Evaluation of Compton attenuation and photoelectric absorption coefficients by convolution of scattering and primary functions and counts ratio on energy spectra

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Purpose: Estimation of Compton attenuation and the photoelectric absorption coefficients were explored at ABSTRACT various depths. Methods: A new method was proposed for estimating the depth based on the convolution of two exponential functions, namely convolution of scattering and primary functions (CSPF), which the convolved result will conform to the photopeak region of energy spectrum with the variable energy-window widths (EWWs) and a theory on the scattering cross-section. The triple energy-windows (TEW) and extended triple energy-windows scatter correction (ETEW) methods were used to estimate the scattered and primary photons according to the energy spectra at various depths due to a better performance than the other methods in nuclear medicine. For this purpose, the energy spectra were employed, and a distinct phantom along with a technetium-99 m source was simulated by Monte Carlo method. Results: The simulated results indicate that the EWW, used to calculate the scattered and primary counts in terms of the integral operators on the functions, was proportional to the depth as an exponential function. The depth will be calculated by the combination of either TEW or ETEW and proposed method resulting in the distinct energy-window. The EWWs for primary photons were in good agreement with those of scattered photons at the same as depths. The average errors between these windows for both methods TEW, and ETEW were 7.25% and 6.03% at different depths, respectively. The EWW value for functions of scattered and primary photons was reduced by increasing the depth in the CSPF method. Conclusions: This coefficient may be an index for the scattering cross-section.

Keywords: Compton, convolution, depth, Monte Carlo method, photopeak

INTRODUCTION

Compton scattering is one of the most important processes on assessment interaction of radiation with material in which the ray will be attenuated quantitatively as Compton attenuation coefficient. This coefficient is a practical and applicable parameter for improving the images in nuclear medicine as well as decreasing the absorbed radiation dose along with the photoelectric

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absorption one. Also, each organ in the body is at a specific situation where acts the functional and physiological engagements in order to obtain health and high growth of cells. Every change in this spatial situation leads to appear abnormalities, lesions and loss of performance as well. Therefore, estimation of the organs' depth plays a key role in medicine due to providing helpful information for evaluating their performance, early diagnosis, as well as the better treatment. While those are deformed, the depth may be changed, and this can be probably utilized to detect the type and degree of disease. Clearly, the calculation of the depth depends on the peripheral tissues, the attenuation coefficient and the organ size that were considered in the existing methods.^[1-7]

Nosil *et al.* employed two radioisotopes that their method was independent of the size, but assuming the attenuation coefficient of water to be constant.^[3] On the other hand, Starck and Carlsson

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proposed a method in which the modulation transform function and the mean linear attenuation coefficient used, in contrast with depending size.^[5] These methods were related to either the size or the total attenuation coefficient, as it has been done comparison of the methods on this matter, but all of them related to the other variable distinct parameters.

In this study, a new method proposed free from all aforementioned parameters in which convolution of the scattering and primary photons functions (CSPF) along with the triple energy-window (TEW) and extended triple energy-window (ETEW) methods is used for estimating depth of organs along with estimating Compton scattering attenuation coefficient and the photoelectric absorption coefficient using the energy spectra.

METHODS

The radiopharmaceutical, containing the nuclear materials, is used in order to evaluation of metabolic performance and physiological parameters of the tissues.^[11,12] Due to the nature of nuclear radiation, several interactions between radiation and tissue occur according to different energies of the radiopharmaceutical. Photoelectric and Compton scattering are usually dominant phenomena in diagnostic nuclear medicine.^[13] Although, the scattered gamma ray reduces image contrast,^[14] but we have demonstrated that the depth can be found by this and some mathematical concepts, and then the Compton attenuation and the photoelectric absorption coefficients will be estimated.

