Review article

Stability analysis of a metapopulation model for the dynamics of malaria's spread including climatic factors

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ABSTRACT

Eradicating malaria remains a big challenge for computer scientists, mathematicians, epidemiologists, entomologists, physicians and many others. Their approaches range from recovering patients to eradicating the disease. However, collaboration, not always efficient between all these scientists, leads to the implementation of incomplete prototypes or to an under-exploitation of their results. Environmental and climatic factors are part of these elements that are usually omitted by computer scientists and mathematicians in the modelling of the malaria spread dynamic. Tropical countries, most affected by the disease are also mostly underdeveloped or developing countries, and therefore, statistical data are often lacking or difficult to access. Populations are constantly in motion over ecosystems with different environmental and climatic conditions, from a region to another. In this paper, we analyse the global asymptotic stability at the disease-free equilibrium of a metapopulation model including climatic factors.

1. Introduction

Malaria is still a major problem in almost all countries of the tropical area. According to the World Health Organization (WHO), approximately half the world's population is exposed to the disease, 95 countries were affected by the transmission of malaria in 2021 and recorded 247 million cases of malaria and 619,000 deaths (www.who.int). The *Lowy Institute* (Australia), through a study, predicts a prevalence of malaria that can be multiplied by 4 in 2050, compared to 1990 [9]. By the end of this century, the world's population living in areas where malaria is endemic could drop from 45% to 60%.

One of the main questions of epidemiologists is whether or not there is or will be an epidemic. Mathematicians and computer scientists in their modelling and predictive analytics have as mission, among others, to provide answers. In epidemic, the number of new patients (infected, infectious) increases. Studying the global stability of the proposed model is important for its reliability. Study the stability of a system informs us about the disease evolution, in order especially to know if after a time, the disease will be contained, the number of new patients will stabilize around a point of equilibrium or if the disease will spread and therefore, if the healthy population will tend to disappear in favour of a population almost totally infected. This stability study is the focal point of our work.

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Table 1
Parameters and description.

Parameters	Description
n	number of patches
$\varphi_{ij} \geq 0,$	rate of human migration from the patch i to the patch j
μ_{H_i}	natural mortality rate of humans in the patch i
μ_{Vi}	natural mortality rate of mosquitoes in the patch i
β_i	probability of infection of susceptible humans per mosquito bite inside the patch i. k_i is the average number of such contacts. $b_i = k_i \beta_i$
ω_i	probability of infection of susceptible mosquitoes per mosquito bite of the infected human inside the patch i. f_i is the average number of such contacts
δ_{H_i}	rate of infected humans that become infectious in the patch i
ρ_{H_i}	rate of infectious humans that become susceptible in the patch i
δ_{V_i}	rate of infected mosquitoes that become infectious in the patch i
α_{H_i}	recovery rate of infectious humans in the patch i
d_{H_i}	death-rate of infectious humans due to the disease
ε_{H_i}	recovery rate of recovered humans (eventually immunised) become susceptible later
Λ_{H_i}	the average recruitment of humans (by birth) in the patch i
Λ_{V_i}	the average recruitment of mosquitoes (by birth) in the patch i

2. Survey

Literature informs us about several models developed to fight against malaria and shows us that the importance and nuisance capacity of that disease still as complex to define. Equation approaches are important for the global forecast of the future behaviour of the system because they are formalized. Through an application of optimal control theory, Tchoumi et al. [11] showed that a combination of personal protection, treatment and vaccination of children under-five produces excellent results for fighting against malaria's spread. But their model does not take into account climatic factors and human migrations. Only few of them integrated climatic factors in their modelling; Gaudart et al. [4] proposed a model which includes the *NDVI* vegetation index is an interesting lead, but this index, as the authors said, is very rarely available, especially in the tropical regions which are nevertheless the most concerned by malaria. Furthermore, they did not integrate human migrations and assumed as constant, human and mosquito sizes (births and deaths are neglected). Tsanou [10] proposed a metapopulation model, and showed that there is a threshold under which the disease disappears and above which the disease remains. In the Tsanou model, demography is neglected (populations sizes are constant), epidemiological parameters are the same for all patches. More, Tsanou did not take into consideration climatic factors. In our previous work [7], after an extensive survey on malaria and existing models, we proposed a metapopulation model including climatic factors and human migrations (without analysing the global asymptotic stability).

3. Our model

We analyze, in this paper, the global asymptotic stability of our model firstly defined in [7] and represented here by system of equations (1). The word *patch* that we will use represents a geographical location (a city, a region, a country, etc.)

Let N_{H_i} (respectively N_{V_i}) be the total humans population (respectively vectors) of the patch i. We also denote by $\varphi_{ij} S_{H_i}$ the susceptible residents from the patch i who are moving to the patch j. $\varphi_{ij} E_{H_i}$ is the infected residents of patch i who are moving to the patch j, $\varphi_{ij} R_{H_i}$ the recovered and immunized residents from patch i who are moving to the patch j, S_{V_i} represents the susceptible mosquitoes residents in the patch i, E_{V_i} the infected mosquitoes residents in the patch i, and I_{V_i} the infectious mosquitoes residents in the patch i. Table 1 shows different parameters and their description.

Our system has the form:

$$
\begin{cases}
\dot{S}_{H_i} = \Lambda_{H_i} + \epsilon_{H_i} R_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{H_j} - S_{H_i} \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} - k_i \beta_i \frac{S_{H_i}}{N_{H_i}} I_{V_i} - \mu_{H_i} S_{H_i}, \\
\dot{E}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} E_{H_j} - E_{H_i} \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} + k_i \beta_i \frac{S_{H_i}}{N_{H_i}} I_{V_i} - \delta_{H_i} E_{H_i} - \mu_{H_i} E_{H_i}, \\
\dot{I}_{H_i} = \delta_{H_i} E_{H_i} - \alpha_{H_i} I_{H_i} - d_{H_i} I_{H_i} - \mu_{H_i} I_{H_i}, \\
\dot{R}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} R_{H_j} + \alpha_{H_i} I_{H_i} - R_{H_i} \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} - \epsilon_{H_i} R_{H_i} - \mu_{H_i} R_{H_i}, \\
\dot{S}_{V_i} = \Lambda_{V_i} - f_i \omega_i \frac{S_{V_i}}{N_{H_i}} I_{H_i} - \pi_{V_i} S_{V_i}, \\
\dot{E}_{V_i} = f_i \omega_i \frac{S_{V_i}}{N_{H_i}} I_{H_i} - \delta_{V_i} E_{V_i} - \pi_{V_i} E_{V_i}, \\
\dot{I}_{V_i} = \delta_{V_i} E_{V_i} - \pi_{V_i} I_{V_i}.
\end{cases}
\tag{1}
$$

With initial conditions $(S_{H_i}(0), E_{H_i}(0), I_{H_i}(0), R_{H_i}(0), S_{V_i}(0), E_{V_i}(0), I_{V_i}(0))$.

Table 2

Average monthly minimum temperatures and average monthly relative humidity for Yaounde city.

Source: www.climatemps.com.

Climatic factors	January	February	March	April
Average minimum temperature	17.1	20	13.9	19.7
Average relative humidity	62	62	65	67

Climatic factors	May	June	July	August
Average minimum temperature	19.5	19.3	19	19
Average relative humidity	70	73	74	75

Climatic factors	September	October	November	December
Average minimum temperature	19	18.9	19.2	19.1
Average relative humidity	73	72	66	60

Where

$$\pi_{V_i} = -\ln(p_i(T, RH)). \tag{2}$$

And

$$p(T, RH) = \exp(\frac{-1}{T^2 \times \beta_2 + T \times \beta_1 + \beta_0}). \tag{3}$$

With

$$\begin{cases} \beta_0 = 0.00113 \times RH^2 - 0.158 \times RH - 6.61, \\ \beta_1 = -2.32 \times 10^{-4} \times RH^2 + 0.0515 \times RH + 1.06, \\ \beta_2 = 4 \times 10^{-6} \times RH^2 - 1.09 \times 10^{-3} \times RH - 0.0255. \end{cases}$$

π_{V_i} is the mortality rate for a mosquito resident in the patch i having temperature T and relative humidity RH at a given time. This mortality rate is estimated from [6].

$p(T, RH)$ is the probability of survival of mosquitoes at temperature T and relative humidity RH, defined by [8].

Studying the global asymptotic stability of a system consists on determine the conditions in which the system remains in an equilibrium state or oscillates around this state. Two cases are usually considered: the disease-free equilibrium and the endemic equilibrium. Our study focuses on the case of disease-free equilibrium (DFE). We proceed with a 5-step approach:

(a) Study evolution of mosquitoes mortality depending on temperature and relative humidity;
(b) Define the basic model properties (positivity and boundedness of the solutions);
(c) Study the unicity of solutions at the disease-free equilibrium;
(d) Determine the basic reproduction number R_0;
(e) Study the global asymptotic stability at the disease-free equilibrium.

4. Study of evolution of mosquito mortality depending on temperature and relative humidity

The system (1) shows a mosquitoes mortality $\pi_{V_i} = -\ln(p_i(T, RH))$ depending on temperature T and relative humidity RH. It is important to observe the mortality evolution as a function depending on temperature and relative humidity. Table 2 shows the average monthly minimum temperatures and average monthly relative humidities of Yaounde city in Cameroon, a city corresponding to a $SEIRS$ model. We note that mosquitoes are usually active by night, when the temperature is usually minimum. The *MATLAB* software is the one we used for our observations.

4.1. Mosquito survival depending on temperature and relative humidity

Remember that the probability of mosquito survival is given by the equation (3)

Fig. 1 shows that the probability of mosquito survival in Yaounde city is low when temperature and relative humidity are both low. On the other hand, this probability increases when the temperature is minimum and the humidity rise.

4.2. Mosquito mortality rate depending on temperature and relative humidity

Remember that the mortality rate as we presented it is given $\pi_{V_i} = -\ln(p_i(T, RH))$.

