(t) \tag{1}$$

where $\mu(t)$ denotes the specific growth rate, biomass concentration indicated by $x(t)$ and the time is expressed by t, and the substrate concentration is significant since it allows the microorganism growth. The substrate concentration rate is transformed into biomass with some constant,

$$s^{'}(t) = -\frac{1}{Y_{x/s}}x^{'}(t) \tag{2}$$

where $Y_{x/s}$ is yield coefficient.

2.2. Specific growth rate

The specific growth rate $\mu(t)$ was computed by the natural logarithm (ln) of biomass versus time, as in the following equation:

$$\mu = \frac{\ln x_2 - \ln x_1}{t_2 - t_1} \tag{3}$$

where, x_1 and x_2 stands for the concentrations of biomass at times t_1 and t_2, respectively.

2.3. Yield coefficient

The yield coefficient ($Y_{x/s}$) computed according to the following equation,

$$Y_{x/s} = \frac{x - x_0}{s_0 - s} \tag{4}$$

where x denotes the biomass concentration (g/l) at the final time, x_0 indicates the initial biomass concentration at the initial time, s (g/l) denotes the final concentration of substrate at the final time and, s_0(g/l) represents the initial concentration of substrate at the initial time.

2.4. Half saturation parameter

The parameter, k_s called the half-saturation parameter, is the substrate concentration at half of the maximum specific growth rate. In the Monod model, if the substrate concentration value is less than k_s, then the specific growth rate is linearly proportional to substrate concentration, and the growth rate is approximately a first-order kinetic equation. The value of the half-saturation parameter must be greater than zero, so the value of $\frac{s}{s+k_s} < 1$ and accordingly μ_{max} is higher than the specific growth rate.

2.5. Maximum specific growth rate

The parameter μ_{max} denotes the maximum specific growth rate. It is a function of environmental factors, such as temperature and light intensity (Goldman and Edward, 1974). In this study, we consider μ_{max}

from 0.30 to 0.50. In this case, it indirectly depends on environmental factors.

In Figure 1, the specific growth rate depending on the following values of the parameter $\mu_{\max} = 0.39\,(\mathrm{h}^{-1})$, $k_s = 12.5\,(\mathrm{mg/l})$, $k_i = 158$ (mg/l), and $p = 0.001$ (curve fitting unitless parameter). Several models have been recommended intending to describe the method of growth kinetics. Some popular models are discussed in the following section.

2.6. Model description

Using (1) and (2) along with the following models, we get (5, 6, 7, 8, 9, 10, 11, 12) (detail description of the microbial growth rate and eight studied biomass kinetic models are provided in the Supplementary Material).

A) Monod model (Monod, 1949)

$$t = \frac{1}{(x_0 + s_0 Y_{x/s})\mu_{\max}}\left\{(x_0 + s_0 Y_{x/s})\ln\left(\frac{x}{x_0}\right) + k_s Y_{x/s}\ln\left(\frac{x}{x_0}\frac{s_0 Y_{x/s}}{x_0 + s_0 Y_{x/s}}\right)\right\} \tag{5}$$

B) Haldane model (Haldane, 1930)

$$\begin{aligned} t = &\frac{1}{(x_0 + s_0 Y_{x/s})\mu_{\max}}\left\{(x_0 + s_0 Y_{x/s})\ln\left(\frac{x}{x_0}\right) + k_s Y_{x/s}\ln\left(\frac{x}{x_0}\frac{s_0 Y_{x/s}}{x_0 + s_0 Y_{x/s} - x}\right)\right\} \\ &+ \frac{1}{\mu_{\max}k_i Y_{x/s}}\left\{(x_0 - x) + (x_0 + s_0 Y_{x/s})\ln\left(\frac{x}{x_0}\right)\right\} \end{aligned} \tag{6}$$

where $c = s - px$

C) Andrews & Noack model (Andrews, 1968)

$$\begin{aligned} t = &\frac{1}{\mu_{\max}}\left\{\left(\frac{1}{c} + \frac{1}{k_i} + \frac{k_s}{c^2}\right)\ln\left(\frac{x}{x_0}\right) - \left(\frac{1}{c} + \frac{k_s}{c^2}\right)\ln\left(\frac{px + c}{px_0 + c}\right)\right. \\ &\left. + \frac{k_s p(x_0 - x)}{c(px + c)(px_0 + c)}\right\} \end{aligned} \tag{7}$$

D) Hinshelwood model (Hinshelwood, 1947)

$$t = \frac{1}{\mu_{\max}(1 - k_i q)c}\left\{(c + k_s)\ln\left(\frac{x}{x_0}\right) - k_s\ln\left(\frac{\frac{-x}{Y_{x/s}} + c}{\frac{-x_0}{Y_{x/s}} + c}\right)\right\} \tag{8}$$

