On Assessing the Temporal Characteristics
of Reaching the Milestone by a Swarm

Eugene Larkin and Maxim Antonov[(✉)]

Tula State University, Tula 300012, Russia
elarkin@mail.ru, max0594@yandex.ru

Abstract. The physical swarm operation is investigated. Swarm units are considered as three-wheeled mobile robots, moving through rough terrain. For the longitudinal movement of such type vehicle the dynamic model is obtained. The article discusses the issue of forecasting the time a physical swarm reaches a milestone and proposes a hypothesis on the form of the law of the distribution of time during which a swarm unit reaches a milestone. Obtaining this time distribution is carried out with use the Petri-Markov net fundamental apparatus. With use Petri-Markov nets time densities of reaching the milestone both one unit and swarm as a whole are obtained. More common formula of distribution of time of milestone reaching by l units of K is obtained too. To confirm the hypothesis about the type of theoretical time distribution a computer experiment was carried out using the Monte Carlo method.

Keywords: Swarm · Swarm unit · Longitudinal movement · Milestone temporal characteristics · Petri-Markov net · Monte Carlo method

1 Introduction

At the present time, the physical swarm is widely used in various fields of human activity, such as industry, defense, ecology [1], as well as to perform tasks of locating objects on the ground, reconnaissance, counter-terrorism, mine clearance operations, etc.

The study of a physical swarm and modeling of its behavior is a field of active scientific research, popular for more than two decades. After the work of Reynolds [2], devoted to the study of the behavior of a flock of birds in flight, extensive studies were carried out to model and analyze the behavior of the swarm, as well as to develop algorithms for controlling and coordinating the swarm [3].

Of greatest interest are the tasks of swarm coordination, requiring that the milestone be reached by all units of the physical swarm at the same time or with a minimum time spread. Such a need arises, for example when it is necessary to surround the target and not miss it. In real conditions, each unit of a physical swarm moves along its own unique trajectory, determined by the relief and micro-relief of the terrain, weather conditions, etc.

Therefore, the achievement of a milestone by each unit of a physical swarm occurs with random time, which in turn, is determined by robot design and properties of terrain.

Known methods for modeling the behavior of the swarm [4, 5], as well as algorithms for coordinating the swarm on the ground [6, 7] have a significant drawback, which at the stage of designing a physical swarm, not allow to evaluate the temporal and probabilistic characteristics of its functioning in real conditions, therefore, they do not allow to solve the problem of reaching the milestone with all units of the physical swarm with a minimum time spread. Therefore, the investigation, carried out below are relevant, and important for the practice of swarm design.

2 Dynamics of Swarm Unit Longitudinal Movement

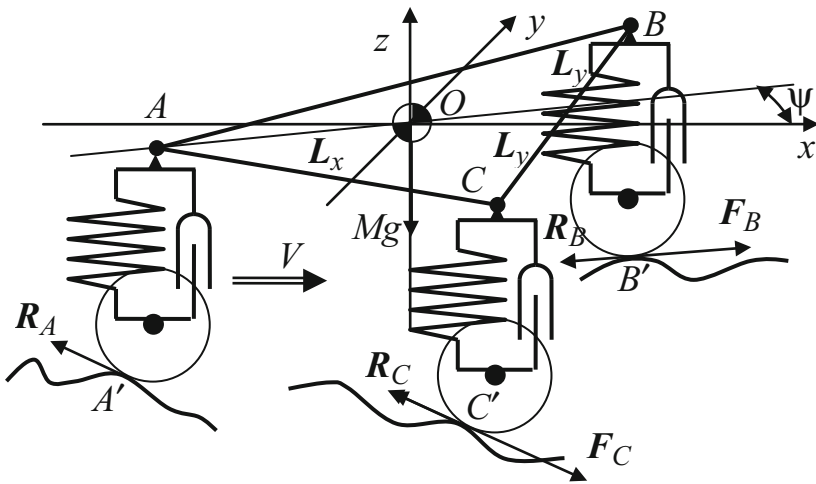Structure of swarm unit, as a transport vehicle, is shown on the Fig. 1.

Fig. 1. Kinematics of swarm unit.