Fig. 2 shows that in Yaounde city, the mortality rate is low when temperature and relative humidity are both high. On the other hand, this mortality increases when temperature and the relative humidity are both low.
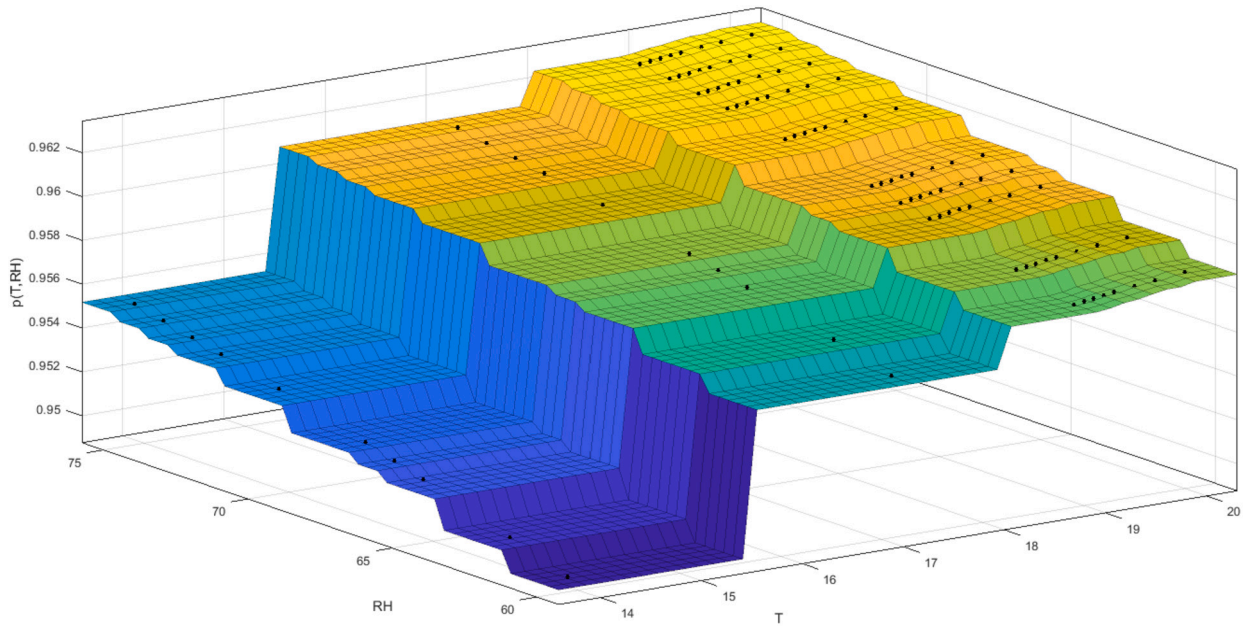
Fig. 1. Probability of survival $p(T, RH)$ mosquitoes depending on the temperature T and the relative humidity RH in Yaounde city.
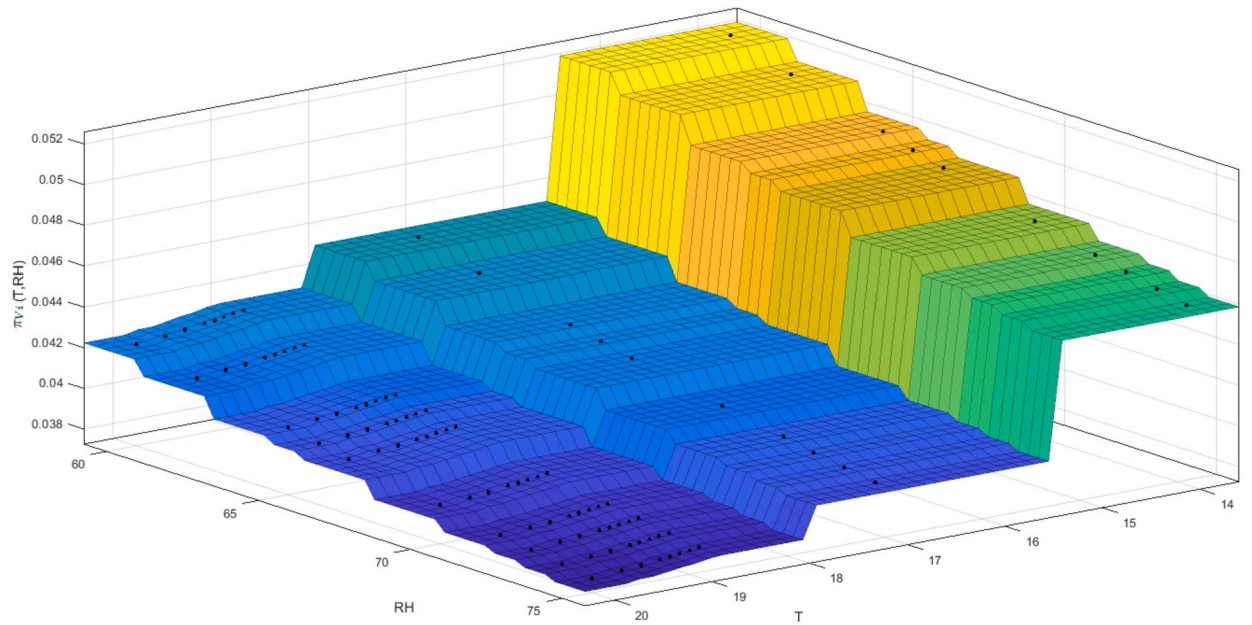

Fig. 2. Mortality rate $\pi_{V_i}(T, RH)$ mosquitoes depending on the temperature t and relative humidity rh in Yaounde city.

Mosquitoes mortality rate, as observed, is a function of temperature and relative humidity. These climatic factors are time dependent, therefore variable and should not be used as is for the mathematical analysis of our models. Thus, for the study of systems equilibrium, we propose to find analytical expressions for temperature and relative humidity as a function of time.

4.3. Analytical expression of temperature

We have to find an analytical expression which adjusts the minimum temperature in Yaounde city. Fig. 3 presents a staircase representation of the monthly minimum temperatures of Yaounde city over 24 months.

We can observe through the Fig. 3 that the temperature function is a periodic function of period $\theta = 12$. Temperature being a time dependent function, Bacaer's work [1] inspires us with an analytical expression of form
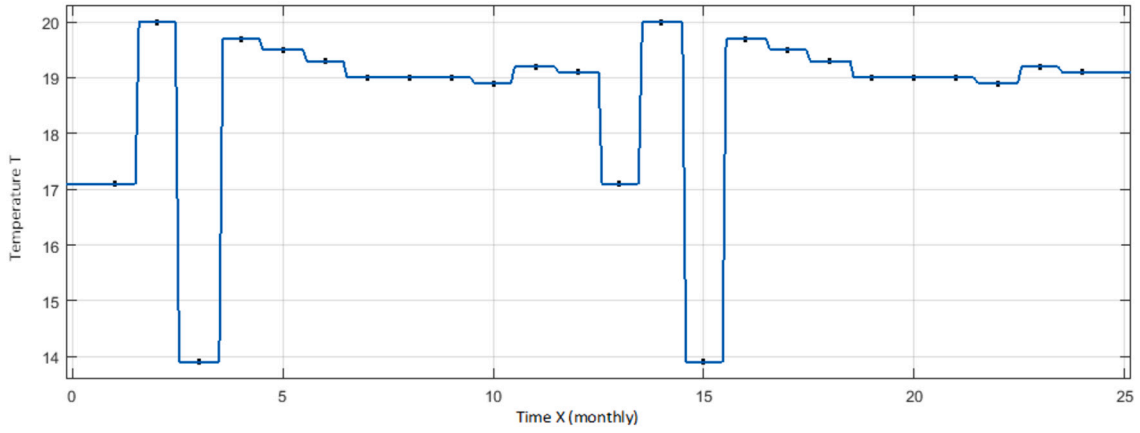
Fig. 3. Minimum temperature T as a function of time X for the city of Yaounde.
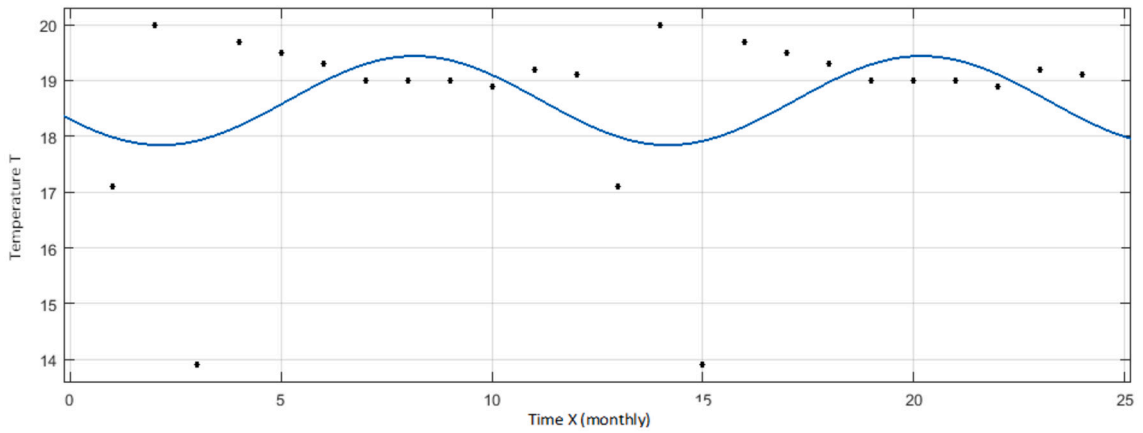

Fig. 4. A calibration of the monthly minimum temperatures of Yaounde city.

$$T(X) = T_0[1 + \varepsilon cos(\omega X - \psi)]. \tag{4}$$

Where $\omega = \frac{2\pi}{\theta}$ and T_0 is the average temperature. In fact, this equation is the equivalent of the Fourrier series development of the Temperature function T to the order 1 which has the form $T(X) = a_0 + a_1 cos(\omega X) + b_1 sin(\omega X)$.

Fig. 4 presents the result of a calibration of the monthly minimum temperatures for Yaounde city performed with the equation (4), using $cftool$ function of $Matlab$. With this equation, we have, at 95% degree of confidence:

$$\omega = \frac{2\pi}{\theta} = \frac{2\pi}{12} = \frac{\pi}{6}, \quad T_0 = 18.64, \quad \varepsilon = -0.04278, \quad \psi = 1.131$$

4.4. Analytical expression of relative humidity

We have to find an analytical expression that adjusts the minimum temperature in Yaounde city. Fig. 5 shows a staircase representation of the monthly humidity in Yaounde city over 24 months.

Fig. 5 shows that the relative humidity function is a periodic function of period $\theta = 12$. The equation inspired by Bacaer's work becomes

$$RH(X) = RH_0[1 + \varepsilon cos(\omega X - \psi)]. \tag{5}$$

RH_0 is the monthly relative humidities average, $RH(X)$ being the equivalent of the Fourrier series development of the relative humidity function RH to the order 1 which has the form $RH(X) = a_0 + a_1 cos(\omega X) + b_1 sin(\omega X)$.