Scatter estimation using triple energy-window

The TEW method,^[8] has better performance than other methods in nuclear medicine,^[9,10] and is based on the energy spectrum. The subtraction and trapezoidal laws are used in this method which the subtraction is carried out using two sets of data: One set is acquired with a main window centered at photopeak energy, and the other is acquired with two subwindows on both sides of the main window. The scattered photons (C_{scal}) included in



Figure 1: The simulated total, primary, and scatter energy spectra with technetium-99m in phantom

the main window are estimated from the counts acquired with the subwindows and then they are subtracted from the count acquired with the main window. As shown in Figure 1, the count of primary photons ($C_{\rm prim}$) is given by,^[15]

$$C_{\text{prim}} = C_{\text{total}} - C_{\text{scat}} \tag{1}$$

The C_{scat} is estimated from the count data C_{left} and C_{right} acquired with the two subwindows that are located at both sides of the main window, as shown in Figure 2. Assuming that the width of the main window as W_{m} and that of the subwindow as W_{s} the C_{scat} can be estimated from a trapezoidal region having a left height of $C_{\text{left}}/W_{\text{s}}$, a right height of $C_{\text{right}}/W_{\text{s}}$, and a base of W_{m} as follows:

$$C_{\text{scat}} \cong \left(\frac{C_{\text{left}}}{W_{\text{s}}} + \frac{C_{\text{right}}}{W_{\text{s}}}\right) W_{\text{m}} / 2 \tag{2}$$

The C_{prim} can be calculated using Eqs. (1, 2). The choice of the energy-window width (EWW) for the values of W_{m} and W_{s} s is critical in these methods because the accuracy is related severely to the EWW value and the detector system. The photopeak point has spatial shift more to the left side on the spectrum when the C_{scat} is increased. This shift may also be considered as a preliminary estimation of the C_{scat} which this is beyond the scope of this study, and it is not considered at accounting process here. This matter may be an error source in CSPF method.^[16]

Scatter estimation using extended triple energy-window

Bong *et al.* proposed this method in order to improve the quality of the nuclear medicine images.^[17] The ETEW method estimates scatter counts with the trapezoidal approximation as follows,

$$C_{\rm s} = \left(\frac{C_{\rm left}}{W_{\rm left}} - \frac{C_{\rm right}}{W_{\rm right}}\right) \left(W_{\rm 1} + W_{\rm 2}\right) \frac{W_{\rm m}}{2W} + \frac{C_{\rm right}}{W_{\rm right}} W_{\rm m}$$
(3)



Figure 2: The energy spectrum used at the triple energy-windows method for estimating the scattered and primary counts along with the chosen energywindows

Where *W* is difference between the centers of the right and left subwindows, W_1 is difference between the center of the right subwindow and lower bound of the main window, and W_2 is difference between the center of the right subwindow and upper bound of the main window, as shown in Figure 3.

Convolved scatter and primary functions method

The aim was to present a new method for calculating the organ depth independent of the linear attenuation coefficient. Estimation of the depth is useful for measuring the amount of radioactive tracker taken up by an organ in the body. This method is based on the mathematical relations as convolution of two exponential functions that both the related parameters of these functions and its result mapping on the spectrum curve which they were determined by the Monte Carlo method and Matlab software. Obviously, the convolution operator is the effectiveness of a function on the other function with progressing some known variables. In the interaction of radiation with matter, the energy value is decreased gradually, and it seems that these energies will act together totally on the system so that one may consider the CSPF appeared in the energy spectrum. However, the spectrum is a final result from these interactions. In CSPF method, two pseudo-analogical expressions are introduced on the spectra. The first, the spectrum function (C(E)) is the convolution of two exponential functions, and the second, the scattered ($C_s(E)$) and primary ($C_p(E)$) photon functions are as exponential functions, as indicated in Figure 4 in which the origin of coordinates is metaphorical. One may obtain the function of C(E) as follows,

$$C(E) = C_{S}(E) \circledast C_{P}(E) = \varphi e^{-\alpha E} \circledast \omega e^{-\beta E}$$
$$= \frac{\varphi \omega}{\beta - \alpha} \left(e^{-\alpha E} - e^{-\beta E} \right)$$
(4)