Unit is considered as the absolutely rigid trunk, based on viscose/elastic supports, placed at isosceles triangle vertices A, B, C. Swarm unit moves on the rugged terrain and every wheel is in its individual road conditions. Wheels B and C are active [8]. The combined moment of these wheels provides the longitudinal movement of the unit, and the differential moment provides maneuvering on a terrain. Wheel A is strongly passive. It provides third fulcrum only, and is installed on the support, which in turn, rotates at hinge. Axis of rotation is perpendicular to the ABC-plane. The trunk has no longitudinal not transverse mobility degree relatively to supports, in such a way centers of wheels may move the straights line perpendicular ABC-plane only. Lengths of supports counterbalance the weight of trunk Mg, where M is the trunk mass; g is the gravity acceleration. Wheels touch the terrain at points A', B', C'. In points of touch forces tangential component provides both resistance R_A, R_B, R_C, and driving forces F_B, F_C.

To describe the influence of terrain relief on the time of getting by swarm unit milestone one should work out the differential equation, describing robot longitudinal

movement. Differential equation, which describes angle velocity of B and C electric engines shafts are as follows:

$$\begin{cases} T\dot{\omega}_B + \omega_B + K_\mu\mu_B + K_\nu\nu_B = K_U U_B; \\ T\dot{\omega}_C + \omega_C + K_\mu\mu_C + K_\nu\nu_C = K_U U_C, \end{cases} \tag{1}$$

where ω_B, ω_C are rotation speeds of B and C motor shafts; T is the time constant of motors; μ_B, μ_C are payload moments on B and C motor shafts; ν_B, ν_C are moments of dry friction in the bears of engine; U_B, U_C are control actions applied to B and C motors; K_μ, K_ν, K_U are proportionality factors.

If necessary, motors are equipped with reduction gears with transmitting coefficient i (when motor have no reducer, then $i = 1$). So moments and rotation speeds at reduction gear output shafts are as follows:

$$\begin{cases} \frac{{}^r\mu_{B,C}}{i} = \mu_{B,C}; \\ {}^r\omega_{B,C}i = \omega_{B,C}, \end{cases} \tag{2}$$

where ${}^r\omega_B$ ${}^r\omega_C$ are rotation speeds on reducer output shafts B and C, respectively; ${}^r\mu_B$, ${}^r\mu_C$ are payload moments on reducers B and C output shafts.

Reducers are loaded on the drive wheels, so wheel drive circumferential speeds, which coincide with the speeds V_B, V_c of supports B and C, are as follows:

$$V_{B,C} = {}^r\omega_{B,C}q, \tag{3}$$

where q is the wheel radius.

Longitudinal speed and azimuth rotation speed of swarm unit are as follows:

$$\begin{cases} V = \dfrac{V_B + V_c}{2}; \\ \dot{\psi} = \dfrac{V_B - V_c}{2L_y}, \end{cases} \tag{4}$$

where $2L_y$ is width between supports B and C; $\dot{\psi} = \frac{d\psi}{dt}$.

Swarm unit moves forward and rotates around z axis under the action of driving and resistance forces. Center of rotation is situated on the straight line, passing through points B and C.

Differential equation system, which describes dynamic of longitudinal movement and azimuth angle rotation of unit, is as follows:

$$\begin{cases} M\dot{V} + \eta_V V = (F_B - R_B)\cos\alpha_B + (F_C - R_C)\cos\alpha_C - R_A\cos\alpha_A; \\ J\ddot{\psi} + \eta_\psi\dot{\psi} = [(F_B - R_B)\cos\alpha_B - (F_C - R_C)\cos\alpha_C]L_y - R_A L_x\cos\alpha_A\sin\phi, \end{cases} \tag{5}$$

where M is the unit mass; J is the moment of inertia of swarm unit relative to center of rotation; η_V and η_ψ are coefficients, describing dissipative forces, affected on the unit (f.e. viscous friction); F_B, F_C are driving forces; R_A, R_B, R_C are resistance

forces; α_A, α_B, α_C are angles between horizontal plane and tangential force in points A', B', C' respectively; φ is angle of the passive wheel A rotation.

