Research article

Gravity measurement to probe the depth of African-continental crust over a north-south profile: theory and modeling

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HIGHLIGHTS

- A computer code is developed to calculate thickness of crust from gravity data.
- The calculated thickness of the African continental crust ranges from 36 to 44 km.
- The results of the developed computer code were tested with a forward model.

ABSTRACT

Based upon gravity measurements and calculations, the depth of the African continental crust is estimated. Taking as constraints the mass and radius of earth, and measured gravity, this theoretical method explores the use of gravitational potential to calculate the absolute gravity at three locations in Africa (e.g., Cape Town at latitude -34°, central Africa at latitude 0, and Benghazi at latitude 32°). The computational method uses as input a continental crust density $\rho_1 = 2.65$–2.75 g/cm^3 while compromising the oceanic crust density ρ_2 to maintain the average crust density of the planet fixed at $<\rho_{12}> = 2.60$ g/cm^3. Crustal depth is assumed uniform around the earth and kept as a free parameter to adjust for the best fitting of gravity but using values of less than 100 km. A solid angle α_o is a solid angle whose vertex is at the center of earth used to separate continental and oceanic crusts ($\alpha_o = 10°, 20°, 35°$). The results obtained for the continental crust were H = 36 km near continental edges at both Benghazi and Cape Town, whereas H = 44.4 km at the center of continent. These results are in excellent agreement with those reported by Tedla and coworkers (H = 39 ± 5 km) using an Euler deconvolution method. Our theoretical results from the developed code are also corroborated by results of numerical forward modeling supporting our code's reliability for further geoscience explorations.

1. Introduction

The African tectonic plate is a complex assembly of many accreted terranes and collision zones. The African plate is composed of several old stable cratons deeply rooted in the subcontinental lithospheric mantle, which are connected by highly deformed orogenic belts. As a result of late Proterozoic assembly, Africa was part of the Pangaea supercontinent, with South America on one side and Eurasia and India on the remaining sides. The breakup and dispersal of Pangaea led to the Atlantic and Indian Ocean rifted and passive margins. The ongoing Cenozoic to Recent collision of Africa with the Eurasian plate has resulted in the highly active seismic regions bordering the Mediterranean Sea. The break-up of the NE margin of Gondwanaland separated the Arabian Plate from Africa, and formed the Red Sea rift and Gulf of Aden. Active rifting of oceanic and continental lithosphere has occurred along much of the eastern side of the African Plate. All these tectonic complexities make the African-

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continental crust a geological nexus for the scientific community, where many structural studies that can improve our current understanding of the plate tectonic mechanism are being conducted.

In this perspective, estimating the African crustal thickness is of great importance as crustal thickness varies significantly across the continent and mapping these variations is important to understanding the different continental rifting processes in Africa. Crustal thickness can provide key elements for large scale structural studies (Pasyanos et al., 2014), tectonic delineation (Tamura et al., 2016), geothermal modeling (Pastorutti and Braitenberg, 2019) as well as subduction systems (Kind et al., 2002) and bring a better insight to the lithospheric structure and its composition in depth.

Several studies have been carried out to reveal the most adequate crustal model of the African continent. Tedla et al. (2011) proposed a continental scale crustal model for Africa by modelling the free-air gravity anomaly. Delph et al. (2015) helped to characterize the crustal thickness, composition and Moho impedance contrast across the Kaapvaal and Zimbabwe cratons southern Africa using seismic data. Ghomsi (2020) estimated the crustal thickness beneath the Atlas region in the north. Ebinger (2017) summarized seismic and magnetotelluric constraints on crustal thickness, and also discussed the role of magmatism in shaping the crustal structure at the active rift African zones, such as the Cameroon volcanic line or the east African rifting system. Casten and Snopek (2006) presented a 3D gravity interpretation software named 3GRAINS which models the structure of lithosphere and asthenosphere of Earth. Bagherbandi (2012) developed a MatLab code called MohoIso which calculates the lithosphere thickness using a gravimetric-isostatic mode. Kaban et al. (2004) presented a new isostatic model of the lithosphere using gravity. There are also other geophysical investigations using seismic waves and helped to estimate the Moho discontinuity of several regions in Africa such as Kimberley craton (Abbott et al., 2013; Szwillus et al., 2019). Zandersons and Karušs (2020) applied the Parker–Oldenburg algorithm to the gravity data to understand the Moho depth under Latvia. Gozzard et al. (2019) applied an inversion to the satellite gravity data to map the crustal thickness in the South China Sea. Chisenga et al. (2019) inverted the gravity data using the regularized Bott's inversion method in the spherical approximation to model the crustal thickness of Antarctica. Lenczuk et al. (2019) investigated the crustal structure of central Europe using satellite gravity gradients. Motta et al. (2019) studied the Amazonian Craton using forward and inverse modeling of the satellite gravity data to reveal its crustal structure. Kusznir et al. (2018) investigated the crustal structure of the Equatorial Atlantic using 3D gravity inversion.