Where α , β and ω are the constant parameters, which are determined by the some distinct points on the spectrum curve using Matlab software (MathWorks, Inc., 3 Apple Hill Drive, Natick, MA 01760-2098 USA). The C_{scat} and C_{prim} are calculated by the TEW and ETEW methods at various depths. In this method, these values are the integrand of the $C_{\text{s}}(E)$ and $C_{\text{p}}(E)$ functions over the ΔE_{CS} and ΔE_{CP} windows as follows,

$$C_{\text{Scat}} = \int C_{\text{s}}(E) dE = \int_{140-0.5\Delta E_{\text{CS}}}^{140+0.5\Delta E_{\text{CS}}} \varphi e^{-\alpha E} dE$$
(5)
$$C_{\text{Scat}} = \int C_{\text{s}}(E) dE = \int_{140-0.5\Delta E_{\text{CP}}}^{140+0.5\Delta E_{\text{CP}}} \varphi e^{-\beta E} dE$$
(6)

The energies are in terms of keV and the technetium-99m (Tc-99m) source has 140-keV gamma ray. The ΔE_{CS} and ΔE_{CP} values will be calculated from the energy spectra at different depths by solving the Eqs. (5, 6) using Matlab software. The

distance from the detector, namely depth (d), is related to the energy-windows of ΔE_{CS} and ΔE_{CP} as follows,

$$\Delta E_{\rm cs} \cong \Delta E_{\rm cp} = \sigma e^{-\tau \times d} \tag{7}$$

The aim is the determination of the σ and τ unknown parameters. Therefore, one may estimate the depth value using Eq. (7) in which the energy-window calculated by both the ETEW and CSPF methods on the energy spectrum obtained in nuclear imaging with applying the flowchart as indicated in Figure 5. This flowchart shows the step by step for the calculation of these unknown parameters. Finally, with characterizing Eq. (7), one the first obtains the spectrum and procedures for determining the $\Delta E_{\rm CS}$ or $\Delta E_{\rm CP}$ values, and then the depth will be determined.

Determination of the Compton attenuation coefficient

The second, the relationship between the counts ratio and the Compton attenuation coefficient has been evaluated. The radiopharmaceuticals concentrated in the special organs may be considered as the volumetric sources along with a special pharmaceutical signal to noise ratio (S/N) in which the rays employed to determine the anatomical and functional parameters. For instance, a Tc-99m point source is positioned beyond an attenuator that decreases both the number and energy of the rays at the different situations, as shown in Figure 6. Some of the rays can pass through the material without scattering to be recorded under the photopeak region. The ray on the straight path does not react with the matter that will pass from the hole of collimator and will reach into the detector. With regard to the resolution and related parameters on the detection, the rays will be detected and recorded under the photopeak region on the energy spectrum. In contrast, the ray passing through the attenuator with the small angle (θ) relative to normal line interacts at the distance of x from the source at the dx thickness included in the attenuator due to its Compton attenuation coefficient that will be scattered so that Compton phenomenon will act as priority one. The scattered photon will travel through the distance of L and will be recorded in the Compton region on the spectrum. The primary counts (C_{n}) has been calculated by,

$$C_{\rm prim} = k I_0 e^{-\mu d} \tag{8}$$

Where k, I_0 , and μ are the buildup factor along with the other parameters of the detector, the primary intensity of the source, and total linear attenuation coefficient, respectively. To estimate theoretically the number of the scattered photons recorded in the Compton region of the spectrum, a thickness of dx away from a distance of x from the source is considered. The number of descending photons to this thickness is as follows,

$$k' I_0 e^{-\mu x} \tag{9}$$

That the number of the scattered photons with respect to the Compton scattering attenuation coefficient (μ_{sc}) and the thickness of dx is as follows,



Figure 3: The energy spectrum used at the extended triple energy-windows scatter correction method for estimating the scattered and primary counts along with the chosen energy-windows