Joint solution of Eqs. (3)–(5) gives following system, describing longitudinal movement and azimuth angle maneuvering of three wheels swarm unit:

$$\delta_V(\kappa) \cdot D_{VV}(\kappa) + \delta_{\dot\psi}(\kappa) \cdot D_{V\dot\psi}(\kappa) = \delta_{FV}(\kappa); \tag{6}$$

$$\delta_V(\kappa) \cdot D_{\dot\psi V}(\kappa) + \delta_{\dot\psi}(\kappa) \cdot D_{\dot\psi\dot\psi}(\kappa) = \delta_{F\dot\psi}(\kappa),$$

where κ is the Laplace variable; $\delta_V(\kappa)$ - is increment of longitudinal speed; $\delta_{\dot\psi}(\kappa)$ - is increment of azimuth rotation speed; δ_{FV} is deviation of driving force, that provides longitudinal speed; $\delta_{F\psi}$ is deviation of azimuth rotation force, that provides azimuth rotation speed.

$$D_{VV}(\kappa) = D_{\dot\psi V}(\kappa) = \kappa\left(M + \frac{2Ti^2}{K_\mu q^2}\right) + \left(\eta_V + \frac{2i^2}{K_\mu q^2}\right);$$

$$D_{V\dot\psi}(\kappa) = -D_{\dot\psi\dot\psi}(\kappa) = \kappa\left(\frac{J_z}{L_y} + \frac{2TL_yi^2}{K_\mu q^2}\right) + \left(\frac{\eta_\psi}{L_y} + \frac{2L_yi^2}{K_\mu q^2}\right).$$

Solving the system (6) gives

$$\delta_V(\kappa) = \frac{k_U\delta_{UB}}{T_{SU}\kappa + 1} + \frac{k_U\delta_{UC}}{T_{SU}\kappa + 1} - \frac{k_{\alpha A}\delta_{\alpha A}}{T_{SU}\kappa + 1} - \frac{k_{\alpha BC}\delta_{\alpha B}}{T_{SU}\kappa + 1} - \frac{k_{\alpha BC}\delta_{\alpha C}}{T_{SU}\kappa + 1}, \tag{7}$$

where k_U, $k_{\alpha A}$, $k_{\alpha BC}$ are coefficients of proportionality; T_{SU} is the speedup time constant of swarm unit; $\delta_{\alpha A}, \delta_{\alpha B}, \delta_{\alpha C}$ is deviations of angles α_A, α_B, α_C between horizontal plane and tangential forces.

It is necessary to admit, that (7) is valid, when wheels does not lose mechanical contact with a road and transverse slip of the wheels on the surface is absent. Expressions (7) shows, that dynamics of longitudinal movement of 3-wheel swarm unit depends on characteristics of motors, gear ratio, design of chassis etc. Current swarm unit speed depends on deviations δ_{UB}, δ_{UC} of control actions U_B, U_C, applied to B and C motors, and deviations $\delta_{\alpha A}, \delta_{\alpha B}, \delta_{\alpha C}$. So, deviations of angles cause the deviation of swarm unit velocity from pre-determined level, and slowing down reaching the milestone by the unit. If such parameter of road, as profile correlation function, is known, time density of milestone achievement may be obtained.

3 Reaching the Milestone by the Swarm Unit

Let the swarm unit move to the milestone located at a distance S from it. When unit runs on the perfect flat surface, without hollows and/or hedges, the milestone may be reached at the time T, value of which is determined by the maximum speed developed by unit. At the same time, as it follows from (7), every obstacle on the movement trace increases this time. For a simulation of passing the distance by vehicle approach, based on Petri-Markov net (PMN) [9–12] conception may be proposed. Structure of PMN is

shown on the Fig. 2. To obtain such structure the distance S from starting point till the milestone is divided onto elementary pieces s. After passing every piece swarm unit immediately starts to pass next piece. Petri-Markov net is described with the following set:

$$\Pi = \{\{a_1, a_1, a_2, \ldots, a_j, \ldots\}, \{z_1, z_2, \ldots, z_j, \ldots\},$$
$$\{I(z_1) = \emptyset, I(z_j) = a_{i-1}, i = 0, \ldots j, \ldots\}, \{O(z_j) = a_j, i = 0, \ldots j, \ldots\}\}, \quad (8)$$