Several geophysical methods allow the exploration of the lithospheric crustal structures such as seismic refraction and wide angle reflection, receiver functions (Parera-Portell et al., 2021), magnetotelluric exploration or potential fields methods. The gravity method remains one of the most affordable and suitable for deep crustal mapping (Grushinsky et al., 2007; Tenzer and Gladkikh, 2014; Bai et al., 2014). In fact, satellite-derived gravity models have already shown promising results in delineating deep structural features since the gravity field anomalies due to variations in crustal thickness are among the largest signals sensed by satellite-borne gravity measurements. We need to state the non-uniqueness of gravity methods, and the limitations imposed by assumptions of crustal density (Blakely, 2009).

The objective of this study is to provide an open-source program that approximates the crustal thickness of African continent at any given location based on density and solid angle data values. The solid angle is used to cover the location where the gravity is measured and around it within an area of solid angle (alpha x R^2). Also the aim is to introduce an easy method and computationally fast to calculate the thickness of crust at any location on Earth and the developed method yield reliable results. Ebinger et al. (1989) and Moucha and Forte (2011) pointing out density variations in the convecting asthenosphere beneath Africa, however this study has an assumptions that the density of this zone is constant.

The present investigation consists of a combination of experimental data and theoretical method used to fit the experimental data by adjusting the crust depth, mass density, and solid angle centered at the gravity measurement stations. In the computation process, each of the 3 variables has range of variations of physical values. The theoretical method is based on gravitational potential from which gravity field can be calculated at the surface of the earth. The method explores a model, in which the planet earth is assumed to be composed of crust and an inner sphere representing the mantle and the core together (i.e., having a uniform constant density $\rho_0 = 5.545$ g/cm^3). This is calculated to normalize the total mass of Earth to attain the total real mass of Earth after varying the densities of crust (oceanic and continental). We mention also that density of mantle is constant. The model splits the crust into continental and oceanic (i.e., having two different mass densities $\rho_1 > \rho_2$, respectively). For any location on the surface of continent, the model uses a solid angle to separate land from ocean in focusing the calculation of gravity at that location.

The theoretical model and method are explained in next section. The experimental data collection and conversion are explained in section 3. The experimental data used in this paper is a public gravity dataset and obtained from Earth Gravitational Model (EGM) 2008 (Bonvalot et al., 2012) and represent Bouguer anomalies in mGal. The computational modeling is explained in detail in section 4 and followed by the forward modeling. Forward modeling is introduced in order to compare current proposed method with a forward model developed using a commercial software named Geosoft and presented in section 5. The last section summarizes our main findings.

2. Theoretical model and method

From a physics perspective, the concept of potential energy is more fundamental than the concept of force. Actually, the conservative force should derive from potential energy function. Similarly, the gravity (i.e., g, normalized force per unit mass) should derive from gravitational potential (i.e., V, normalized potential energy per unit mass). The approach of so-called analytical mechanics is very powerful and has the ability to solve a broad ensemble of problems (Kumar-Roy 2008; Jacoby and Kumar-Roy 2009). So, we decided to undertake the approach to first calculate "V" then derive it to get "g" at the surface of the earth and specifically on the African continent. To take account of the infrastructure of the planet earth and specifically the crust, we have generated code to take as input data some variables (parameters like: H = crust depth, its mass density in continental and oceanic districts ρ_1 and ρ_2, respectively, and the solid angle of solid value α_0, see Figure 1). So, by giving these parameters some realistic values and recognizing the constraint of the total mass of the earth, one could adjust the value of H to yield gravity at any location which is close to the experimental value. We obtain H within a certain range justified as estimates with inclusion of theoretical error bars.

2.1. Original problem

To study the effect of variation of mass density of the crust on the gravity at a certain point "P" on surface of earth we need to find g_p.