Fig. 6 shows the result of a calibration of the monthly relative humidity for Yaounde city carried out with the equation (5), using $cftool$ function of $Matlab$. We have, at 95% degree of confidence:

$$\omega = \frac{2\pi}{\theta} = \frac{2\pi}{12} = \frac{\pi}{6}; \qquad RH_0 = 68.24$$
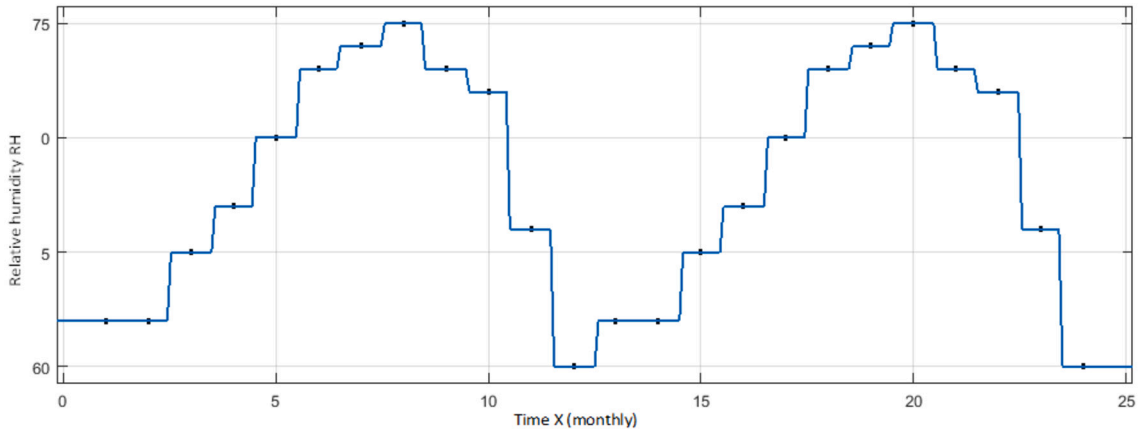$$\varepsilon = -0.1011, \qquad \psi = 0.7213$$

Fig. 5. Relative humidity RH as a function of time X for Yaounde city.
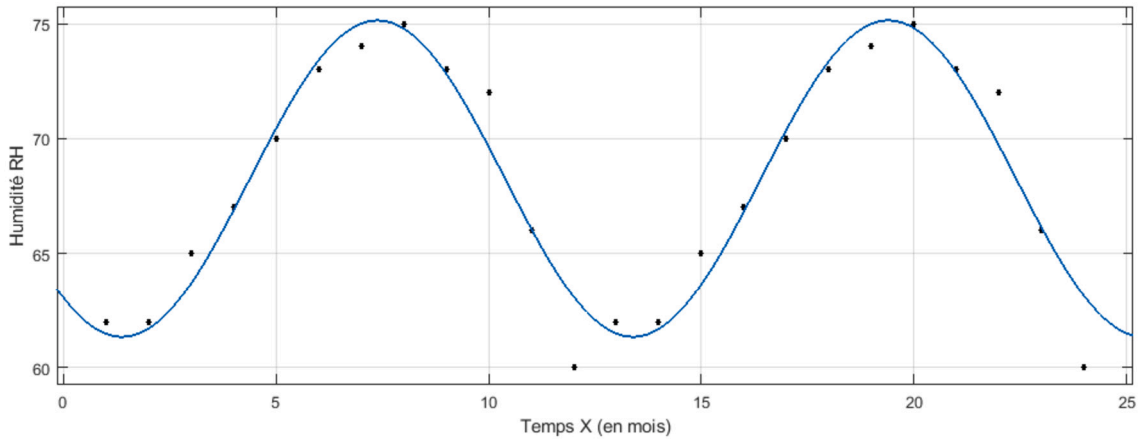

Fig. 6. A calibration of the monthly relative humidity of Yaounde city.

5. Basic model properties

5.1. Positivity and boundedness of the solutions

We study in this part positivity and boundedness of the solutions which correspond to the various compartments.

5.1.1. Positivity of the solutions

Proposition 5.1. *For all $i = 1, 2, ..., n$ and $t \geq 0$,*

$$S_{H_i}(t) \geq 0, \ E_{H_i}(t) \geq 0, \ I_{H_i}(t) \geq 0, \ R_{H_i}(t) \geq 0, \ S_{V_i}(t) \geq 0, \ E_{V_i}(t) \geq 0, \ I_{V_i}(t) \geq 0.$$

Proof. At the initial conditions, we have:

$$S_{H_i}(0) = \psi_1^i \geq 0, \ E_{H_i}(0) = \psi_2^i \geq 0, \ I_{H_i}(0) = \psi_3^i \geq 0, \ R_{H_i}(0) = \psi_4^i \geq 0,$$
$$S_{V_i}(0) = \psi_5^i \geq 0, \ E_{V_i}(0) = \psi_6^i \geq 0, \ I_{V_i}(0) = \psi_7^i \geq 0.$$

- Suppose there is a positive t_1 and an integer i_1 such as
  $S_{H_{i_1}}(t_1) = 0$ and $S_{H_i}(t) > 0, \ E_{H_i}(t) > 0, \ I_{H_i}(t) > 0, \ R_{H_i}(t) > 0, \ S_{V_i}(t) > 0, \ E_{H_i}(t) > 0, \ I_{H_i}(t) > 0$, for all $i = 1, 2, ..., n$ and $0 \leq t \leq t_1$.
  Furthermore, with (1), we have $\dot{S}_{H_{i_1}}(t_1) \geq \Lambda_{H_{i_1}} > 0$, which contradicts the initial hypothesis $S_{H_{i_1}}(t) > 0 = S_{H_{i_1}}(t_1)$.
- Suppose there is a positive t_2 and an integer i_2 such as
  $E_{H_{i_2}}(t_2) = 0$ and $S_{H_i}(t) > 0, \ E_{H_i}(t) > 0, \ I_{H_i}(t) > 0, \ R_{H_i}(t) > 0$, for all $i = 1, 2, ..., n$ and $0 \leq t \leq t_2$.
  According to (1), we have $\dot{E}_{H_{i_2}}(t_2) > 0$, which contradicts the initial hypothesis $E_{H_{i_2}}(t) > 0 = E_{H_{i_2}}(t_2)$.

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- Suppose there is a positive t_3 and an integer i_3 such as

  $I_{H_{i_3}}(t_3) = 0$ and $S_{H_i}(t) > 0$, $E_{H_i}(t) > 0$, $I_{H_i}(t) > 0$, $R_{H_i}(t) > 0$, for all $i = 1, 2, ..., n$ and $0 \le t \le t_3$.

  According to (1), we have $\dot{I}_{H_{i_3}}(t_3) > 0$, which contradicts the initial hypothesis $I_{H_{i_3}}(t) > 0 = I_{H_{i_3}}(t_3)$.

- Suppose there is a positive t_4 and an integer i_4 such as

  $R_{H_{i_4}}(t_4) = 0$ and $S_{H_i}(t) > 0$, $E_{H_i}(t) > 0$, $I_{H_i}(t) > 0$, $R_{H_i}(t) > 0$, for all $i = 1, 2, ..., n$ and $0 \le t \le t_4$.

  Furthermore, with (1), on a $\dot{R}_{H_{i_4}}(t_4) \ge \Lambda_{H_{i_4}} > 0$, which contradicts the initial hypothesis $R_{H_{i_4}}(t) > 0 = R_{H_{i_4}}(t_4)$.

  We proceed in an analogous way to show that $S_{V_i}(t) > 0$, $E_{V_i}(t) > 0$ and $I_{V_i}(t) > 0$, for all $i = 1, 2, ..., n$ and $t \ge 0$.

5.1.2. Boundedness of the solutions

We must show that the total populations of mosquitoes and humans are bounded.

The dynamic of mosquitoes population in each patch i is given by equation (6).

$$\dot{N}_{V_i} = \Lambda_{V_i} - \pi_{V_i} N_{V_i}. \tag{6}$$

Since $\pi_{i_{min}} \le \pi_{V i} \le \pi_{i_{max}}$, with:

$\pi_{i_{min}}$ as the minimum mortality rate of mosquitoes in the patch i,

$\pi_{i_{max}}$ as the maximum rate of mosquito mortality in the patch i.

We have the inequality (7):

$$\Lambda_{V_i} - \pi_{i_{max}} N_{V_i} \le \dot{N}_{V_i} \le \Lambda_{V_i} - \pi_{i_{min}} N_{V_i}. \tag{7}$$

Equation $\dot{N}_{V_i} = \Lambda_{V_i} - \pi_{i_{max}} N_{V_i}$ has solution $N_{V_i}(t) = N_{V_i}^0 e^{-\pi_{imax} t} + \frac{\Lambda_{V_i}}{\pi_{imax}}(1 - e^{-\pi_{imax} t})$, and equation $\dot{N}_{V_i} = \Lambda_{V_i} - \pi_{i_{min}} N_{V_i}$ has solution

$N_{V_i}(t) = N_{V_i}^0 e^{-\pi_{imin} t} + \frac{\Lambda_{V_i}}{\pi_{imax}}(1 - e^{-\pi_{imin} t})$.

Applying Grönwall's inequality, we obtain:

$$N_{V_i}^0 e^{-\pi_{imax} t} + \frac{\Lambda_{V_i}}{\pi_{imax}}(1 - e^{-\pi_{imax} t}) \le N_{V_i}(t) \le N_{V_i}^0 e^{-\pi_{imin} t} + \frac{\Lambda_{V_i}}{\pi_{imin}}(1 - e^{-\pi_{imin} t}).$$

At the limit crossing, we have the inequality (8):

$$\frac{\Lambda_{V_i}}{\pi_{i_{max}}} \le \lim_{t \to +\infty} N_{V_i}(t) \le \frac{\Lambda_{V_i}}{\pi_{i_{min}}}. \tag{8}$$

The system of equations (9) defines Φ_i^1 and Φ_i^2 as:

$$\begin{cases} \Phi_i^1 = \phi_{ji} = \sum_{\substack{j=1 \\ j \ne i}}^{n} \varphi_{ji}(S_{H_j} + E_{H_j} + R_{H_j}), \\ \Phi_i^2 = \phi_{ij} = (S_{H_i} + E_{H_i} + R_{H_i}) \sum_{\substack{j=1 \\ j \ne i}}^{n} \varphi_{ij}. \end{cases} \tag{9}$$

The dynamic of humans population for each patch i is given by equation (10):

$$\dot{N}_{H_i} = \Lambda_{H_i} + \Phi_i^1 - \Phi_i^2 - d_{H_i} I_{H_i} - \mu_{H_i} N_{H_i}. \tag{10}$$

To prove that humans population in each patch is bounded, we choose to show that the total humans population in all patches is bounded as defined in equation (11).

$$\sum_{i=1}^{n} \dot{N}_{H_i} = \sum_{i=1}^{n} \Lambda_{H_i} + \sum_{i=1}^{n} \Phi_i^1 - \sum_{i=1}^{n} \Phi_i^2 - \sum_{i=1}^{n} d_{H_i} I_{H_i} - \sum_{i=1}^{n} \mu_{H_i} N_{H_i}. \tag{11}$$

Proposition 5.2. *The system (1) has the form of a closed graph, all migrations between different patches cancel each other out.*

$$\sum_{i=1}^{n} \Phi_i^2 = \sum_{i=1}^{n} \Phi_i^1.$$

Proof. We set $\Delta_{\Phi^1} = \sum_{i=1}^{n} \Phi_i^2 - \sum_{i=1}^{n} \Phi_i^1$.