where a_j are places; z_j are transitions; $I(\ldots)$ are transition input functions $O(\ldots)$ are transition output functions; $\emptyset$ is the zero set.
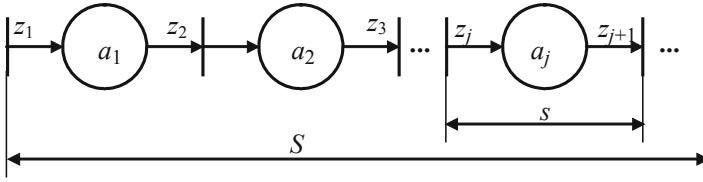

Fig. 2. PMN modeling the process of achieving the milestone.

Transition z_1 is the starting one, and simulates beginning swarm unit movement. Transition z_j simulates achievement the end of stage s. When $j \to \infty$ it simulates the end of the distance S. Places simulate generation of time intervals when swarm unit passes the stages.

Every stage may be passed by swarm unit with current velocity $V(t)$. Time, during of which unit passes the stage, is occasional and equal to

$$t_s = \frac{s}{V(t)} = T_s + \Delta(t), \qquad T_s = \frac{s}{V + \delta_V} = \bar{T}_s + \delta_{Ts} \quad (9)$$

where V is mean swarm unit velocity, caused by control actions U_B, U_C, applied to B and C motors according to Eq. (7); δ_V is a deviation of velocity from mean value, caused by roughness of terrain under wheels A, B, C; $\bar{T}_s$ is the mean time of passing the stage by swarm unit; δ_s is a deviation of time from mean value.

When deviation δ_V is occasional value, then deviation δ_{Ts} is occasional value too. It would be considered, that δ_{Ts} has zero mean value. So density of time of passing the stage is as follows:

$$f_s(t) = \delta(t - T_s) * f_\Delta(t), f_s(t) = \delta(t - \bar{T}_s) * \bar{f}_s(t) \quad (10)$$

where $\delta(t - \bar{T}_s)$ is the Dirac δ-function; $\bar{f}_s(t)$ is the time deviation density from mean value.

Accordingly to [13]

$$f_S(t) = \lim_{\substack{J \to \infty, \\ s \to 0}} L^{-1}\left[\prod_{j=1}^{J} L[\delta(t - \bar{T}_s) * \bar{f}_s(t)]\right] = \delta(t - \bar{T}_S) * \lim_{\substack{J \to \infty, \\ s \to 0}} L^{-1}\left[\prod_{j=1}^{J} L[\bar{f}_s(t)]\right] \quad (11)$$

where $L[\ldots]$ and $L^{-1}[\ldots]$ are direct and inverse Laplace transforms; $\bar{T}_S$ is the mean time spent by swarm unit for passing the whole distance S with pre-determined velocity V;

$$\bar{T}_S = \frac{S}{V}. \tag{12}$$

It is necessary to admit, that

$$\bar{f}_S(t) = \lim_{\substack{J \to \infty, \\ s \to 0}} L^{-1}\left[\prod_{j=1}^{J} L[\bar{f}_s(t)]\right] \tag{13}$$

is the convolution, describing sum of great number of small random values, which equally contribute to resulting random value. Accordingly to central limit theorem [14, 15] $\bar{f}_S(t)$ is normally distributed. Also swarm unit cannot reach the milestone earlier than at the time τ when driving the whole distance with maximal velocity $V_{\max}$;

$$\tau = \frac{S}{V_{\max}} \tag{14}$$

Taking into account the above circumstances, finally the time density of reaching the milestone by swarm unit takes the following form:

$$f_S(t) = \begin{cases} \dfrac{exp\left[-\frac{(t-T_S)^2}{2D_\Delta}\right] \cdot \eta(t-\tau)}{\int\limits_{\tau}^{\infty} exp\left[-\frac{(t-T_S)^2}{2D_\Delta}\right] dt}, & when \ t \geq \tau; \\ 0 \ otherwise \end{cases} \tag{15}$$

where T_S is expectation of unclipped normal distribution; D_S is dispersion of density $\bar{f}_S(t)$; $\eta(t-\tau)$ is Heaviside function.