We assume the density to be given as functional of radial distance (r) and solid angle (θ) to be $\rho(r, \theta)$ as follows (Figure 1):

$$\rho(r, \theta) = \begin{cases} \rho_0 \text{ if } r < r_0 \\ \rho_1 \text{ if } r_0 < r < R \text{ and } \theta \leq \alpha_0 \\ \rho_2 \text{ if } r_0 < r < R \text{ and } \alpha_0 < \theta \leq \pi \end{cases} \tag{1}$$

R = average radius of Earth ($\cong$ 6378 Km).

α_0 = solid angle limit to define the solid angle of the top layer having distinct mass density (ρ_2). This layer is to model the crust and has a thickness H ($H \leq 100$ Km).

ρ_0 = average mass density of the earth w/o crust.
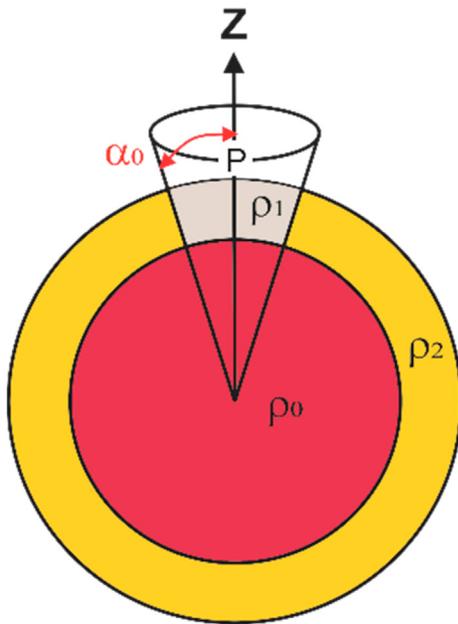
Figure 1. The theoretical model using a solid angle α_0 to represent the continental crust with a density ρ_1, whereas the rest of the crust has a density ρ_2 and the mantle and core with a density ρ_0. The radius is 6378 km.

ρ_1 = average mass density of the cap (within the continental crust).
ρ_2 = average mass density of the crust without cap. Cap is the portion of the continental crust.

2.2. Solution

Background: (1) For a homogenous mass distribution in a sphere of radius R and a total mass, M, we know that: $\overrightarrow{g} = -g(r)\hat{r}$

$$g(r) = \begin{cases} \dfrac{GM}{r^2} \ldots if\ r > R \\ \dfrac{GM}{R^3}r \ldots if\ r \leq R \end{cases} \tag{2}$$

whereas, the gravitational potential (Figure 2) should be given by:

$$V(r) = \begin{cases} -\dfrac{GM}{r} \ldots if\ r > R \\ -\dfrac{GM}{2R^3}\left(3R^2 - r^2\right) \ldots if\ r \leq R \end{cases} \tag{3}$$

It should be emphasized that Eqs. (2) and (3) to be valid under the assumption of having a sphere of uniform mass density (Kumar-Roy, 2008; and Blakely, 2009). Namely, in the present investigation, we will treat the mantle and the inner structure of the earth using a similar model with mass density ρ_0. Whereas, the crust is treated in distinct and having two different mass densities ρ_1 and ρ_2 to correspond to continental and oceanic crusts, respectively.

(2) We will derive gravity $g(r)$ from gravitational potential $V(r)$.

So, the infinitesimally small element of V(r) due to the contribution of dm is:

$$dV = -G\frac{dm}{S} \tag{4}$$

where S is the distance between dm and point P shown in Figure 3, and $dm = \rho.d\Omega = \rho(r^2 \sin\theta dr d\theta\ d\Phi)$ in spherical coordinates.