$$\Delta_{\Phi^1} = \sum_{i=1}^{n} [(S_{H_i} + E_{H_i} + R_{H_i})(\varphi_{i1} + \varphi_{i2} + ... + \varphi_{in})]$$

$$- \sum_{i=1}^{n} [\varphi_{1i}(S_{H_1} + E_{H_1} + R_{H_1}) + \phi_{2i}(S_{H_2} + E_{H_2} + R_{H_2}) + ... + \varphi_{ni}(S_{H_n} + E_{H_n} + R_{H_n})]$$

$$= \sum_{i=1}^{n}[\varphi_{i1}(S_{H_i} + E_{H_i} + R_{H_i}) + \varphi_{i2}(S_{H_i} + E_{H_i} + R_{H_i}) + ... + \phi_{in}(S_{H_i} + E_{H_i} + R_{H_i})]$$

$$- \sum_{i=1}^{n}[\varphi_{1i}(S_{H_1} + E_{H_1} + R_{H_1}) + \varphi_{2i}(S_{H_2} + E_{H_2} + R_{H_2}) + ... + \varphi_{ni}(S_{H_n} + E_{H_n} + R_{H_n})]$$

$$\Delta_{\Phi} = \varphi_{11}(S_{H_1} + E_{H_1} + R_{H_1}) + \varphi_{12}(S_{H_1} + E_{H_1} + R_{H_1}) + ... + \varphi_{1n}(S_{H_1} + E_{H_1} + R_{H_1})$$

$$+ \varphi_{21}(S_{H_2} + E_{H_2} + R_{H_2}) + \varphi_{22}(S_{H_2} + E_{H_2} + R_{H_2}) + ... + \varphi_{2n}(S_{H_2} + E_{H_2} + R_{H_2})$$

$$+ ...$$

$$+ \varphi_{n1}(S_{H_n} + E_{H_n} + R_{H_n}) + \varphi_{n2}(S_{H_n} + E_{H_n} + R_{H_n}) + ... + \varphi_{nn}(S_{H_n} + E_{H_n} + R_{H_n})$$

$$- \varphi_{11}(S_{H_1} + E_{H_1} + R_{H_1}) - \varphi_{21}(S_{H_2} + E_{H_2} + R_{H_2}) - ... - \varphi_{n1}(S_{H_n} + E_{H_n} + R_{H_n})$$

$$- \varphi_{12}(S_{H_1} + E_{H_1} + R_{H_1}) - \varphi_{22}(S_{H_2} + E_{H_2} + R_{H_2}) + ... - \varphi_{n2}(S_{H_n} + E_{H_n} + R_{H_n})$$

$$- ...$$

$$- \varphi_{1n}(S_{H_1} + E_{H_1} + R_{H_1}) - \varphi_{2n}(S_{H_2} + E_{H_2} + R_{H_2}) + ... - \varphi_{nn}(S_{H_n} + E_{H_n} + R_{H_n})$$

$$= 0.$$

Equation (11) becomes:

$$\sum_{i=1}^{n} \dot{N}_{H_i} = \sum_{i=1}^{n} \Lambda_{H_i} - \sum_{i=1}^{n} d_{H_i} I_{H_i} - \sum_{i=1}^{n} \mu_{H_i} N_{H_i}. \tag{12}$$

Knowing that $-d_{H_i} N_{H_i} \le -d_{H_i} I_{H_i}$, with equation (12), we have inequality (13):

$$\sum_{i=1}^{n} \Lambda_{H_i} - \sum_{i=1}^{n} d_{H_i} N_{H_i} - \sum_{i=1}^{n} \mu_{H_i} N_{H_i} \le \sum_{i=1}^{n} \dot{N}_{H_i} \le \sum_{i=1}^{n} \Lambda_{H_i} - \sum_{i=1}^{n} \mu_{H_i} N_{H_i}, \tag{13}$$

i.e.

$$\sum_{i=1}^{n} \Lambda_{H_i} - \sum_{i=1}^{n} (d_{H_i} + \mu_{H_i}) N_{H_i} \le \sum_{i=1}^{n} \dot{N}_{H_i} \le \sum_{i=1}^{n} \Lambda_{H_i} - \sum_{i=1}^{n} \mu_{H_i} N_{H_i}. \tag{14}$$

With inequality (14) we have:

$$\sum_{i=1}^{n} \Lambda_{H_i} - n(max(d_{H_i}) + max(\mu_{H_i})) \sum_{i=1}^{n} N_{H_i} \le \sum_{i=1}^{n} \dot{N}_{H_i} \le \sum_{i=1}^{n} \Lambda_{H_i} - nmin(\mu_{H_i}) \sum_{i=1}^{n} N_{H_i}. \tag{15}$$

We set

$$\Gamma_{H_i} = \sum_{i=1}^{n} \Lambda_{H_i}, \quad H_i = \sum_{i=1}^{n} N_{H_i}, \quad \dot{H}_i = \sum_{i=1}^{n} \dot{N}_{H_i} \text{ with } H_i^0 = \sum_{i=1}^{n} N_{H_i}^0.$$

Inequality (15) becomes inequality (16):

$$\Gamma_{H_i} - n(max(d_{H_i}) + max(\mu_{H_i})) H_i \le \dot{H}_i \le \Gamma_{H_i} - nmin(\mu_{H_i}) H_i. \tag{16}$$

Equation $\dot{H}_i = \Gamma_{H_i} - n(max(d_{H_i}) + max(\mu_{H_i})) H_i$ has as solution:

$$H_i(t) = H_i^0 e^{-n(max(d_{H_i}) + max(\mu_{H_i}))t} + \frac{\Gamma_{H_i}}{n(max(d_{H_i}) + max(\mu_{H_i}))}(1 - e^{-n(max(d_{H_i}) + max(\mu_{H_i}))t})$$

and equation $\dot{H}_i = \Gamma_{H_i} - nmin(\mu_{H_i}) H_i$ has as solution

$$H_i(t) = H_i^0 e^{-nmin(\mu_{H_i})t} + \frac{\Gamma_{H_i}}{n(min\mu_{H_i})}(1 - e^{-nmin(\mu_{H_i})t})$$

Applying Grönwall's inequality, we obtain:

$$H_i^0 e^{-n(max(d_{H_i}) + max(\mu_{H_i}))t} + \frac{\Gamma_{H_i}}{n(max(d_{H_i}) + max(\mu_{H_i}))}(1 - e^{-n(max(d_{H_i}) + max(\mu_{H_i}))t})$$

$$\le H_i(t) \le H_i^0 e^{-nmin(\mu_{H_i})t} + \frac{\Gamma_{H_i}}{nmin(\mu_{H_i})}(1 - e^{-nmin(\mu_{H_i})t})$$

At the limit crossing, we have inequality (17):

$$\frac{\Gamma_{H_i}}{n(max(d_{H_i}) + max(\mu_{H_i}))} \leq \lim_{t \to +\infty} H_i(t) \leq \frac{\Gamma_{H_i}}{nmin(\mu_{H_i})}, \tag{17}$$

i.e.

$$\frac{\sum_{i=1}^{n} \Lambda_{H_i}}{n(max(d_{H_i}) + max(\mu_{H_i}))} \leq \lim_{t \to +\infty} \sum_{i=1}^{n} N_{H_i}(t) \leq \frac{\sum_{i=1}^{n} \Lambda_{H_i}}{nmin(\mu_{H_i})}. \tag{18}$$

Inequalities (8) and (18) show that mosquitoes and humans populations are bounded.

6. Disease-free equilibrium (DFE)

The dynamic of the system (1) is given by:

$$\dot{N}_{H_i} = \Lambda_{H_i} - \mu_{H_i} N_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji}(S_{H_j} + E_{H_j} + R_{H_j}) - (S_{H_i} + E_{H_i} + R_{H_i}) \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} - d_{H_i} I_{H_i} \text{ and } \dot{N}_{V_i} = \Lambda_{V_i} - \pi_{V_i} N_{V_i}.$$

Where $\pi_{V_i} = -\ln(p_i(T, RH))$, with $p_i(T, RH) = \exp(\frac{-1}{T^2 \times \beta_2 + T \times \beta_1 + \beta_0})$.

p_i is a probability if: $T^2\beta_2 + T\beta_1 + \beta_0 > 0$.
We discretize space time X such as $X = \{t_k, k = 1, 2, ..., m\}$, where m is the number of subdivisions of the period studied.
Hence, p_i is defined for all couples (T_{t_k}, RH_{t_k}) of the patch i checking inequality (19):

$$T_{t_k}^2 \beta_{2_{t_k}} + T_{t_k} \beta_{1_{t_k}} + \beta_{0_{t_k}} > 0. \tag{19}$$

With

$$\begin{cases} \beta_{0_{t_k}} = 0.00113 \times RH_{t_k}^2 - 0.158 \times RH_{t_k} - 6.61 \\ \beta_{1_{t_k}} = -2.32 \times 10^{-4} \times RH_{t_k}^2 + 0.0515 \times RH_{t_k} + 1.06 \\ \beta_{2_{t_k}} = 4 \times 10^{-6} \times RH_{t_k}^2 - 1.09 \times 10^{-3} \times RH_{t_k} - 0.0255 \end{cases}$$

The disease-free equilibrium corresponds to the state where we only have healthy individuals. We have: $E_H = I_H = R_H = E_V = I_V = 0$.
The system (1) at the DFE gives:

$$\dot{N}_{H_i} = \Lambda_{H_i} - \mu_{H_i} N_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{H_j} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} S_{H_i} \text{ and } \dot{N}_{V_i} = \Lambda_{V_i} - \pi_{V_i} N_{V_i}.$$

Humans population grows if the numbers of births and immigrants are greater than the numbers of deaths and emigrants; it decreases if the numbers of births and immigrants are lower than the numbers of deaths and emigrants; it stabilizes when they are equals. Mosquitoes population increases if the number of births is greater than the number of deaths, decreases if the number of births is less than the number of deaths and stabilizes when they are equals.

At the DFE, we have the system of equations (20):

$$\begin{cases} \Lambda_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{H_j} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} S_{H_i} - \mu_{H_i} S_{H_i} = 0, \\ \Lambda_{V_i} - \pi_{V_i} S_{V_i} = 0. \end{cases} \tag{20}$$

With $i = 1, 2, ..., n$.
I.e.

$$\begin{cases} \mu_{H_i} S_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} S_{H_i} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{H_j} = \Lambda_{H_i}, \\ \pi_{V_i} S_{V_i} = \Lambda_{V_i}. \end{cases} \tag{21}$$

In matrix form, the system (21) becomes the system of equations (22):

$$\begin{cases} D_H S_H = \Lambda_H, \\ D_V S_V = \Lambda_V, \end{cases} \tag{22}$$

where

$$D_H = \begin{pmatrix} (\mu_{H_1} + \sum_{\substack{j=1 \\ j\neq 1}}^{n} \varphi_{1j}) & -\varphi_{21} & -\varphi_{31} & \cdots & -\varphi_{n1} \\[2em] -\varphi_{12} & (\mu_{H_2} + \sum_{\substack{j=1 \\ j\neq 2}}^{n} \varphi_{2j}) & -\varphi_{32} & \cdots & -\varphi_{n2} \\[1em] \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\[1em] -\varphi_{1n} & -\varphi_{2n} & -\varphi_{3n} & \cdots & (\mu_{H_n} + \sum_{\substack{j=1 \\ j\neq n}}^{n} \varphi_{nj}) \end{pmatrix}$$

and

$$D_V = \begin{pmatrix} \pi_{V_1} & 0 & 0 & \cdots & 0 \\ 0 & \pi_{V_2} & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & \pi_{V_n} \end{pmatrix}$$

$$S_H = (S_{H_1}, S_{H_2}, ..., S_{H_n})^T \text{ and } \Lambda_H = (\Lambda_{H_1}, \Lambda_{H_2}, ..., \Lambda_{H_n})^T.$$

$$S_V = (S_{V_1}, S_{V_2}, ..., S_{V_n})^T \text{ and } \Lambda_V = (\Lambda_{V_1}, \Lambda_{V_2}, ..., \Lambda_{V_n})^T.$$

All non-diagonal elements of D_H being negative and all the principal minors strictly positive, D_H is then a reversible $M - Matrix$, and $D_H^{-1} \geq 0$. D_V being a diagonal matrix with positive values, it is therefore reversible. So the system (20) admits per unit of time, a unique solution $S^0 = ((S_{H_1}^0, S_{H_2}^0, ..., S_{H_n}^0)^T, (S_{V_1}^0, S_{V_2}^0, ..., S_{V_n}^0)^T) = (D_H^{-1}\Lambda_H, D_V^{-1}\Lambda_V) > 0$. Therefore, at the Disease-free equilibrium, the system (1) admits per unit of time, a single solution $P_i^0 = (S_{H_i}^0, 0, 0, 0, S_{V_i}^0, 0, 0)$.