Figure 5: Flowchart of implementation of the proposed method

$$k' I_0 e^{-\mu x} \times \mu_{sc} \times dx \tag{10}$$

After the attenuation process, the fraction of these photons detected by the detector will be as follows,

$$k' \mathbf{I}_{0} e^{-\mu \mathbf{x}} \times \mu_{\mathrm{sc}} \times d\mathbf{x} \times e^{-\mu \mathbf{l}}$$
(11)

Where *K* is the same as *K*, but without considering parameters of the detector. Regarding to be small angle of θ less than 5° ($\theta < 5^{\circ}$), one may obtain the following approximation,

$$l \cong d - x \tag{12}$$

That it is true. Therefore, the number of the scattered photons (dC) at the thickness of dx will be calculated as follows,

$$dC_{\rm s} = k I_0 e^{-\mu x} \times \mu_{\rm sc} \times dx \times e^{-\mu(d-x)} = k I_0 e^{-\mu d} \times \mu_{\rm sc} \times dx \qquad (13)$$



Figure 4: Convolution of two exponential functions



Figure 6: Schematic of presentation for calculation of the Compton attenuation coefficient

Because there is a probability of gamma ray interaction and scattering process in the total distance (d), for computing the total number of the scattered photons (C_y) in the thickness of d, one may obtain an integration on the distance from 0 to d as follows,

$$C_{\rm s} = \int_{0}^{a} dC_{\rm s} = k I_{0} e^{-\mu d} \times \mu_{\rm sc} \times d \tag{14}$$

By dividing the Eq. (14) to (8), it is shown that the ratio of the number of scattered to primary photons is proportional to the d and μ_{sc} as follows,

$$\frac{C_{\rm s}}{C_{\rm p}} = \frac{k' I_0 e^{-\mu d} \times \mu_{sc} \times d}{k' I_0 e^{-\mu d}} = \frac{k'}{k} \times \mu_{sc} \times d \tag{15}$$

Where the k, k are assumed as invariable and constant values. Finally, one may estimate the μ_{sc} as follows,

$$\mu_{\rm sc} \cong \kappa' / \kappa \times \frac{C_{\rm s} / C_{\rm p}}{d} \tag{16}$$

Therefore, the μ_{sc} is proportional to the scattered/primary ratio, and inversely with the depth.

Simulation method and computation of the parameters

To study the effects of interactions, Monte Carlo N-Particle version 4C (MCNP4C) code was used here. The input files

would specify geometry of source objects, collimators and detector planes for a distinct aim, which input files and geometry specifications are often complex and can be very cumbersome.^[18,19] This code was modified to simulate spectra from phantom including the Tc-99m source. The NaI (Tl) crystal with the size of 39 cm \times 39 cm, the thickness of 0.9525 cm and a low energy general purpose collimator with the parallel hexagonal holes at the size of 0.145 cm of small diameter, 0.02 cm inter-hole spacing, and 2.41 cm thickness are used in simulation as shown in Table 1.

The phantom is a cylinder, the diameter of 20 cm and the height of 30 cm, full of water placed at the front of the detector, and the spherical source with 2-cm diameter was inserted at distance of 5 cm from the center of the cylinder and 5 cm from the detector, as shown in Figure 7, then it was prepared for MCNP4C code input, which it run by F8 tally (energy distribution of pulses created in the detector as pulse height). The trajectories of 10⁹ photons for the phantom were calculated using the workstation. The validity of the simulation was also confirmed with a spectrum of a distinct source.

The detector is to be closed to the phantom for improving signal to noise ratio (SNR). Source is placed at distances of 1 cm through 20 cm from the detector. The depths were calculated relying on energy spectra taken from MCNP4C code.