It should be further emphasized that, of course $\overrightarrow{g} = -\overrightarrow{\nabla}V$
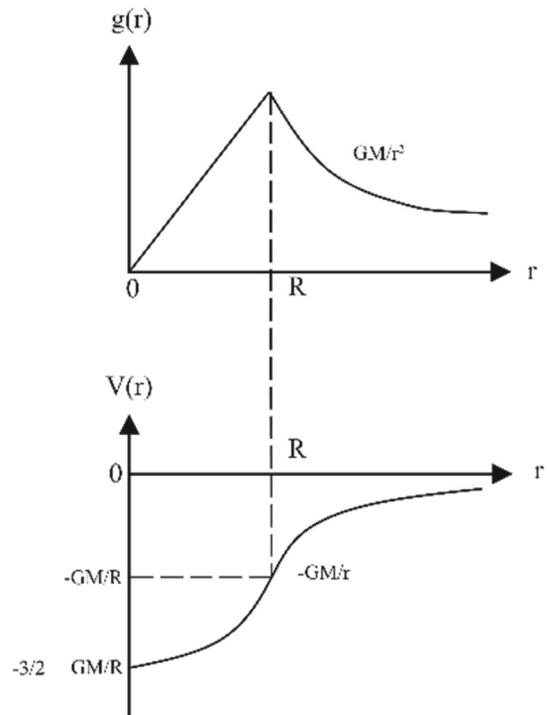
Figure 2. Gravitational field and potential versus the radial distance from the Earth's center.

Hence, one gets:
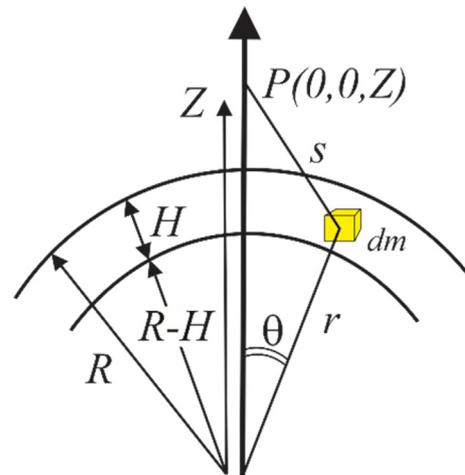
Figure 3. Calculation of gravity at point P near the Earth's surface based on elementary volume of mass dm taken from within the crust.

$$dV = -G\frac{\rho(r^2 \sin\theta dr d\theta\ d\Phi)}{S} \tag{5}$$

Consider an arbitrary point "P" at fixed radial distance Z from center of the Earth (i.e. $Z > R$). The Rule of Cosines would yield:

$$S^2 = r^2 + Z^2 - 2rZ\cos\theta \tag{6}$$

$$dV(r,\theta,\Phi) = -G\frac{\rho(r^2 \sin\theta dr d\theta\ d\Phi)}{\sqrt{r^2 + z^2 - 2rZ\cos\theta}} \tag{7}$$

Now:

$$dV(r,\theta) = \int_0^{2\pi} dV(r,\theta,\phi)d\phi \tag{8}$$

$$dV(r,\theta) = -2\pi G\rho \frac{r^2 \sin\theta \, dr d\theta}{\sqrt{r^2 + z^2 - 2rz\cos\theta}} \tag{9}$$

We will use this to find the contribution of the cap and the rest of crust in V_p.

We divide the Earth into 3 zones of respective densities ρ_0, ρ_1 and ρ_2. So, the gravitational potential at point "P" due to the contributions from these 3 zones would be:

$$Vp = V_0 + V_1 + V_2. \tag{10}$$

(a) Zone of ρ_0:

For a point outside a sphere, the potential varies as $= -G\frac{M}{r}$. So, simply:

$$V_0 = -G\frac{\rho_0 4\pi}{3Z}(R-H)^3 \tag{11}$$

(b). Zone of ρ_1:

(C). Zone of ρ_2:

$$V_2 = -\frac{4\pi G\rho_2}{3Z}\left(R^3 - (R-H)^3\right) - V_1(\rho_2) \tag{13}$$

The integration in Eq. (12) is carried out numerically using Simpson's rule. For checking, one can put $\rho_1 = \rho_2 = \rho_0$ then check for $Z = R$ to get $g = 9.81$ m/s^2.

We calculate $Vp = V_0 + V_1 + V_2 = V(Z)$, then derive it to get:

$$g(Z) = -\frac{dV}{dZ} \tag{14}$$

3. Application to real data

3.1. Relative gravity data along a bisector profile line going north-south in Africa

The data were collected along the line shown in Figure 4. The data were interpolated from Bouguer anomaly map of Africa, which was derived from the EGM 2008 and was released by the National Geospatial-Intelligence Agency (NGA, USA) (Bonvalot et al., 2012). The Bouguer anomaly map with average over 2.5 × 2.5 arc-minutes is computed from the EGM2008 spherical harmonic coefficients (Pavlis et al., 2008). The map itself is comprised of positive and negative anomalies with various