7. Basic reproduction number R_0

Proposition 7.1. *The basic reproduction number*

$$R_0 = \frac{1}{N_{H_i}^*} \times \sqrt{\frac{\delta_{H_i} \delta_{V_i} k_i \beta_i f_i w_i S_{H_i}^* S_{V_i}^*}{\pi_{0_i}(\sum_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i})(\alpha_{H_i} + d_{H_i} + \mu_{H_i})(\delta_{V_i} + \pi_{V_i})}}$$

Where $\pi_{0_i} = -\ln(p_i(T_0, RH_0))$ is the average mortality rate of mosquitoes in the patch i, from calibrations of T and RH presented in sections 4.3 and 4.4.

Proof. The *Next generation method* proposed by Diekmann et al. [2], then Driessche and Watmough [3] is the one chosen for our demonstration.
We use here the average mosquitoes mortality rate in the patch i, $\pi_{0_i} = -\ln(p_i(T_0, RH_0))$.

$$F = \begin{pmatrix} 0 & 0 & 0 & k_i\beta_i \dfrac{S_{H_i}^*}{N_{H_i}^*} \\[1em] 0 & 0 & f_i w_i \dfrac{S_{V_i}^*}{N_{H_i}^*} & 0 \\[1em] 0 & 0 & 0 & 0 \\[0.5em] 0 & 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} -(\sum_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) & 0 & 0 & 0 \\[1.5em] 0 & -(\delta_{V_i} + \pi_{0_i}) & 0 & 0 \\[0.5em] \delta_{H_i} & 0 & -(\alpha_{H_i} + d_{H_i} + \mu_{H_i}) & 0 \\[0.5em] 0 & \delta_{V_i} & 0 & -\pi_{0_i} \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} -\dfrac{1}{(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij}+\delta_{H_i}+\mu_{H_i})} & 0 & 0 & 0 \\ 0 & -\dfrac{1}{(\delta_{V_i}+\pi_{0_i})} & 0 & 0 \\ -\dfrac{\delta_{H_i}}{(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij}+\delta_{H_i}+\mu_{H_i})(\alpha_{H_i}+d_{H_i}+\mu_{H_i})} & 0 & -\dfrac{1}{(\alpha_{H_i}+d_{H_i}+\mu_{H_i})} & 0 \\ 0 & -\dfrac{\delta_{V_i}}{\pi_{0_i}(\delta_{V_i}+\pi_{0_i})} & 0 & -\dfrac{1}{\pi_{0_i}} \end{pmatrix}$$

$$F * V^{-1} = \begin{pmatrix} 0 & -\dfrac{\delta_{V_i}k_i\beta_i S_{H_i}^*}{\pi_{0_i}(\delta_{V_i}+\pi_{0_i})N_{H_i}^*} & 0 & -\dfrac{k_i\beta_i S_{H_i}^*}{\pi_{0_i}N_{H_i}^*} \\ -\dfrac{\delta_{H_i}f_i w_i S_{V_i}^*}{(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij}+\delta_{H_i}+\mu_{H_i})(\alpha_{H_i}+d_{H_i}+\mu_{H_i})N_{H_i}^*} & 0 & -\dfrac{f_i w_i S_{V_i}^*}{(\alpha_{H_i}+d_{H_i}+\mu_{H_i})N_{H_i}^*} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P(X) = |XI - FV^{-1}|$$

$$P(X) = \begin{vmatrix} X & -\dfrac{\delta_{V_i}k_i\beta_i S_{H_i}^*}{\pi_{0_i}(\delta_{V_i}+\pi_{0_i})N_{H_i}^*} & 0 & -\dfrac{k_i\beta_i S_{H_i}^*}{\pi_{0_i}N_{H_i}^*} \\ -\dfrac{\delta_{H_i}f_i w_i S_{V_i}^*}{(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij}+\delta_{H_i}+\mu_{H_i})(\alpha_{H_i}+d_{H_i}+\mu_{H_i})N_{H_i}^*} & X & -\dfrac{f_i w_i S_{V_i}^*}{(\alpha_{H_i}+d_{H_i}+\mu_{H_i})N_{H_i}^*} & 0 \\ 0 & 0 & X & 0 \\ 0 & 0 & 0 & X \end{vmatrix}$$

$$P(X) = X^4 - \dfrac{\delta_{H_i}\delta_{V_i}k_i\beta_i f_i w_i S_{H_i}^* S_{V_i}^*}{\pi_{0_i}(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij}+\delta_{H_i}+\mu_{H_i})(\alpha_{H_i}+d_{H_i}+\mu_{H_i})(\delta_{V_i}+\pi_{0_i})N_{H_i}^{*2}}X^2$$

We obtain:

$$R_0 = \rho(FV^{-1}) = \dfrac{1}{N_{H_i}^*} \times \sqrt{\dfrac{\delta_{H_i}\delta_{V_i}k_i\beta_i f_i w_i S_{H_i}^* S_{V_i}^*}{\pi_{0_i}(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij}+\delta_{H_i}+\mu_{H_i})(\alpha_{H_i}+d_{H_i}+\mu_{H_i})(\delta_{V_i}+\pi_{0_i})}}$$

8. Global asymptotic stability at the disease-free equilibrium (DFE)

In this section, we have to show that when $R_0 < 1$, there is a positively invariant set where our system (1) is globally asymptotically stable at the DFE.

We consider, as mosquitoes mortality rate in the patch i, the average mosquitoes mortality rate π_{0_i} for the patch i, from calibrations of T and RH presented in sections 4.3 and 4.4. We set $\pi_{V_i} = \pi_{0_i} = -ln(p_i(T_0, RH_0))$.

The system (1) can be rewritten in linear form as well. We obtain the system (23)

$$\dot{x} = A(x)x + b(x) \leftrightarrow \begin{cases} \dot{x}_{S_i} = A_{S_i}(x).x_{S_i} + A_{S_i I_i}(x).x_{I_i} + b_{S_i}(x), \\ \dot{x}_{I_i} = A_{I_i}(x).x_{I_i}. \end{cases} \tag{23}$$

The system (23) is defined on the positively invariant orthant set $\Omega \subset \mathbb{R}_+^7$. $x_{S_i} = (S_{H_i}; S_{V_i}; R_{H_i})$ represents susceptible and recovered populations and $x_{I_i} = (E_{H_i}; E_{V_i}; I_{H_i}; I_{V_i})$, infected and infectious populations.

A_{S_i} matrix can be rewritten as

$$A_{S_i}(x) = \begin{pmatrix} -(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij}+\mu_{H_i}+k_i\beta_i \dfrac{I_{V_i}}{N_{H_i}}) & 0 & \epsilon_{H_i} \\ 0 & -(f_i w_i \dfrac{I_{H_i}}{N_{H_i}}+\pi_{V_i}) & 0 \\ 0 & 0 & -(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij}+\mu_{H_i}) \end{pmatrix}$$

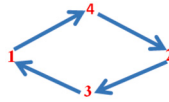$A_{S_i I_i}$ matrix can be rewritten as

Fig. 7. Graph associated to $A_I(x)$ matrix.

$$A_{S_i I_i}(x) = \begin{pmatrix} 0 & 0 & 0 & -k_i \beta_i \frac{I_{V_i}}{N_{H_i}} \\ 0 & 0 & -f_i w_i \frac{I_{H_i}}{N_{H_i}} & 0 \\ 0 & 0 & \alpha_{H_i} & 0 \end{pmatrix}$$

$A_{I_i}(x)$ matrix can be rewritten as $A_{I_i}(x) = \begin{pmatrix} A_{I_{E_i}}(x) & A_{I_{E_i,I_i}}(x) \\ A_{I_{I_i,E_i}}(x) & A_{I_I}(x) \end{pmatrix}$, where:

$$A_{I_{E_i}}(x) = \begin{pmatrix} -(\sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) & 0 \\ 0 & -(\delta_{V_i} + \pi_{V_i}) \end{pmatrix},$$

$$A_{I_{E_i,I_i}}(x) = \begin{pmatrix} 0 & k_i \beta_i \frac{S_{H_i}}{N_{H_i}} \\ f_i w_i \frac{S_{V_i}}{N_{H_i}} & 0 \end{pmatrix},$$

$$A_{I_{I_i,E_i}}(x) = \begin{pmatrix} \delta_{H_i} & 0 \\ 0 & \delta_{V_i} \end{pmatrix}, \qquad A_{I_{I_i}}(x) = \begin{pmatrix} -(\alpha_{H_i} + d_{H_i} + \mu_{H_i}) & 0 \\ 0 & -\pi_{V_i} \end{pmatrix}.$$

$\forall x \in \mathbb{R}^7_+$, matrices $A(x), A_{S_i}(x), A_{I_i}(x)$ are Metzler matrices.

Theorem 8.1. *Let $\tau = \frac{\mu_{H_i}}{d_{H_i} + \mu_{H_i}}$ $(\tau \leq 1)$ and $\tilde{\Omega} = \{x \in \Omega : x \neq 0\}$ a positively invariant set. When $R_0 < \tau$, the system (1) at the DFE is globally asymptotically stable (GAS) in $\{x \in \tilde{\Omega} : x_I = 0\}$.*

Proof. We use Theorem 4.3 of Kamgang and Sallet [5] which establishes global asymptotic stability by respecting 5 conditions $(H_1 - H_5)$. We have to prove that for our system (1), these 5 conditions are satisfied when $R_0 < \tau$.

(H_1) Our system (1) must be dissipative on Ω.

Our system (1) is defined on the positively invariant orthant set $\Omega \subset \mathbb{R}^7_+$. It is a dissipative system on Ω.

(H_2) Show that when there is no disease, the population will stabilize at the DFE.