E) Teissier model (Yurt et al., 2002)

$$\begin{aligned} \Rightarrow t = &\frac{1}{\mu_{\max}}\left\{\frac{1}{1 - e^{\frac{-c}{k_s}}}\ln x - \frac{k_s}{c}\ln\left(\frac{1 - e^{\frac{-(px+c)}{k_s}}}{e^{\frac{-(px+c)}{k_s}}}\right)\right\} \\ &- \frac{1}{\mu_{\max}}\left\{\frac{\ln x_0}{1 - e^{\frac{-c}{k_s}}} - \frac{k_s}{(s_0 - px_0)}\ln\left(\frac{1 - e^{\frac{-s_0}{k_s}}}{e^{\frac{-s_0}{k_s}}}\right)\right\} \end{aligned} \tag{9}$$

F) Webb model (Webb, 1963)

$$t = \frac{1}{\mu_{\max}}\left\{\frac{c + k_s + \frac{c^2}{k_i}}{c\left(1 + \frac{c}{k_i}\right)}\ln\left(\frac{x}{x_0}\right) - \frac{k_s}{c}\ln\left(\frac{(px + c)}{(px_0 + c)}\right) + \frac{k_s}{c + k_i}\ln\left(\frac{k_i + px + c}{k_i + px_0 + c}\right)\right\} \tag{10}$$

G) Yano & Koga model (Yano et al., 1966)

$$\begin{aligned} \Rightarrow t = &\frac{1}{\mu_{\max}}\left\{\left(1 + \frac{k_s}{c} + \frac{c}{k_i} + \frac{c^2}{kk_i}\right)\ln\left(\frac{x}{x_0}\right) - \frac{k_s}{c}\ln\left(\frac{px + c}{px_0 + c}\right)\right. \\ &\left. + \frac{p(x - x_0)}{k_i}\left(1 + \frac{2c}{k}\right) + \frac{p^2(x^2 - x_0^2)}{2kk_i}\right\} \end{aligned} \tag{11}$$

H) Aiba model (Aiba et al., 1968)

$$\begin{aligned} \Rightarrow t = &\frac{k_i}{\mu_{\max}}\left\{\frac{c + k_s}{c(k_i - c)}\ln\left(\frac{x}{x_0}\right) - \frac{1}{c}\frac{k_s}{k_i}\ln\left(\frac{px + c}{px_0 + c}\right)\right. \\ &\left. - \frac{k_s + k_i}{(k_i - c)^2}\ln\left(\frac{k_i - px - c}{k_i - px_0 - c}\right)\right\} \end{aligned} \tag{12}$$
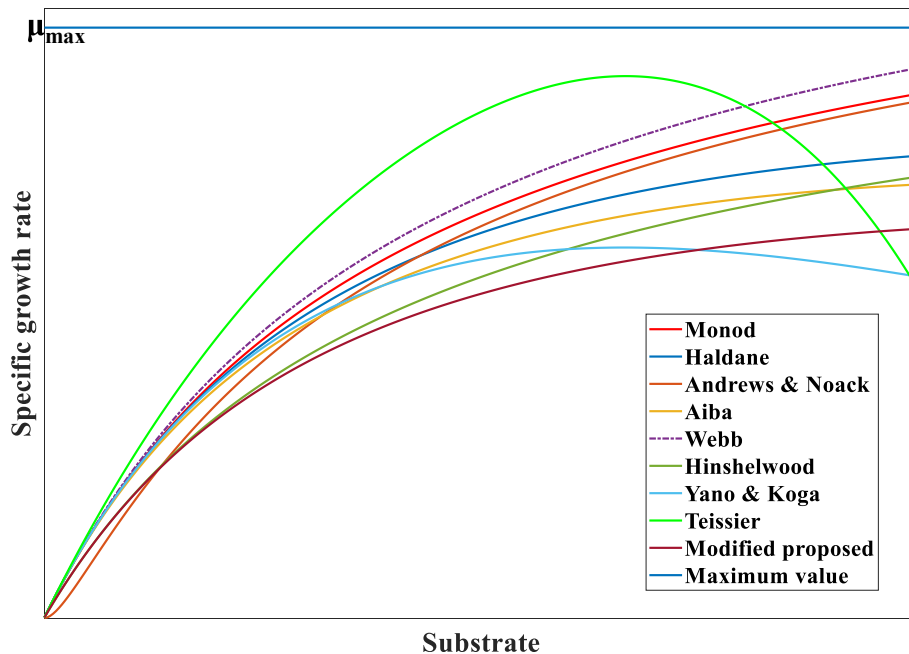

Figure 1. Biomass specific growth rate versus substrate according to the Monod, Haldane, Andrews & Noack, Aiba, Webb, Hinshelwood, Yano & Koga, Teissier model, modified proposed model, and maximum specific growth.