$$V_1 = \frac{2\pi G\rho_1}{Z}\left(\frac{Z}{2}R^2 - \frac{R^3}{3} - \frac{Z}{2}(R-H)^2 + \frac{1}{3}(R-H)^3\right) - \frac{2\pi G\rho_1}{3Z}\left(\left[R^2 + Z^2 - 2RZ\cos\alpha_0\right]^{\frac{3}{2}} - \left[(R-H)^2 + Z^2 - 2Z(R-H)\cos\alpha\right]^{\frac{3}{2}}\right)$$
$$-2\pi G\rho_1 \cos\alpha \int_{R-H}^{R} dr\sqrt{r^2 + Z^2 - 2rZ\cos\alpha_0} \tag{12}$$
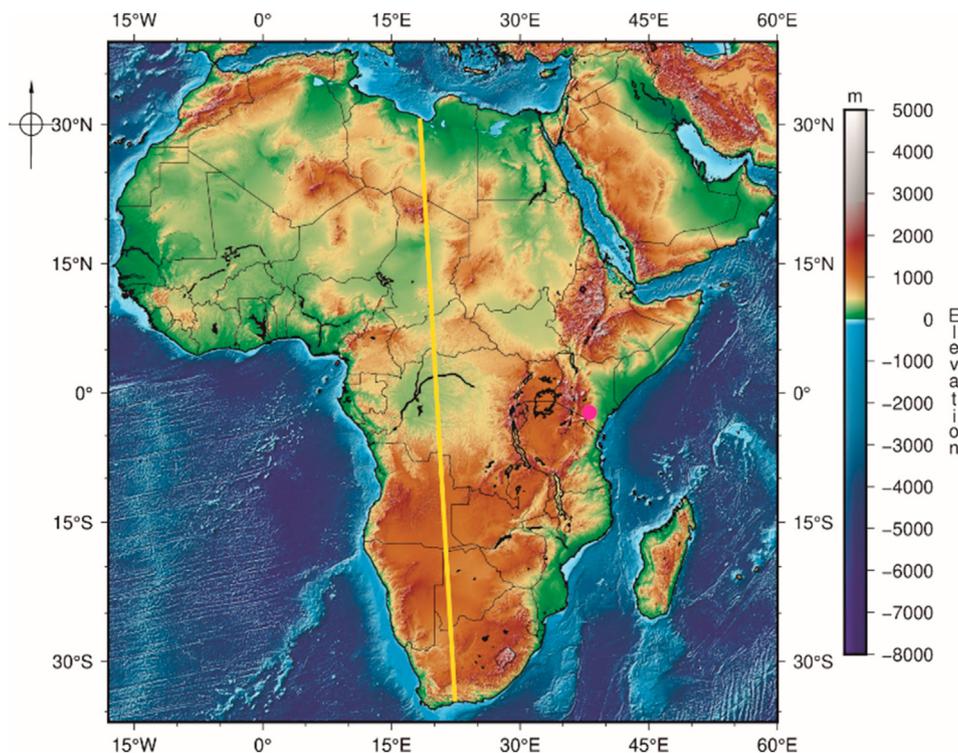
Figure 4. Elevation map of Africa showing the gravity profile crossing Africa from South to North. Pink dot shows the location of the tied absolute gravity reference station.
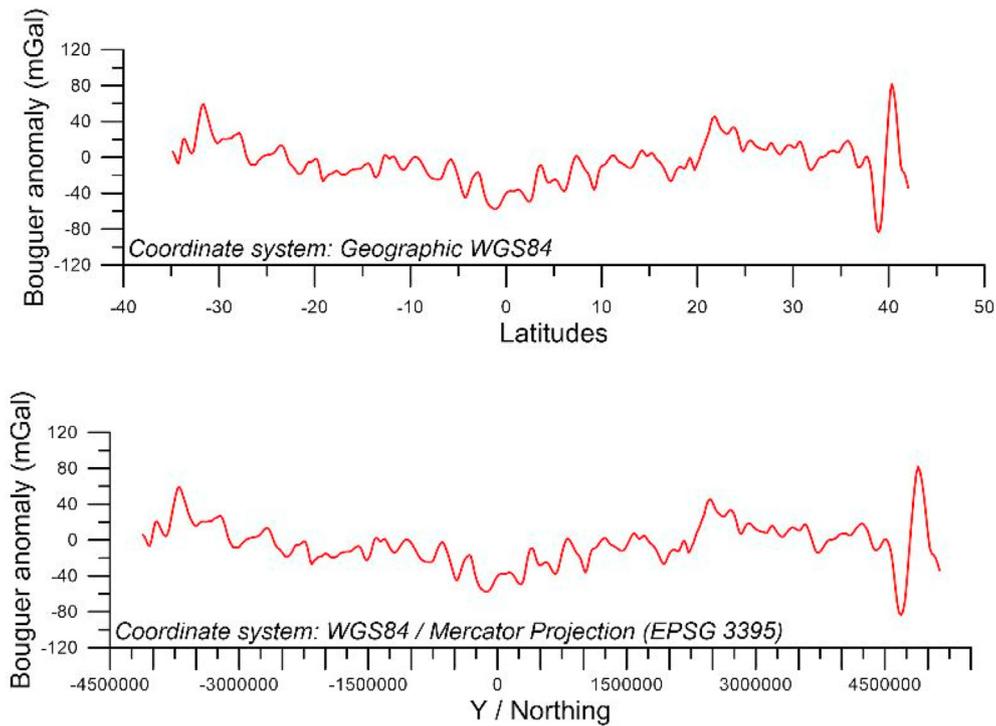
Figure 5. Bouguer anomaly through the profile shown in Figure 4 in both latitudes (degrees) and northing (m). The left side of the figure refers to South Africa and right side of the figure refers to North Africa.