The subsystem $\dot{x}_S = A_S(x_S, 0)(x - x_S^*)$ has the linear form defined by the system of equations (24):

$$\begin{cases} \dot{S}_{H_i} = (\Lambda_{H_i} + \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{H_j}) - (\sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} + \mu_{H_i}) S_{H_i} + \epsilon_{H_i} R_{H_i}, \\ \dot{S}_{V_i} = \Lambda_{V_i} - \pi_{V_i} S_{V_i}. \end{cases} \tag{24}$$

This subsystem is globally asymptotically stable at the DFE $\begin{pmatrix} \dfrac{(\Lambda_{H_i} + \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{H_j})}{(\sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} + \mu_{H_i})}, \dfrac{\Lambda_{V_i}}{\pi_{V_i}} \end{pmatrix}$. The DFE satisfies the condition (H_2).

(H_3) All non-diagonal elements of $A_I(x)$ matrix have to be non-positive and there is no block of compartments which does not interact with others.

$A_I(x)$ matrix presented in (23) is a Metzler matrix, which can be represented at Fig. 7, where each node represents the different phases of infection.

The two properties necessary to satisfy the condition (H_3) are respected: all non-diagonal elements of $A_I(x)$ matrix non-positive and the graph associated with the $A_I(x)$ matrix is strongly connected, which shows that the $A_I(x)$ matrix is irreducible.

(H_4) There is an upper-bound for $A_I(x)$ matrix.

Knowing that $\frac{1}{N_{H_i}^\phi} \geq \frac{1}{N_{H_i}}, S_{H_i}^* \geq S_{H_i}$ and $S_{V_i}^* \geq S_{V_i}$, we get as the upper bound of $A_I(x)$,

$$\bar{A}_I(x) = \begin{pmatrix} M & N \\ P & Q \end{pmatrix}$$

Where

$$M = \begin{pmatrix} -(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) & 0 \\ 0 & -(\delta_{V_i} + \pi_{V_i}) \end{pmatrix}, \ N = \begin{pmatrix} 0 & k_i \beta_i \dfrac{S_{H_i}^*}{N_{H_i}^\phi} \\ f_i w_i \dfrac{S_{V_i}^*}{N_{H_i}^\phi} & 0 \end{pmatrix},$$

$$P = \begin{pmatrix} \delta_{H_i} & 0 \\ 0 & \delta_{V_i} \end{pmatrix} \text{ and } Q = \begin{pmatrix} -(\alpha_{H_i} + d_{H_i} + \mu_{H_i}) & 0 \\ 0 & -\pi_{V_i} \end{pmatrix}.$$

$A_I(x) \leq \bar{A}_I(x)$ for all $x \in \Omega$ and $A_I(x^*) = \bar{A}_I(x)$ for all $x \in \tilde{\Omega}$. The condition (H_4) is satisfied.

(H_5) **Show that** $\alpha(\bar{A}_I) < 0$.

$$\alpha(\bar{A}_I) < 0 \iff \alpha(Q - PM^{-1}N) < 0.$$

Let: $T = Q - PM^{-1}N$

$$PM^{-1} = \begin{pmatrix} -\dfrac{\delta_{H_i}}{\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}} & 0 \\ 0 & -\dfrac{\delta_{V_i}}{\delta_{V_i} + \pi_{V_i}} \end{pmatrix}$$

$$PM^{-1}N = \begin{pmatrix} 0 & -\dfrac{k_i \beta_i \delta_{H_i} S_{H_i}^*}{(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) N_{H_i}^\phi} \\ -\dfrac{f_i w_i \delta_{V_i} S_{V_i}^*}{(\delta_{V_i} + \pi_{V_i}) N_{H_i}^\phi} & 0 \end{pmatrix}$$

$$T = \begin{pmatrix} -(\alpha_{H_i} + d_{H_i} + \mu_{H_i}) & -\dfrac{k_i \beta_i \delta_{H_i} S_{H_i}^*}{(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) N_{H_i}^\phi} \\ -\dfrac{f_i w_i \delta_{V_i} S_{V_i}^*}{(\delta_{V_i} + \pi_{V_i}) N_{H_i}^\phi} & -\pi_{V_i} \end{pmatrix}$$

T is stable if T_{11} stable and $(T_{22} - T_{21}T_{11}^{-1}T_{12})$ stable.
$T_{11} = -(\alpha_{H_i} + d_{H_i} + \mu_{H_i}) < 0$; T_{11} is stable.

Let $\chi = T_{22} - T_{21}T_{11}^{-1}T_{12} = -\pi_{V_i} + \dfrac{f_i w_i k_i \beta_i \delta_{H_i} \delta_{V_i} S_{H_i}^* S_{V_i}^*}{(\delta_{V_i} + \pi_{V_i})(\alpha_{H_i} + d_{H_i} + \mu_{H_i})(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) N_{H_i}^{\phi 2}}$.

χ is stable if $\chi < 0$.

$$\chi < 0 \Leftrightarrow \dfrac{f_i w_i k_i \beta_i \delta_{H_i} \delta_{V_i} S_{H_i}^* S_{V_i}^*}{\pi_{V_i}(\delta_{V_i} + \pi_{V_i})(\alpha_{H_i} + d_{H_i} + \mu_{H_i})(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) N_{H_i}^{\phi 2}} < 1$$

$$N_{H_i}^* = \dfrac{N_{H_i}}{\mu_{H_i}} \text{ and } N_{H_i}^\phi = \dfrac{N_{H_i}}{(d_{H_i} + \mu_{H_i})}, \qquad \text{i.e.} \quad N_{H_i}^\phi = \dfrac{\mu_{H_i}}{d_{H_i} + \mu_{H_i}} N_{H_i}^*$$

Hence, we have: $\dfrac{f_i w_i k_i \beta_i \delta_{H_i} \delta_{V_i} S_{H_i}^* S_{V_i}^*}{\pi_{V_i}(\delta_{V_i} + \pi_{V_i})(\alpha_{H_i} + d_{H_i} + \mu_{H_i})(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) N_{H_i}^{*2} (\frac{\mu_{H_i}}{d_{H_i} + \mu_{H_i}})^2} < 1$.

I.e. $\dfrac{f_i w_i k_i \beta_i \delta_{H_i} \delta_{V_i} S_{H_i}^* S_{V_i}^*}{\pi_{V_i}(\delta_{V_i} + \pi_{V_i})(\alpha_{H_i} + d_{H_i} + \mu_{H_i})(\sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{ij} + \delta_{H_i} + \mu_{H_i}) N_{H_i}^{*2}} < (\dfrac{\mu_{H_i}}{d_{H_i} + \mu_{H_i}})^2$.

I.e. $R_0^2 < (\dfrac{\mu_{H_i}}{d_{H_i} + \mu_{H_i}})^2$.

Thus, $\alpha(\bar{A}_I) < 0 \Leftrightarrow R_0 < \dfrac{\mu_{H_i}}{d_{H_i} + \mu_{H_i}}$, which is the starting hypothesis. So, condition (H_5) is satisfied.

The conditions mentioned in Theorem 4.3 of Kamgang and Sallet [5] being satisfied, we deduce our system (1) is globally asymptotically stable (GAS) at the DFE when $R_0 < \dfrac{\mu_{H_i}}{d_{H_i} + \mu_{H_i}}$.
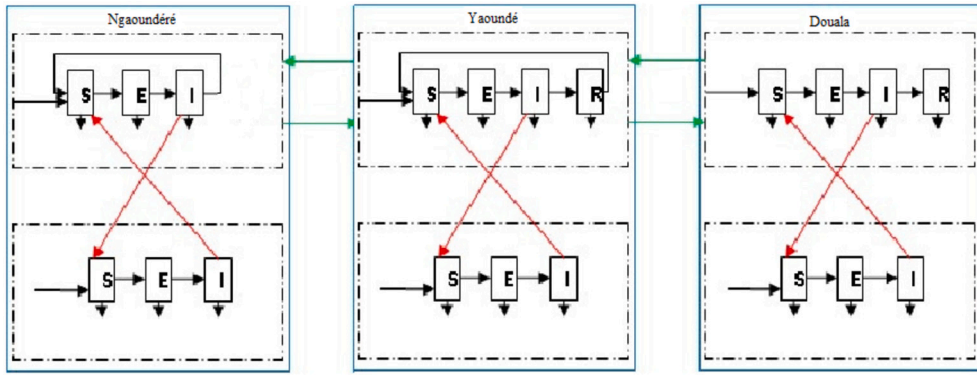
Fig. 8. Representation of the compartmental models of Douala (right), Yaounde (middle) and Ngaoundere (left).

9. Numerical simulation

9.1. Environment

We consider three cities (Douala, Yaounde and Ngaoundere) of Cameroon country. The report of the National Strategic Plan for fighting against Malaria of the Ministry of Public Health in Cameroon [12] reveals that the transmission of malaria in Douala is continuous over the year, individuals who reside there benefit a permanent immunity after a time; we model this city by a SEIR model. The transmission is continuous for a major part of the year in Yaounde, residents get a temporary immunity after a time; we model by a SEIRS model (as defined by the system (1)). The malaria transmission is seasonal in Ngaoundere, residents have no immunity; we model by a SEIS model. Fig. 8 shows three compartment models for Douala, Yaounde and Ngaoundere cities.

The SEIR model corresponding to the city of Douala can be represented by the system (25):

$$
\begin{cases}
\dot{S}_{H_i} = \Lambda_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{Hj} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} S_{Hi} - k_i \beta_i \frac{S_{Hi}}{N_{Hi}} I_{Vi} - \mu_{Hi} S_{Hi}, \\
\dot{E}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} E_{Hj} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} E_{Hi} + k_i \beta_i \frac{S_{Hi}}{N_{Hi}} I_{Vi} - \delta_{Hi} E_{Hi} - \mu_{Hi} E_{Hi}, \\
\dot{I}_{H_i} = \delta_{Hi} E_{Hi} - \alpha_{Hi} I_{Hi} - d_{Hi} I_{Hi} - \mu_{Hi} I_{Hi}, \\
\dot{R}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} R_{Hj} + \alpha_{Hi} I_{Hi} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} R_{Hi} - \mu_{Hi} R_{Hi}, \\
\dot{S}_{V_i} = \Lambda_{V_i} - f_i \omega_i \frac{S_{Vi}}{N_{Hi}} I_{Hi} - \pi_{V_i} S_{Vi}, \\
\dot{E}_{V_i} = f_i \omega_i \frac{S_{Vi}}{N_{Hi}} I_{Hi} - \delta_{Vi} E_{Vi} - \pi_{V_i} E_{Vi}, \\
\dot{I}_{V_i} = \delta_{Vi} E_{Vi} - \pi_{V_i} I_{Vi}.
\end{cases}
\tag{25}
$$

Where π_{V_i} is the mortality rate including climatic factors defined at the equation (2).
Initial conditions are $(S_{H_i}(0), E_{H_i}(0), I_{H_i}(0), R_{H_i}(0), S_{V_i}(0), E_{V_i}(0), I_{V_i}(0))$.