RESULTS

Septal width

The response of the detector is simulated with the F8 tally and E8 card. For instance, the output file from the modified code

Table 1: Geometric characteristics of parallel beam collimator (in terms of cm)	
Collimator	Parallel
Hole shape	Hexagonal
Central hole length	2.41
Hole size	0.145

0.02



Figure 7: Parallel Beam collimator with hexagonal holes and point source in scatter medium in front of collimator at z (depth) = 5 cm

for geometric shape is shown in Figure 8. The obtained spectra in each depth are shown in Figure 9. The Compton scattering counts at the depths <4 cm are similar together because of the concept of mean free path value at which the attenuation coefficient of water plays a key role. In contrast, the primary counts were decreased with increasing depths. When the depth is increased, the ratio of scattered to primary counts will be increased.

The simulated values of photopeak point (C_m, E_{C_m}) , 2/3 of its point $(2/3C_m, E_2/_3C_m)$, and ϕ by the Compton edge valley on the energy spectrum were calculated for estimating the α , β and ω parameters. These parameters used for calculating the $C_{\rm a}$ and $C_{\rm c}$ functions were determined from Eqs. (2,3) These calculated parameters at various depths are indicated in Table 2. The amounts of scattered and primary photons were measured at various depths by the TEW and ETEW methods in the phantom. We used a main window (W: 126-154 keV [20% of 140 keV]) and subwindows (Ws: 3 keV centered at 126 and 154keV) on the simulated spectrum in TEW method, and a main window (W: 126-154 keV [20% of 140 keV]) and subwindows (Ws: 3 keV centered at 123.5 and 156.5 keV) in ETEW method. The widths of the energy-windows (ΔE_{CP} and ΔE_{CS}) were calculated using the Table 2 and Eqs. (4, 5), as shown in Table 3. The EWWs for primary photons are in good agreement with those of scattered photons in the CSPF method. The average error between these windows for TEW and ETEW methods were 7.25% and 6.03% at all depths, respectively. Therefore, we may use the EWW of scattering instead of that of primary and vice versa.

The EWW value for functions of scattered and primary photons is reduced by increasing the depth in the CSPF method, as indicated in Figure 10. The EWWs at different depths were fitted by the exponential curve as follows,

EWW =
$$18.610e^{-0.229d}$$
, for TEW method (17)

$$EWW = 20.299e^{-0.239d}, \text{ for ETEW method}$$
(18)



Figure 8: A schematic representation of the simulated gamma camera, (a) collimator with the parallel hexagonal holes, (b) and phantom and detector (c)



Figure 9: The simulated energy spectra at different depths

At d = 0, the EWW may be a criterion for detector performance, and the simulated spectrum is shown in Figure 11. The more the EWW, the less energy resolution of the detector will be.

As shown in Figures 12 and 13, the scattered to primary counts ratio as a linear function of depth are determined as $C_s/C_p = 0.1122 \ d + 1.4235 \ (R^2 = 0.9130)$ at the TEW method, and as $C_s/C_p = 0.1140 \ d + 0.9079 \ (R^2 = 0.9517)$ at the ETEW method using data in Table 3. The Compton attenuation coefficients were 0.1122 and 0.1140 cm⁻¹ for these methods for Tc-99m in water phantom, respectively. The total attenuation coefficient for this state was 0.15 cm⁻¹ so that the photoelectric absorption coefficient was 0.0378 and 0.0360 for TEW and ETEW methods, respectively.

DISCUSSION

In this study, the μ_{sc} and the depth were investigated by new methods along with reputable proposed assumptions using Monte Carlo method. As known, the choice of energy-window at the energy spectra is important for the SNR and image contrast. We have demonstrated that the EWW is proportional to the depth with respect to the primary and scattered photon counts. While the energy spectra obtained both as experimentally by the detector systems and as theoretically either by the simulation using the Monte Carlo method or the calculation by the existing formulas are accessible, one may extract the more information on tracer, detector system, and the geometrical specifications of organ.

The TEW and ETEW methods have been used for estimating the scattered and primary counts accurately. Though the spectra of the scattered photons vary with object size, source distribution, and source energy, estimation of the scattered photons as a trapezoid is good approximation. The other problem is the energy value used in the field of imaging that leads to appearing the Compton scattering and photoelectric absorption processes. Emitted gamma radiation interacts with the body based on these processes, producing a significant attenuation in the primary beam at energies. These mechanisms are well known, and those were a basis in our method.