4 Reaching the Milestone by a Swarm

Let there is a swarm of mobile robots moving towards the milestone. The process of functioning of the swarm when reaching milestone can be described by the PMN [14–16], shown on the Fig. 3:

$$\begin{aligned} \Pi = \{&\{a_1, \ldots, a_k, \ldots, a_K\}, \{z_b, z_e\}, \\ &\{O(z_b) = \{a_1, \ldots, a_k, \ldots, a_K\}, O(z_e) = \emptyset, \\ &\{I(z_b) = \emptyset; I(z_e) = \{a_1, \ldots, a_k, \ldots, a_K\}\}\}. \end{aligned} \tag{16}$$

On the Fig. 3, transition z_b simulates the start of swarm movement; transition z_e simulates the reaching the milestone by the swarm units and the swarm as a whole; places a_1,..., a_k,..., a_K simulate process of movement units toward the milestone.

There is the competition between units for not to be last when passing the distance [17]. Time density of event of gathering the whole swarm at the milestone is as follows:

$$f_g(t) = \frac{d \prod\limits_{k=1}^{K} F_k(t)}{dt} = \sum_{k=1}^{K} f_k(t) \prod_{\substack{i=1, \\ i \neq k}}^{K} F_i(t), \tag{17}$$
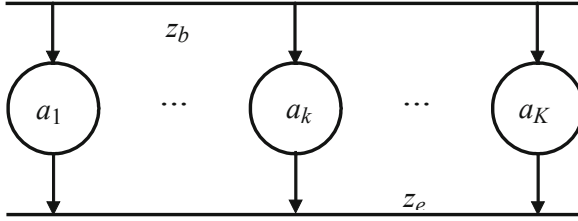
Fig. 3. PMN, that simulates the swarm functioning.

where $f_k(t)$ is the time density of reaching the milestone by k-th swarm unit, described with formula (15); $F_k(t) = \int_0^t f_k(\theta)d\theta$; θ is auxiliary variable.

When $f_k(t)$ is described with formula (15), then

$$F_S(t) = \begin{cases} \dfrac{\int_\tau^t exp\left[-\dfrac{(\theta-T_S)^2}{2D_\Delta}\right]d\theta}{\int_\tau^\infty exp\left[-\dfrac{(t-T_S)^2}{2D_\Delta}\right]dt} when\ t \geq \tau; \\ 0\ otherwise. \end{cases} \tag{18}$$

Probability and pure time density, that namely k-th unit of swarm finishes the distance the last is as follows [13]:

$$\tilde{p}_k = \int_0^\infty f_k(t) \prod_{\substack{i=1,\\i\neq k}}^K F_i(t)dt; \tag{19}$$

$$\tilde{f}_k(t) = \frac{f_k(t) \prod_{\substack{i=1,\\i\neq k}}^K F_i(t)dt}{\tilde{p}_k}. \tag{20}$$

Probability and pure time density, that namely k-th unit of swarm finishes the distance the first is as follows [13]:

$$\hat{p}_k = \int_0^\infty f_k(t) \prod_{\substack{i=1,\\i\neq k}}^K [1 - F_i(t)]dt; \tag{21}$$

$$\hat{f}_k(t) = \frac{f_k(t) \prod_{\substack{i=1,\\i\neq k}}^K [1 - F_i(t)]dt}{\hat{p}_k}. \tag{22}$$

Let us introduce K-digit binary number $N(K) = \langle n_1, \ldots, n_k, \ldots, n_K \rangle$ and select from N^K those numbers, which contain l zeros and $K - l$ ones. Quantity of such combinations is

$$C(K, l) = \frac{K!}{(K-l)! \cdot l!}. \tag{23}$$

Let us introduce function

$$\Phi_k[t, c(K, l)] = \begin{cases} F_k(t), & when \ n_k[c(K, l)] = 0; \\ 1 - F_k(t), & when \ n_k[c(K, l)] = 0, \end{cases} \quad (24)$$

where $c(K, l)$ is number of combination of binary number, which contain l zeros and $K - l$ ones; $n_k[c(K, l)]$ is k-th binary digit in $c(K, l)$-th combination.