The SEIS model corresponding to the city of Ngaoundere can be represented by the system (26):

$$
\begin{cases}
\dot{S}_{H_i} = \Lambda_{H_i} + \rho_{Hi} I_{Hi} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{Hj} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} S_{Hi} - k_i \beta_i \frac{S_{Hi}}{N_{Hi}} I_{Vi} - \mu_{Hi} S_{Hi}, \\
\dot{E}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} E_{Hj} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} E_{Hi} + k_i \beta_i \frac{S_{Hi}}{N_{Hi}} I_{Vi} - \delta_{Hi} E_{Hi} - \mu_{Hi} E_{Hi}, \\
\dot{I}_{H_i} = \delta_{Hi} E_{Hi} - d_{Hi} I_{Hi} - \mu_{Hi} I_{Hi} - \rho_{Hi} I_{Hi}, \\
\dot{S}_{V_i} = \Lambda_{V_i} - f_i \omega_i \frac{S_{Vi}}{N_{Hi}} I_{Hi} - \pi_{V_i} S_{Vi}, \\
\dot{E}_{V_i} = f_i \omega_i \frac{S_{Vi}}{N_{Hi}} I_{Hi} - \delta_{Vi} E_{Vi} - \pi_{V_i} E_{Vi}, \\
\dot{I}_{V_i} = \delta_{Vi} E_{Vi} - \pi_{V_i} I_{Vi}.
\end{cases}
\tag{26}
$$

Where π_{V_i} is the mortality rate including climatic factors defined at the equation (2).
At initial conditions, we have $(S_{H_i}(0), E_{H_i}(0), I_{H_i}(0), S_{V_i}(0), E_{V_i}(0), I_{V_i}(0))$.

9.2. Data migration

The report on the General Population Census of Cameroon [24] shows a population of 1,907,479 inhabitants in Douala (N_{Hdla} = 1907479), 1,817,524 inhabitants in Yaounde (N_{Hyde} = 1817524). Ngaoundere has an estimated population of 262,747 inhabitants (N_{Hnd} = 262747) [25]. In 2019, we carried out an investigation in Douala, Yaounde and Ngaoundere inside the railway company *Camrail* and some travel bus agencies like *Garanti express*, *Touristique express* and *Buca voyages*. We got the following information about daily migrations:

$$\phi_{Yaounde \longrightarrow Douala} = \frac{496 + 2700}{N_{Hyde}} = \frac{3196}{1817524} = 0.0018 = 0.18\%;$$

$$\phi_{Douala \longrightarrow Yaounde} = \frac{519 + 2800}{N_{Hdla}} = \frac{3319}{1907479} = 0.0017 = 0.17\%;$$

$$\phi_{Yaounde \longrightarrow Ngaoundere} = \frac{788}{N_{Hyde}} = \frac{788}{1817524} = 0.00043 = 0.043\%;$$

$$\phi_{Ngaoundere \longrightarrow Yaounde} = \frac{860}{N_{Hndere}} = \frac{860}{262747} = 0.0033 = 0.33\%.$$

9.3. Epidemiological data

Table 3 shows monthly temperatures (minimum) and relative humidity (maximum) for three cities (Douala, Yaounde and Ngaoundere) for years 2017 and 2018.

Table 4 (respectively Table 5 and Table 6) shows parameters of Yaounde (respectively Douala and Ngaoundere).

9.4. Influence of climatic factors

We illustrate influence of climatic factors by comparing a model without taking into account climatic factors against a model including climatic factors.

If we neglect climatic factors, our SEIR model could be represented by the system of equations (27):

$$
\begin{cases}
\dot{S}_{H_i} = \Lambda_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{Hj} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} S_{Hi} - k_i \beta_i \frac{S_{Hi}}{N_{Hi}} I_{Vi} - \mu_{Hi} S_{Hi}, \\
\dot{E}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} E_{Hj} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} E_{Hi} + k_i \beta_i \frac{S_{Hi}}{N_{Hi}} I_{Vi} - \delta_{Hi} E_{Hi} - \mu_{Hi} E_{Hi}, \\
\dot{I}_{H_i} = \delta_{Hi} E_{Hi} - \alpha_{Hi} I_{Hi} - d_{Hi} I_{Hi} - \mu_{Hi} I_{Hi}, \\
\dot{R}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} R_{Hj} + \alpha_{Hi} I_{Hi} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} R_{Hi} - \mu_{Hi} R_{Hi}, \\
\dot{S}_{V_i} = \Lambda_{V_i} - f_i \omega_i \frac{S_{Vi}}{N_{Hi}} I_{Hi} - \mu_{Vi} S_{Vi}, \\
\dot{E}_{V_i} = f_i \omega_i \frac{S_{Vi}}{N_{Hi}} I_{Hi} - \delta_{Vi} E_{Vi} - \mu_{Vi} E_{Vi}, \\
\dot{I}_{V_i} = \delta_{Vi} E_{Vi} - \mu_{Vi} I_{Vi}.
\end{cases}
\tag{27}
$$

Initial conditions are $(S_{H_i}(0), E_{H_i}(0), I_{H_i}(0), R_{H_i}(0), S_{V_i}(0), E_{V_i}(0), I_{V_i}(0))$.

If we neglect climatic factors, our SEIRS model could be represented by the system (28):

$$
\begin{cases}
\dot{S}_{H_i} = \Lambda_{H_i} + \epsilon_{H_i} R_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{H_j} - S_{H_i} \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} - k_i \beta_i \frac{S_{H_i}}{N_{H_i}} I_{V_i} - \mu_{H_i} S_{H_i}, \\
\dot{E}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} E_{H_j} - E_{H_i} \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} + k_i \beta_i \frac{S_{H_i}}{N_{H_i}} I_{V_i} - \delta_{H_i} E_{H_i} - \mu_{H_i} E_{H_i}, \\
\dot{I}_{H_i} = \delta_{H_i} E_{H_i} - \alpha_{H_i} I_{H_i} - d_{H_i} I_{H_i} - \mu_{H_i} I_{H_i}, \\
\dot{R}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} R_{H_j} + \alpha_{H_i} I_{H_i} - R_{H_i} \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} - \epsilon_{H_i} R_{H_i} - \mu_{H_i} R_{H_i}, \\
\dot{S}_{V_i} = \Lambda_{V_i} - f_i \omega_i \frac{S_{V_i}}{N_{H_i}} I_{H_i} - \mu_{V_i} S_{V_i}, \\
\dot{E}_{V_i} = f_i \omega_i \frac{S_{V_i}}{N_{H_i}} I_{H_i} - \delta_{V_i} E_{V_i} - \mu_{V_i} E_{V_i}, \\
\dot{I}_{V_i} = \delta_{V_i} E_{V_i} - \mu_{V_i} I_{V_i}.
\end{cases}
\tag{28}
$$

With initial conditions $(S_{H_i}(0), E_{H_i}(0), I_{H_i}(0), R_{H_i}(0), S_{V_i}(0), E_{V_i}(0), I_{V_i}(0))$.

Table 3
Monthly temperatures (minimum) and relative humidity (maximum) for Douala, Yaounde and Ngaoundere for the years 2017 and 2018.

Source: www.wunderground.com.

Climatic factors	January	February	March	April
Temperature (min) of Douala	25(2017)	24(2017)	24(2017)	24(2017)
	24(2018)	24(2018)	23(2018)	24(2018)
Relative humidity (max) of Douala	99(2017)	98(2017)	98(2017)	98(2017)
	99(2018)	97(2018)	97(2018)	98(2018)
Temperature (min) of Yaounde	21(2017)	21(2017)	20(2017)	20(2017)
	20(2018)	20(2018)	21(2018)	20(2018)
Relative humidity (max) of Yaounde	94(2017)	93(2017)	94(2017)	94(2017)
	95(2018)	95(2018)	94(2018)	95(2018)
Temperature (min) of Ngaoundere	14(2017)	17(2017)	20(2017)	20(2017)
	10(2018)	12(2018)	16(2018)	19(2018)
Relative humidity (max) of Ngaoundere	58(2017)	45(2017)	76(2017)	87(2017)
	71(2018)	65(2018)	77(2018)	94(2018)

Climatic factors	May	June	July	August
Temperature (min) of Douala	25(2017)	24(2017)	24(2017)	23(2017)
	24(2018)	24(2018)	23(2018)	22(2018)
Relative humidity (max) of Douala	99(2017)	98(2017)	100(2017)	99(2017)
	99(2018)	97(2018)	100(2018)	100(2018)
Temperature (min) of Yaounde	21(2017)	21(2017)	20(2017)	20(2017)
	20(2018)	20(2018)	20(2018)	20(2018)
Relative humidity (max) of Yaounde	94(2017)	93(2017)	94(2017)	94(2017)
	95(2018)	95(2018)	95(2018)	94(2018)
Temperature (min) of Ngaoundere	14(2017)	17(2017)	17(2017)	17(2017)
	10(2018)	12(2018)	18(2018)	18(2018)
Relative humidity (max) of Ngaoundere	58(2017)	45(2017)	96(2017)	98(2017)
	71(2018)	65(2018)	99(2018)	100(2018)

Climatic factors	September	October	November	December
Temperature (min) of Douala	23(2017)	23(2017)	23(2017)	21(2017)
	23(2018)	23(2018)	24(2018)	24(2018)
Relative humidity (max) of Douala	100(2017)	100(2017)	99(2017)	100(2017)
	100(2018)	100(2018)	100(2018)	99(2018)
Temperature (min) of Yaounde	20(2017)	20(2017)	20(2017)	19(2017)
	20(2018)	20(2018)	20(2018)	20(2018)
Relative humidity (max) of Yaounde	94(2017)	96(2017)	96(2017)	95(2017)
	97(2018)	97(2018)	98(2018)	96(2018)
Temperature (min) of Ngaoundere	17(2017)	17(2017)	14(2017)	13(2017)
	17(2018)	17(2018)	14(2018)	11(2018)
Relative (max) of Ngaoundere	98(2017)	94(2017)	88(2017)	82(2017)
	100(2018)	99(2018)	90(2018)	78(2018)

Table 4
Parameters of Yaounde.

Parameters	Λ_{Hyde}	Λ_{Vyde}	μ_{Hyde}	μ_{Vyde}	b_{Hyde}	ε_{Hyde}
Values	57000	6500	0.009	0.033	0.044	0.002
References	[13]		[14]	[15]	[16]	[17]

Parameters	ω_{yde}	f_{yde}	δ_{Hyde}	δ_{Vyde}	α_{Hyde}	d_{Hyde}
Values	0.11	0.01	0.09	0.091	0.100	3.01%
References	[18]	[16]	[17]	[17]	[17]	[19]

Table 5
Parameters of Douala.

Parameters	Λ_{Hdla}	Λ_{Vdla}	μ_{Hdla}	μ_{Vdla}	b_{Hdla}	ω_{dla}
Values	60000	13000	0.012	0.033	3.94	0.35
References	[13]		[14]	[15]	[20]	[18]

Parameters	f_{dla}	δ_{Hdla}	δ_{Vdla}	α_{Hdla}	d_{Hdla}
Values	0.022	0.09	0.083	0.10	3.01%
References	[20]	[17]	[17]	[17]	[19]

Table 6
Parameters of Ngaoundere.