We also analyze the substrate concentration for all parameters. Substrate analysis is an essential substance for the optimized growth kinetics of biomass. Joshi et al. (2014) shows that *Spirulina platensis* when farmed on varying substrate concentrations, yield better progress. Organic substrates are capable of accelerating the growth of microalgae. In this study, we solve all models for the substrate concentration using Eqs. (1) and (2). The results are:

A) Monod model

$$t = \frac{1}{c\mu_{\max}}\left\{ -k_s \ln\left(\frac{s}{s_0}\right) + (c + k_s)\ln\left(\frac{s-c}{s_0-c}\right)\right\} \tag{13}$$

B) Haldane model

$$t = \frac{1}{k_i\mu_{\max}}\left\{(s-s_0) - \frac{k_sk_i}{c}\ln\left(\frac{s}{s_0}\right) + \frac{ck_i + k_ik_s + c^2}{c}\ln\left(\frac{s-c}{s_0-c}\right)\right\} \tag{14}$$

C) Andrews & Noack model

$$t = \frac{1}{k_i\mu_{\max}}\left\{\frac{k_sk_i}{c}\left(\frac{1}{s} - \frac{1}{s_0}\right) - \frac{k_i(c+k_s)}{c^2}\ln\left(\frac{s}{s_0}\right) + \frac{ck_i + k_ik_s + c^2}{c^2}\ln\left(\frac{s-c}{s_0-c}\right)\right\} \tag{15}$$

D) Teissier model

$$t = \frac{k_s}{c\mu_{\max}}\left\{\ln\left(\frac{s-c}{s_0-c}\right) - \ln\left(\frac{s}{s_0}\right)\right\} \tag{16}$$

E) Hinshelwood model

$$t = \frac{k_s}{c\mu_{\max}(1 - pk_i)}\left\{ -k_s\ln\left(\frac{s}{s_0}\right) + (c + k_s)\ln\left(\frac{s-c}{s_0-c}\right)\right\} \tag{17}$$

F) Yano & Koga model

$$t = \frac{1}{kk_i\mu_{\max}}\left\{\left(\frac{s^2}{2} - \frac{s_0^2}{2}\right) + (c + k_s)(s - s_0) - \frac{kk_sk_i}{c}\ln\left(\frac{s}{s_0}\right) + \frac{c^3 + c^2k + ckk_i + kk_sk_i}{c}\ln\left(\frac{s-c}{s_0-c}\right)\right\} \tag{18}$$

G) Webb model

$$t = \frac{1}{\mu_{\max}}\left\{ -\frac{k_s}{c}\ln\left(\frac{s}{s_0}\right) + \frac{c^2 + ck_i + k_sk_i}{c(c+k_i)}\ln\left(\frac{s-c}{s_0-c}\right) + \frac{k_s}{k_i - c}\ln\left(\frac{s+k_i}{s_0+k_i}\right)\right\} \tag{19}$$

$$t = \frac{1}{(x_0 + s_0y)\mu_{\max}(1 - k_iq)}\left\{(x_0 + s_0y_{x/s})\ln\left(\frac{x}{x_0}\right) + k_sy_{x/s}\ln\left(\frac{x}{x_0}\frac{s_0y_{x/s}}{x_0 + s_0y_{x/s} - x}\right)\right\} + \frac{1}{y_{x/s}\mu_{\max}k_i(1 - k_iq)}$$
$$\left\{(x_0 - x) + (x_0 + s_0y_{x/s})n\left(\frac{x}{x_0}\right)\right\} \tag{21}$$

H) Aiba model

$$t = \frac{k_i}{\mu_{\max}}\left\{ -\frac{k_s}{ck_i}\ln\left(\frac{s}{s_0}\right) + \frac{c+k_s}{c(k_i-c)}\ln\left(\frac{s-c}{s_0-c}\right) - \frac{k_s + k_i}{k_i(k_i-c)}\ln\left(\frac{k_i - s}{k_i - s_0}\right)\right\} \tag{20}$$

From this solution, we can easily describe the substrate concentration for different parameters.

2.7. Proposed modified model

In this study, it is observed that Hinshelwood and Yano & Koga model fits very well with the experimental results. Regarding these pertinent

Table 1. Specific growth rate equation for biomass kinetic models.