Depth (cm) C_m (10 ⁻⁷) Z_{-3} (KeV)E (right) $_{[2/3],cm}$ (KeV)E (left) $_{[2/3],cm}$ (KeV) $p^+(10^{-3})$ ω α β 112. 6128188.408545140.9361149.9311129.78915.839330.0058323149055219.33888839336251019.338888393365634310. 3526486.901766137.0196149.5868127.76665.0149250.0041365235101412.5692097253467212.5692097253467212.569209755379558.55136085.700907136.2879148.3818123.20424.6400350.00281656734409319.599383795864509.5993837958645077.02238814.681592135.0828147.6071117.95353.1663740.0012473782346609.599383795864507.75137443655206777.02238814.681592135.0297147.2628123.37644.2985070.00281656334409319.599383795864509.47622399180072104.91878843.2779193135.0297147.2628123.37644.2985070.001247378234860.4762239918395510.47622399180072124.38191402.921276135.0297147.2628112.05722.1044000.001547378234869.599383795864557.751374436550057143.329266102.2561194135.0297147.2628112.05722.1731780.0001547378234860.4762239918395510.47622399183055162.5561194135.0142135.0297147.2628112.05722.17044000.001247378234860.47662239918395510.4766203739916<		s calculated p		a munude	y MUNT40 COUP					
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	20	1.5798946	1.053263	128.6271	143.9151	106.2167	1.204975	0.00040778763746	-8.736790519397641	-8.736790518344913

Table 3: The calculated scattered and primary photons, EWWs and relative error at both methods												
Depth	TEW method					ETEW method						
(cm)	<i>C</i> _p (10 ⁻⁶)	<i>C</i> _s (10 ⁻⁶)	<i>C</i> _t (10 ⁻⁶)	∆ <i>E_{cP}</i> (KeV)	$\Delta \mathbf{\textit{E}}_{cs}$ (KeV)	Relative error $(\Delta \boldsymbol{E}_{CP}, \Delta \boldsymbol{E}_{CS})$ (%)	<i>C</i> _p (10 ⁻⁶)	<i>C</i> _s (10 ⁻⁶)	<i>C</i> _t (10 ⁻⁶)	∆ <i>E_{cP}</i> (KeV)	∆ <i>E_{cs}</i> (KeV)	Relative error $(\Delta \boldsymbol{E}_{CP}, \Delta \boldsymbol{E}_{CS})$ (%)
1	6.925396	9.398694	16.324100	19.714	20.114	1.99	8.870679	7.453320	16.32400	21.600	20.000	8.00
3	4.781002	8.722003	13.503010	9.852	10.098	2.44	6.306530	7.196470	13.503010	8.800	8.400	4.50
5	3.824005	7.003999	10.82800	5.234	5.800	9.75	4.352830	6.776170	11.129000	5.800	5.600	3.45
7	2.888701	6.225397	9.114098	4.052	4.286	5.46	3.206140	5.907860	9.113997	4.200	4.000	5.00
10	1.706300	4.596704	6.303004	1.462	1.518	3.69	1.847510	4.455490	6.303004	1.583	1.504	5.26
12	1.213000	3.835999	5.048999	0.924	1.008	8.34	1.579250	3.585750	5.164999	1.046	1.055	0.81
14	0.919701	3.005395	3.925096	0.559	0.606	7.75	1.131600	2.777840	3.909440	0.565	0.561	0.71
16	0.754297	2.384705	3.139002	0.398	0.449	11.35	0.804211	2.040790	2.845001	0.424	0.385	9.19
18	0.585696	1.801300	2.386995	0.389	0.351	9.77	0.501870	1.444130	1.946000	0.301	0.350	14.00
20	0.344994	1.241997	1.586991	0.283	0.249	12.01	0.311840	1.008160	1.320000	0.225	0.246	9.34