Parameters	Λ_{Hnd}	Λ_{Vnd}	μ_{Hnd}	μ_{Vnd}	b_{Hnd}	ω_{nd}
Values	7000	6500	0.021	0.033	0.63	0.11
References	[22]		[22]	[15]	[21]	[18]
Parameters	f_{nd}	δ_{Hnd}	δ_{Vnd}		ρ_{Hnd}	d_{Hnd}
Values	0.01	0.10	0.091		0.008	3.01%
References	[21]	[17]	[17]		[17]	[19]
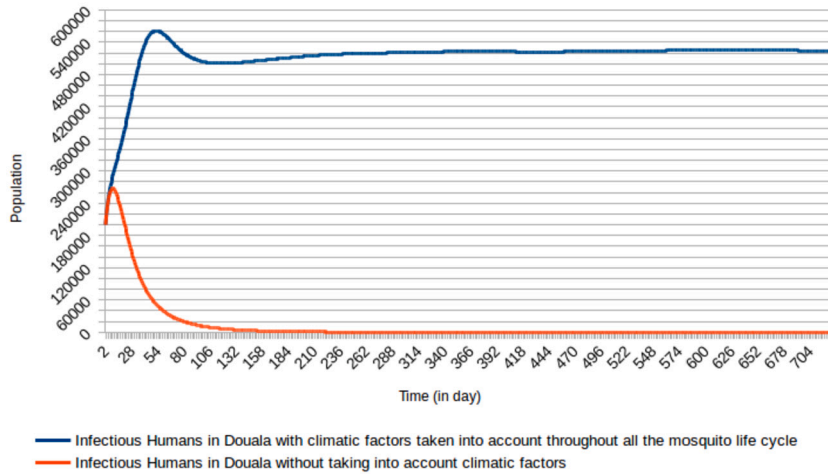
Fig. 9. Comparison of the evolution in Douala of infectious humans including climatic factors with their evolution without considering climatic factors.

Ignoring climatic factors given as a SEIS model, the system (29):

$$\begin{cases} \dot{S}_{H_i} = \Lambda_{H_i} + \rho_{Hi} I_{H_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} S_{H_j} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} S_{H_i} - k_i \beta_i \frac{S_{H_i}}{N_{H_i}} I_{Vi} - \mu_{H_i} S_{H_i}, \\ \dot{E}_{H_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ji} E_{H_j} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{ij} E_{H_i} + k_i \beta_i \frac{S_{H_i}}{N_{H_i}} I_{Vi} - \delta_{H_i} E_{H_i} - \mu_{Hi} E_{H_i}, \\ \dot{I}_{H_i} = \delta_{H_i} E_{H_i} - d_{H_i} I_{H_i} - \mu_{H_i} I_{H_i} - \rho_{Hi} I_{Hi}, \\ \dot{S}_{V_i} = \Lambda_{V_i} - f_i \omega_i \frac{S_{V_i}}{N_{H_i}} I_{Hi} - \mu_{V_i} S_{V_i}, \\ \dot{E}_{V_i} = f_i \omega_i \frac{S_{V_i}}{N_{H_i}} I_{Hi} - \delta_{V_i} E_{V_i} - \mu_{V_i} E_{V_i}, \\ \dot{I}_{V_i} = \delta_{Vi} E_{V_i} - \mu_{V_i} I_{V_i}. \end{cases} \qquad (29)$$

With initial conditions $(S_{H_i}(0), E_{H_i}(0), I_{H_i}(0), S_{V_i}(0), E_{V_i}(0), I_{V_i}(0))$.
Using data described in section 9.2, we have Figs. 9, 10, 11.

We observe, with Fig. 9, that ignoring climatic factors in Douala displays a number of infectious humans below 20,000 after 78 days and stabilizes around 0 (zero) after approximately 601 days. Consideration of climatic factors Douala shows a number of infectious humans stabilizing below 522,000 after 333 days.

In Yaounde (Fig. 10), we have a number of infectious humans below 10,000 after 55 days which stabilizes around 0 after 366 days in the case where we do not consider climatic factors. Taking into account climatic factors shows a number of infectious humans below 20,000 after 64 days which stabilizes around 10,000 after 363 days.

In Ngaoundere (Fig. 11), taking into consideration climatic factors shows a number of infectious humans oscillating below 115,000 with minimum values of 39,326 and 41,299 per 100th (respectively 438th) day and maximum values of 59,711, 114,986 and 106,866 on the 29th (respectively 291st and 668th) day. Ignoring climatic factors, we have a number of infectious humans falling below 5,000 after 100 days and stabilizing around 0 after 401 days.

9.5. Comparison between our results and real data from the hospital: case of Ngaoundere

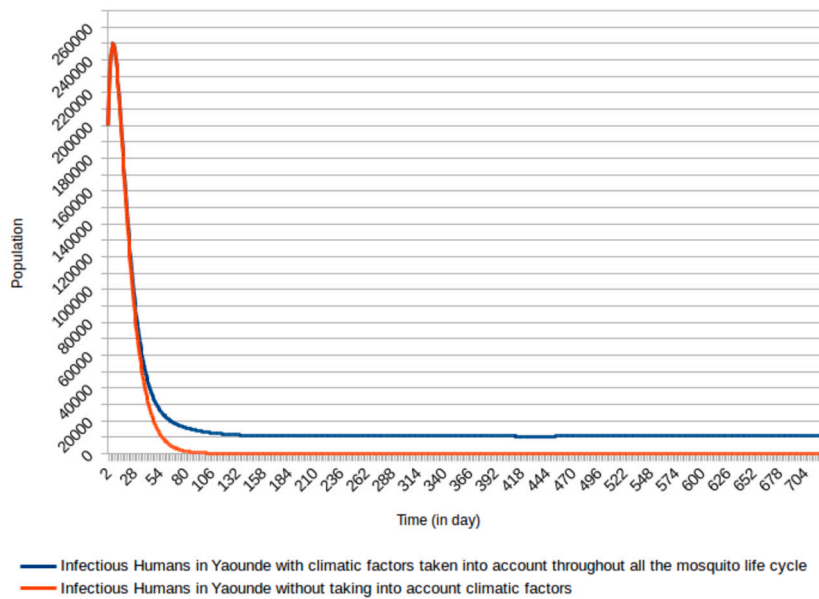Table 7 shows data from the *Protestant Hospital of Ngaoundere*.

Fig. 10. Comparison of the evolution in Yaounde of infectious humans including climatic factors with their evolution without considering climatic factors.
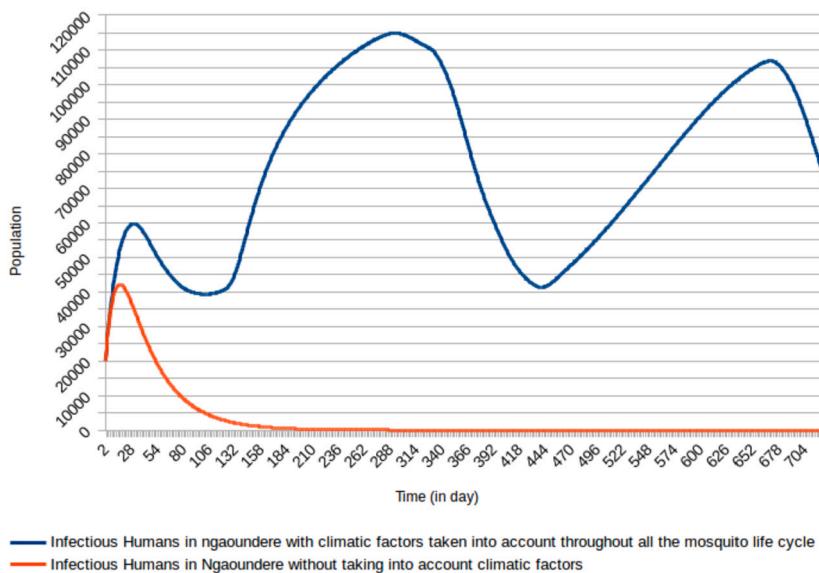

Fig. 11. Comparison of the evolution in Ngaoundere of infectious humans including climatic factors with their evolution without considering climatic factors.

Fig. 12 shows that the number of infectious humans listed among patients fluctuates over the year and is maximum in October. Results with our modelling including climatic factors show that the number of infectious humans also fluctuates over the year and is the maximum on the 291st day of the year (Fig. 11), which also corresponds to a day in October.

10. Conclusion

In population dynamics, analyze the model stability is important, because it allows to validate the modelling, avoid artefacts and also to ensure the robustness of the modelling. This is the reason why we studied the stability of a metapopulation model. This study allowed us to define the conditions of the global asymptotic stability of our model. Our work constitutes an advance in the understanding the dynamics spread of malaria in a metapopulation context, including climatic factors. Our analysis focused on the asymptotic stability in the case of disease-free equilibrium (DFE). It would also be interesting to carry out a mathematical analysis of our model in an endemic situation (EE).

Table 7
Data from the Protestant Hospital of Ngaoundere:
Humans with positive malaria (infectious) for the
year 2013. Noubissi [23].

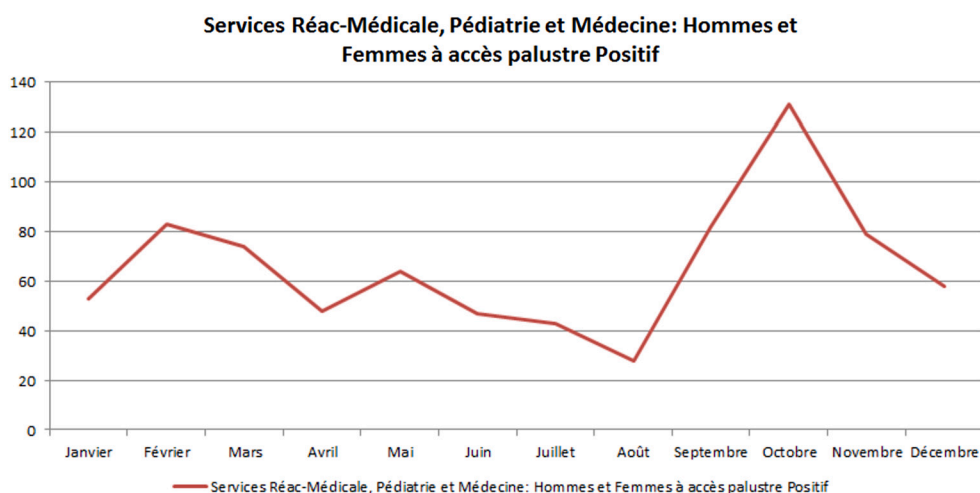| Month	Patients	Infectious patients
January	125	53
February	138	83
March	147	74
April	124	48
May	124	64
June	129	47
July	121	43
August	111	28
September	169	82
October	189	131
November	152	79
December	138	58
Total	**1667**	**790**

Fig. 12. Infectious humans in Ngaoundere.

CRediT authorship contribution statement

Justin-Hervé Noubissi: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Jean Claude Kamgang:** Validation, Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data included in article/supp. material/referenced in article.

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