Model	Growth rate of biomass
Monod (1949)	$\mu = \mu_{\max}\dfrac{s(t)}{s(t) + k_s}$
Haldane (1968)	$\mu = \mu_{\max}\dfrac{s(t)}{s(t) + k_s + \dfrac{s^2}{k_i}}$
Andrews & Noack	$\mu = \mu_{\max}\dfrac{s(t)}{1 + \dfrac{k_s}{s} + \dfrac{s}{k_i}}$
Aiba (1968)	$\mu = \mu_{\max}\dfrac{s(t)}{s(t) + k_s}e^{\frac{-s}{k_i}}$
Webb (Edward, 1970)	$\mu = \mu_{\max}\dfrac{s\left(1 + \dfrac{s}{k}\right)}{s(t) + k_s + \dfrac{s^2}{k_i}}$
Yano and Koga (1969)	$\mu = \mu_{\max}\dfrac{s}{s(t) + k_s + \dfrac{s^2}{k_i}\left(1 + \dfrac{s}{k}\right)}$
Teissier (1970)	$\mu = \mu_{\max}\left(1 - e^{\frac{-s}{k_s}}\right)$
Hinshelwood	$\mu = \mu_{\max}\dfrac{s(t)}{s(t) + k_s}(1 - k_iq)$
Proposed model	$\mu = \mu_{\max}\dfrac{s(t)}{s(t) + k_s + \dfrac{s^2}{k_i}}(1 - k_iq)$

issues, these two models were analyzed rigorously with the half-saturation parameter k_s, yield coefficient $Y_{x/s}$, maximum specific growth rate $\mu_{\max}$, and substrate inhibition parameter k_i. It is noticeable that the above-discussed models have a significant effect when changing the values of the three parameters k_s, $Y_{x/s}$, and $\mu_{\max}$. Surprisingly, the parameter k_i has an impact only on Hinshewood and Yano & Koga models but no effect on the other six models. In literature, there is a lack of discussions about the impact of the substrate inhibition parameter analysis for growth kinetic models. Based on this observation, this study proposed a modified model for *Spirulina* biomass growth, which is furnished in Table 1.

The analytic solution of the proposed modified model by using Eqs. (1) and (2)

3. Statistical analysis

In this study, all models are verified by using root mean square deviation error (RMSD) and coefficient of determination (R^2) statistics. RMSD error can be calculated by using the equation below:

$$RMSD = \sqrt{\frac{\sum_{j=1}^{n}\left(x_{j,p} - x_{j,m}\right)^2}{n}} \tag{22}$$

Where $x_{j,p}$ comes from the model data and $x_{j,m}$ is the experimental value, j is the experimented time step, and n is the experimented time step in the total period. The coefficient of determination (R^2) lies

between 0 and 1. It is a statistical quantity that represents the ratio for an independent variable and a dependent variable in a regression model. Independent variables explain the dependent variables. R-squared describes the intensity of the relationship between dependent and independent variables. R-squared estimates the capability of a model to envisage experimental data. In model selection criteria, AIC and BIC are commonly operated. When we compare the AIC and BIC for different models, it is better to have low AIC and low BIC values. Japanese statistician Hirotugu Akaike (1969) proposed a model selection criterion based on the relation between the relative Kullback-Leiber (K-L) distance and the maximum log-likelihood. The mathematical form of AIC's is as follows:

$$AIC = 2q - 2l \tag{23}$$

where $2l = -n\{\log 2\pi + \log(error) - \log n + 1\}$, n is the number of observations, and q is the parameter numbers of the model.

Schwarz G. (1978) proposed the Bayesian information criterion, which uses posterior probability and Bayesian information. Mathematically, BIC can be evaluated by the following equation-

$$BIC = q \log(n) - 2l \tag{24}$$

4. Results and discussion

This study illustrates *Spirulina platensis* growth for several existing biomass models and compares them with experimental data available in the literature (Çelekli and Yavuzatmaca 2009). We began by considering the influence of the biomass concentration growth rate, as shown in Figures 2, 3 and 4 (and Supplementary Figures S1-S3). Figures 5 and 6 represent the outcomes of substrate concentration.