EWWs: Energy-window widths, TWE: Triple energy-window, ETEW: Extended triple energy-windows



Figure 10: The energy-window width values as function of depth for the scattered and primary counts at the extended triple energy-windows methods



Figure 12: The scattered to primary counts ratio as a linear function of depth at the triple energy-windows method

The simulated results indicate that the EWW value is decreased with increasing depth due to the more attenuation and higher cross-sections. It is found that the relationship between the EWW and depth is as exponential function. This method may be used for estimating energy resolution of detector system. Also, it is estimated a distinct distance that Compton scattering regions at depths lower than this distance (4 cm) are similar to with each other. This distinct depth is probably useful to better compensation of scattering for organs close to the skin.



Figure 11: The simulated energy spectrum at zero depth (without phantom)



Figure 13: The scattered to primary counts ratio as a linear function of depth at the extended triple energy-windows scatter correction method

The scattered photons of the photopeak window are mainly contributed by the first-order Compton scatter.^[20] The Compton scattering which may be identified by the cross-sections that will vary with the energy of gamma ray has a key role in this study, although in the field of imaging is unsuitable and must be compensated in order to having a better diagnosis. The C_{scat} value is important both to improve SNR and to estimate depth because the increase in SNR and the reduction of noise followed by the rejection of scattering that it can be clearly observed as well as to provide better quality in the reconstructed images.^[21]

Corrections for scattering are necessary in order to obtain the higher quantitation accuracy,^[22-41] which at all categories, the depth parameter is not considered. It can be used to compensate some effects due to scattering that is undesirable for forming a qualified image. Also, it seems that the noise is an important factor to accuracy estimation of depth as well as the rigid and flexible motions. To decrease these effects, it must be prepared some methods before obtaining the spectra. Some theoretical formulas could be used to rapidly assess the impact of different scatter correction strategies on image quality.^[42]

Several methods for the effects of scattered events have been proposed.^[38-49] The difference between methods is the way of estimating the scatter contribution. As known, the TEW and ETEW methods can directly calculate the scattered photons in the main window using the subwindows that are located at both sides of the main window, the scatter component corresponds to the source position and the shape of the scatterer, the localization of the source distribution is considered as well as the shape of the scatterer. The errors in estimating scattered photons are due to the center location of the subwindows, which are defined on both sides of the main window. Scattered fractions were estimated correctly within an 8% error using the 26% window and within a 10% error using the 20% window. When we select a 2-6 KeV subwindow, the scatter correction can perform well for this phantom.^[17] In the actual gamma camera system, however, the determination of the optimum location of the center and the width of the subwindow should be based on the results of experiment because the system may differ from the simulation model in the stability of gain and other factors.

One can estimate the count ratios of the subwindows to the main window. From this result, if the radionuclide that has a single photopeak is measured using narrow subwindows and a large main window, the measured counts of the right subwindow will be <5% of the main window. Therefore, the counts of the high-energy subwindow (C_{right}) may be insignificant for the radionuclide, and then one can set the count C_{right} to zero. However, for a radionuclide having multiple photopeaks, or for a combination of radionuclides, the count (C_{right}) may not always be insignificant. In clinical cases, it is difficult to get energy spectra at pixels with good statistics.

The μ_{sc} is a fraction of total attenuation coefficient indicating only the Compton cross-section in a distinct substance. This parameter will be varied with the impurities, crack and cleavages existing in substance. The lower the μ_{sc} , the more stable the C_s/C_p value, thus the effect of the depth value will be insignificant. While the μ_{sc} is increased, this ratio will be unstable in each depth. The advantage of estimating the μ_{sc} is for better compensation of scattered counts in images with forming the μ_{sc} -map.

CONCLUSIONS

The primary and scattered counts were calculated by TEW and ETEW methods at the various depths, and then the data

extracted from these methods were applied in the CSPF method. As a result, the EWW was obtained at each depth. The widths of energy-window calculated with primary photons were in good agreement with those of scattered photons.

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