Then time density of the event, that l drom K swarm units get the milestone is as follow

$$\tilde{f}_{K,l}(t) = \sum_{c(K,l)=1}^{C(K,l)} \sum_{k \in c(K,l)} f_k(t) \prod_{\substack{i=1, \\ i \neq k}}^{K} \Phi_i[t, c(K, L)], \quad (25)$$

where $k \in c(K, l)$ means, that summation should be executed only on all k's, which are in combination of $\langle n_1, \ldots, n_k, \ldots, n_K \rangle$, containing l zeros.

5   Computer Experiment

The direct computer experiment was performed to verify the proposed method. It was performed for swarm unit, which moves over rough terrain with random velocity $V + \delta_V$, where $V = 5$ m/s, δ_V is a random value, which is uniformly distributed at the domain $-0,5 \leq \delta_V \leq 0,5$ m/s. Distance 1 km is divided onto 100 stages of length 10 m. Histogram of reaching the milestone by swarm unit, obtained with use the Monte-Carlo method, is shown on the Fig. 4. Error evaluation of time expectation (200 s) is equal to 1,3%, error of standard deviation evaluation (0,9 s) is equal to 3,4%.
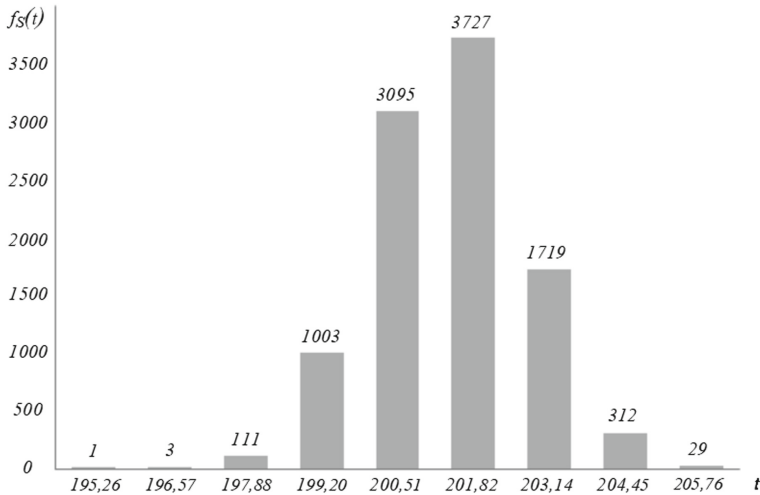

Fig. 4. Histograms of reaching the milestone by swarm unit

Histogram of reaching the milestone by swarm, consisting of 5 units, is shown on the Fig. 5. This histogram illustrates (17). As it follows from named formula, moda of histogram is shifted in comparison with histogram, sown on the Fig. 4. This fact one should took into account when planning operations, executed by swarm.
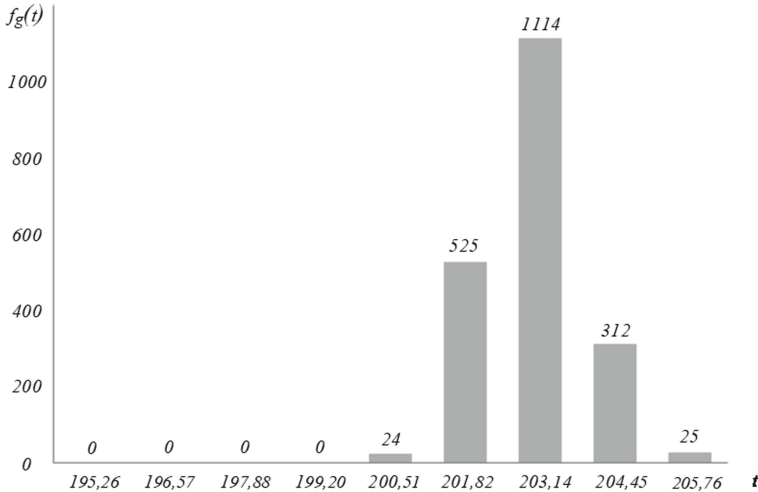

Fig. 5. Histograms of reaching the milestone by swarm as a whole

The result of computer experiment, shown on the Fig. 4 and the Fig. 5, confirms the validity of the hypothesis of the time density of a milestone achievement by the swarm unit.