Figure 2 displays the set of line graphs for six biomass models: (A) Haldane, (B) Andrew (C) Hinshelwood (D) Yano & Koga (E) Webb, and (F) Aiba for inhibition parameter values 120, 140, 160, 180, and 200 respectively. It illustrates that the Haldane, Andrews & Noack, Webb, and Aiba models are independent of the inhibition parameter, while the Hinshelwood and Yano & Koga models are dependent on it. In Yano & Koga model, biomass concentration increases with the increase of inhibition parameter. However, the complete opposite trend is observed for the Hinshelwood model; biomass concentration increases as the
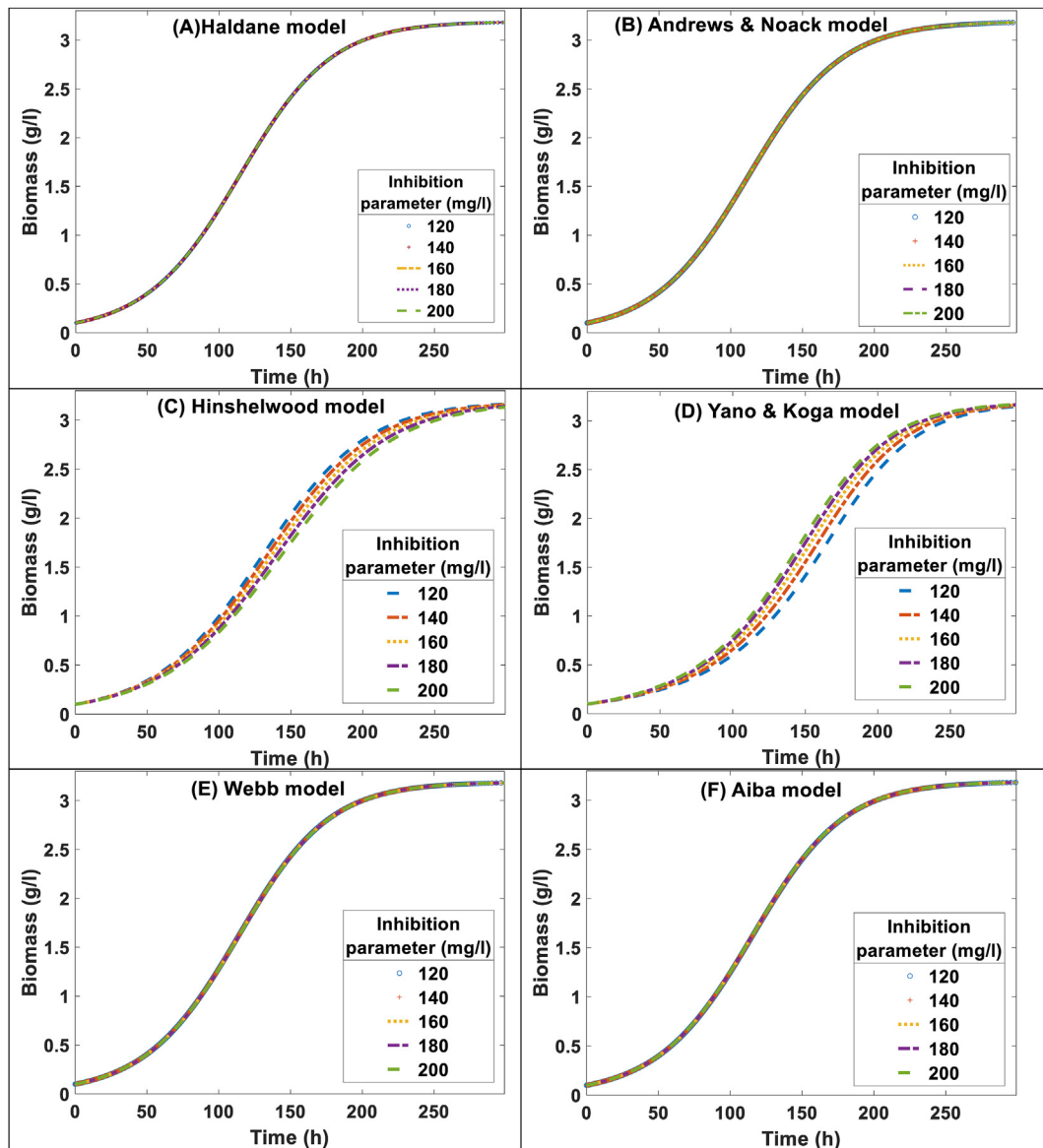

Figure 2. Biomass concentration profiles of (A) Haldane, (B) Andrews & Noack, (C) Hinshelwood, (D) Yano & Koga, (E) Webb, and (F) Aiba model, versus time t for fixed values of μ_{max}= 0.39 h^{-1}, $Y_{x/s}$ = 3.09 (-),k_s = 12.5 mg/l and different values of k_i = 120, 140, 160, 180, and 200 mg/l respectively.
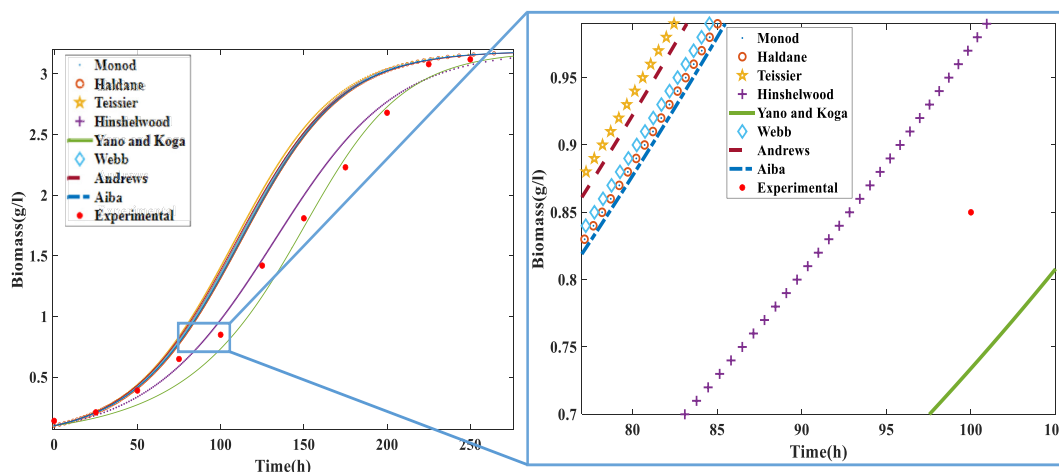
Figure 3. The biomass growth rate of the *Spirulina platensis* cultures measured for Monod, Haldane, Teissier, Hinshelwood, Yano & Koga, Webb, Andrews & Noack, Aiba, and experimental values at different times. The dotted points represent the experimental data and other dashed, star, circle, diamond, and solid line represents the model predictions.
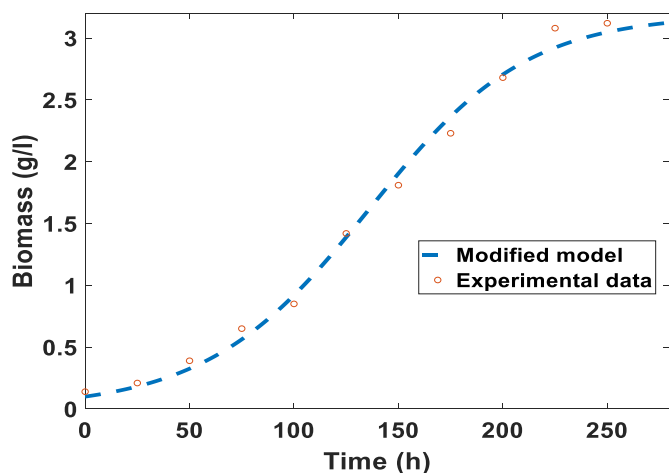

Figure 4. *Spirulina* growth culture compared with the modified model and experimental data. The red dotted points represent the experimental data, and the blue dashed conveys the modified model predictions.

inhibition parameter decreases. Besides, it can be said that the biomass concentration increases when the yield coefficient rises for all considered models (Supplementary Figure S1).

After observing Supplementary Figure S1, we can demonstrate the following remarks: Hinshelwood and Yano & Koga model biomass yield rates are slower than other models. Monod, Haldane, Andrews & Noack, Teissier, Webb, and Aiba model attains 3.29 g/l at approximately 300 days, but Hinshelwood and Yano & Koga model need 352 days and 332 days, respectively. After 250 h, biomass growth exhibits a stationary phase for all models. The specific growth rate of biomass concentration increases with the increase of specific growth rates for kinetic models. Subsequently, biomass concentration remains constant for all models except Hinshelwood and Yano & Koga model. The exponential phase was observed within 50–200 h and then attained the maximum biomass growth rate. It is observed from Supplementary Figure S2, biomass escalations with increases for specific growth rates. Consequently, it is found from Supplementary Figure S3, Yano & Koga, and Hinshelwood model exhibits the higher growth rate of biomass from the other models for the half-saturation parameter. In 150 h, biomass concentration distribution shows more variance for these two models. After 250 h, the biomass growth curve appears more stable for all models.

To show the holistic tendency, we present Figure 3, which represents the biomass concentration rate for all considered models. After 50–200 h, experimental results are close to the Yano & Koga and Hinshelwood model. Monod, Haldane, Andrews & Noack, Teissier, Webb, and Aiba model predicts the higher growth than experimental data as well as the Hinshelwood model. Hinshelwood model exhibits over expected results than empirical data, but Yano & Koga shows under predicted data. The experimental data exhibited an S-shaped curve (Narang 2006), which is like the logistics growth model (Bacaër 2011; Islam et al., 2017). In this study, Logistics, Richards (1959), Schenute (1981), Gompertz (1825), and Stannard models (Kyurkchiev and Iliev 2016) were not considered because these models do not contain any substrate or nutrient related parameters. When we considered the delay time for the logistics equation, it can be transformable to the Monod equation using an approximation on Taylor expansion (Lobry et al., 1992). In this study, from Figure 5, it is apparent that for increasing the inhibition parameter, there is no effect of substrate concentration on the Haldane, Andrews & Noack, Webb, and Aiba model. But Hinshelwood and Yano & Koga's model has significant outcomes. If the inhibition parameter is increasing, substrate concentration decreases slowly for the Hinshelwood model. But for Yano & Koga model, substrate concentration increases when the inhibition parameter increases. These two models exhibit the opposite characteristics for the inhibition parameter. Supplementary Figures S4, S5, S6, and S7 correspond to the concentration of substrate for various values of the particular parameters, maximum specific growth rate (μ_{max}), observed yield coefficient ($Y_{x/s}$), and half-saturation concentration (k_s). From Supplementary Figure S4, it is estimated that the substrate concentration lessens when the specific growth rate escalates. It is also observed that from 50 h to 200 h, substrate concentration decreased rapidly. After 300 h, substrate concentration flattens for Monod, Haldane Andrews & Noack, Teissier, Webb, and Aiba model except for Hinshelwood and Yano & Koga model. From Supplementary Figures S5 and S6, it is stated that the substrate concentration diminishes when the yield coefficient and half-saturation constant increase. It is evident from Figure 6 that the substrate concentration decreases faster for the Teissier model and then Monod, Haldane, and others. After 100 h, the substrate concentration is approximately 0.76 mg/l, 0.68 mg/l, and 0.56 mg/l, corresponding to Yano & Koga model, Hinshelwood model, and Monod model. Similarly, we observed that after 150 h, the pattern of substrate concentration follows the same pattern. For Yano & Koga and Hinshelwood model, substrate concentration decreases slowly than other models.