6   Conclusion

So, the article notes that an important task of swarm operation planning is evaluation of milestone achieving time, both swarm unit, and swarm as a whole. In the case of physical swarm, moving through a rough terrain, it may be done by means of analytical simulation of vehicle longitudinal movement. Proposed approach permits to link design and physical parameters of unit with characteristics of terrain as a road and with time density of reaching the milestone both one unit and l units of K. The fidelity of the proposed method is confirmed by a direct computer experiment.

References

1. Brambilla, M., Ferrante, E., Birattari, M., Dorigo, M.: Swarm robotics: a review from the swarm engineering perspective. Swarm Intell. **7**(1), 1–41 (2013)

2. Reynolds, C.W.: Flocks, herds, and schools: a distributed behavioral model. Comput. Graph. **21**(4), 25–34 (1987)
3. Kececi, E.F., Ceccarelli, M.: Robot Swarms: Dynamics and Control, 1st edn. ASME Press, New York (2015)
4. Balch, T., Arkin, R.C.: Behavior-based formation control for multirobot teams. IEEE Trans. Robot. Autom. **14**(6), 926–939 (1998)
5. Beard, R.W., Lawton, J., Hadaegh, F.Y.: A coordination architecture for spacecraft formation control. IEEE Trans. Control Syst. Technol. **9**(6), 777–790 (2001)
6. Saaj, C.M., Lappas, V., Gazi, V.: Spacecraft swarm navigation and control using artificial potential field and sliding mode control. In: IEEE International Conference on Industrial Technology, Mumbai, pp. 2646–2652. IEEE (2006)
7. Akat, S.B., Gazi, V., Marques, L.: Asynchronous particle swarm optimization based search with a multi-robot system: simulation and implementation on a real robotic system. Turk. J. Electr. Eng. Comput. Sci. **18**(5), 749–764 (2010)
8. Morin, P., Samson, C.: Motion Control of Wheeled Mobile Robots. In: Siciliano, B., Khatib, O. (eds.) Springer Handbook of Robotics, pp. 799–826. Springer, Heidelberg (2008)
9. Ivutin, A., Larkin, E., Kotov, V.: Established routine of swarm monitoring systems functioning. In: Tan, Y., Shi, Y., Buarque, F., Gelbukh, A., Das, S., Engelbrecht, A. (eds.) ICSI 2015. LNCS, vol. 9141, pp. 415–422. Springer, Cham (2015). https://doi.org/10.1007/978-3-319-20472-7_45
10. Larkin, E.V., Ivutin, A.N.: Estimation of latency in embedded real-time systems. In: 3-rd Meditteranean Conference on Embedded Computing (MECO-2014), Budva, pp. 236–239. IEEE (2014)
11. Larkin, E., Ivutin, A., Kotov, V., Privalov, A.: Semi-Markov modelling of commands execution by mobile robot. In: Ronzhin, A., Rigoll, G., Meshcheryakov, R. (eds.) ICR 2016. LNCS (LNAI), vol. 9812, pp. 189–198. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-43955-6_23
12. Korolyuk, V., Swishchuk, A.: Semi-Markov Random Evolutions, 1st edn. Springer Science+Business Media, Dordrecht (1995)
13. Shiryaev, A.N.: Probability-2, 2nd edn. Springer Science+Business Media, New York (2019)
14. Dinov, I.D., Christou, N., Sanchez, J.: Central limit theorem: new SOCR applet and demonstration activity. J. Stat. Educ. **16**(2), 1–5 (2008)
15. Bárány, I., Van, V.: Central limit theorems for Gaussian polytopes. Ann. Prob. **35**(4), 1593–1621 (2007)
16. Limnios, N., Swishchuk, A.: Discrete-time semi-Markov random evolutions and their applications. Adv. Appl. Probab. **45**(1), 214–240 (2013)
17. Bielecki, T.R., Jakubowski, J., Niewęgłowski, M.: Conditional Markov chains: properties, construction and structured dependence. Stochast. Process. Appl. **127**(4), 1125–1170 (2017)