The proposed model consists of two parts: the first part is related to substrate consumption, and the last part is associated with substrate inhibition. The proposed model shows better performance than previously
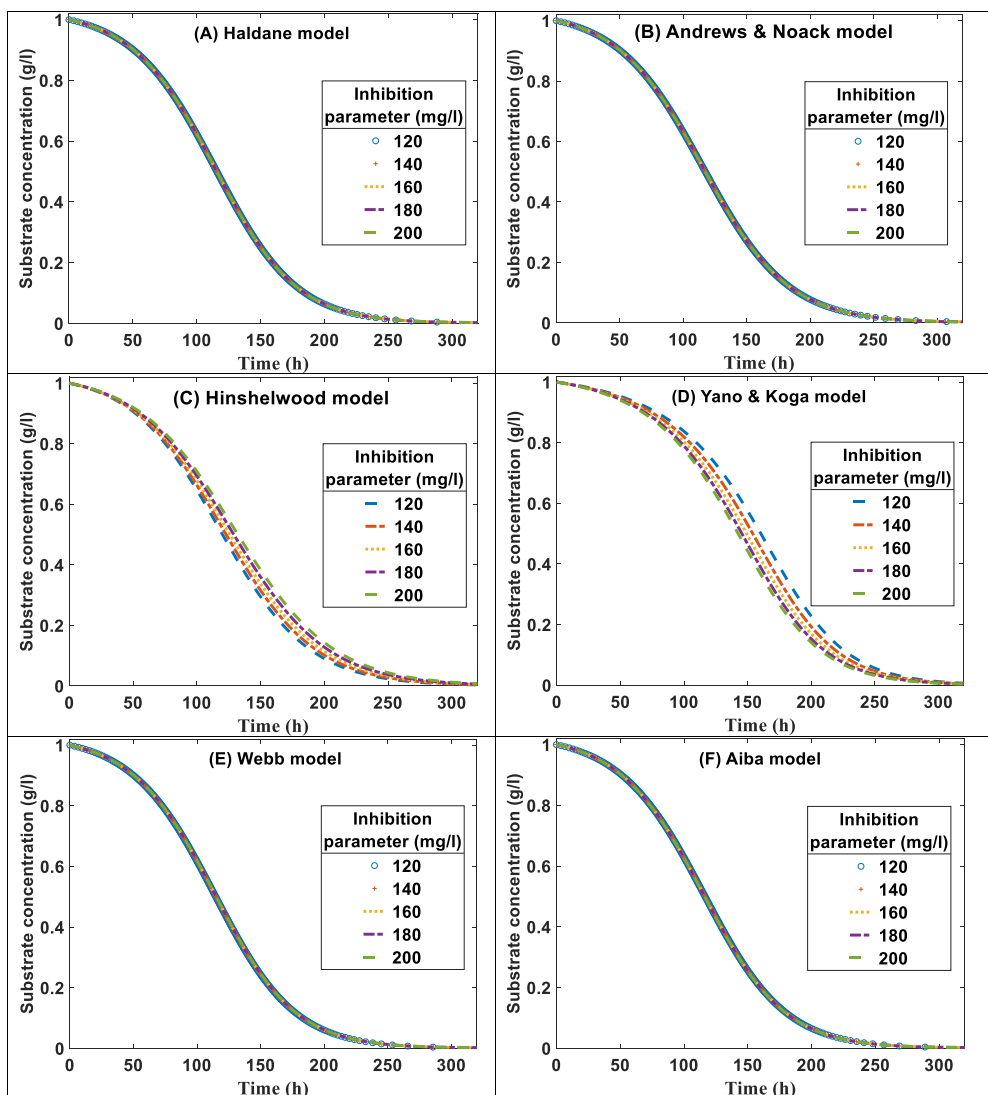
Figure 5. Substrate concentration profiles of (A) Haldane, (B) Andrews & Noack, (C) Hinshelwood, (D) Yano & Koga, (E) Webb, and (F)Aiba model versus time t for fixed values of μ_{max} = 0.39 time^{-1}, $Y_{x/s}$ = 3.09, k_s = 12.5 mg/l, and different values of k_i = 120, 140, 160, 180, and 200 mg/l respectively.
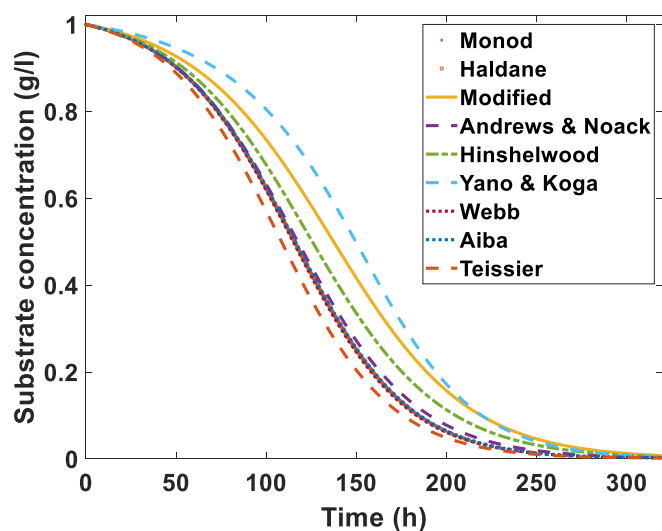

Figure 6. Substrate concentration profiles of the *Spirulina platensis* cultures measured for Monod, Haldane, Teissier, Hinshelwood, Yano & Koga, Webb, Andrews & Noack, and Aiba model at different times.

studied models. The RMSD of the Hinshelwood and Yano & Koga model is 0.1086 and 0.1207, respectively. But other models have higher values than these two models, which is shown in the following Table 2. The R-squared values of the Hinshelwood model and Yano & Koga model are 0.973 and 0.9582, respectively. The R-squared values of the Hinshelwood and Yano & Koga models are higher than that of the other models. The RMSD of the modified proposed model is 0.081393, the R-squared value is 0.9862, AIC is -9.7277, and BIC is -8.2148, whereas the study

Table 2. RMSD, R^2, AIC, and BIC values of the kinetic models.

Model	RMSD	R^2	AIC	BIC
Monod	0.373007	0.9606	1.4954	2.4031
Haldane	0.373262	0.9606	3.5021	4.7125
Andrews & Noack	0.380012	0.9531	3.6814	4.8918
Teissier	0.40717	0.9567	2.3717	3.2795
Hinshelwood	0.108673	0.9727	-6.8372	-5.3243
Yano & Koga	0.155987	0.9618	-5.2229	-4.0125
Webb	0.381455	0.9600	3.7193	4.9297
Aiba	0.376514	0.9612	1.5889	2.4967
Proposed model	0.081393	0.9862	-9.7277	-8.2148

found the best RMSD of 0.1086, the R-squared value is 0.973, AIC is -6.8372, and BIC is -5.3243 from the studied models.

Though Haldane, Webb, Aiba, and Andrews and Noack's models include the inhibition parameter, there is no significant effect on these models when we change the inhibition parameter value. Hinshelwood, and Yano, and Koga models, as well as our proposed hybrid model, have a significant effect when we change the inhibition parameter values (ranges from 120 to 200). The present study showed better fitting of experimental values of algae cultivation with the three mathematical models. Future studies are required to further understand the effect of inhibition parameters on the algae growth model.

5. Conclusions

The present study demonstrates insight into the fast growth of *Spirulina platensis* that incorporates popular eight biomass growth models and experimental data. One of our primary concerns while studying theoretical comparisons is to illustrate the more well-suited model compared with experimental data. We found that Hinshelwood and Yano & Koga model results are more compatible than others. Further, it is observed that the substrate inhibition parameter has significant effects on Hinshelwood and Yano & Koga model. Our proposed models had a high R^2 (0.9862), a low RSS (0.0813), AIC (-9.7277), and BIC (-8.2148) in this investigation, implying that this recommended modified model was considerably matched with the experimental data for *Spirulina platensis* growth compared to the popular eight studied models. Thus, the authors anticipate that this study provides a suitable guide for algae growth kinetics and a profound understanding of various parameters with realistic circumstances.

Declarations

Author contribution statement

Mir Shariful Islam: Conceived and designed the experiments; Performed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.

K M Ariful Kabir: Conceived and designed the experiments; Performed the experiments.

Jun Tanimoto: Contributed reagents, materials, analysis tools or data; Wrote the paper.

Bidyut Baran Saha: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

Supplementary content related to this article has been published online at https://doi.org/10.1016/j.heliyon.2021.e08185.

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