sizes, shapes, patterns, and augmentations separated by zero Bouguer value (see Figure 5).

The traverse line is oriented straight north-south Africa continent, with equal-spacing between data points ($\approx$20 km) based on a set of 500 gravity data points to calculate the most accurate corresponding absolute gravity data points.

The Bouguer anomalies along the profile are of the order of -80 to 80 mGal. The positive nature of the gravity field is most likely because of high-density rocks, which indicates the proximity of high density rocks to the surface. The abrupt changes of the gradients of the gravity field at the north and south ends of the profile, near the coastal areas, are due to the transition from continental crust to the oceanic crust.

3.2. Conversion of relative gravity to absolute gravity

The conversion of relative gravity to absolute gravity is achieved by tying the measurements to a station whose absolute gravity is accurately known through the following equation:

$$G_{abs} = G_{absR} + G_{obs} \qquad (15)$$

where.

G_{abs}: absolute gravity value at the station.

G_{absR}: absolute gravity value at the reference station (known value).

G_{obs}: measured (observed) value at the station (mGal).

The reference station used in this calculation is located in Kajiado County, Kenya (see Figure 4) and has coordinates of 254336 m Easting and 9796593 m Northing with updated absolute gravity value of 977563.2687 mGal (Korir et al., 2016). Figure 6 shows the normal gravity and the absolute gravity calculated at each station using Eq. (15).

It is very clear that the normal gravity depends on the latitude of the station (curved plot) and the absolute values after the survey is tied to the very known absolute gravity (in linear plot).

4. Computational modeling

The aim is to estimate the crust depth of the African continent versus latitude. Toward this aim, we use the experimental data of absolute
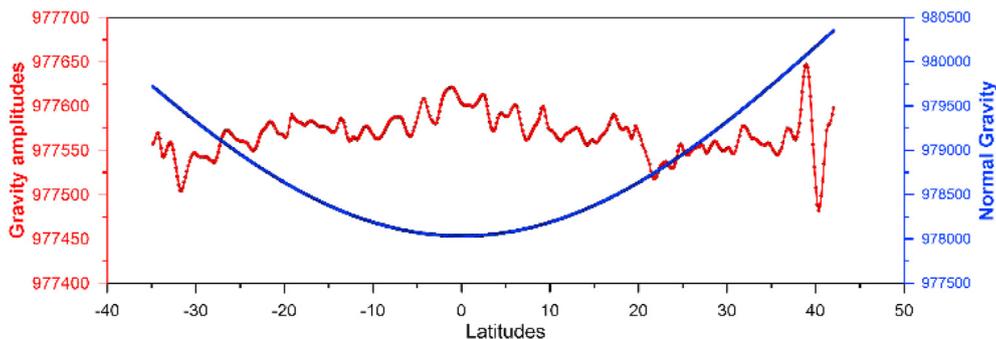
Figure 6. Normal and absolute gravity through the gravity profile.
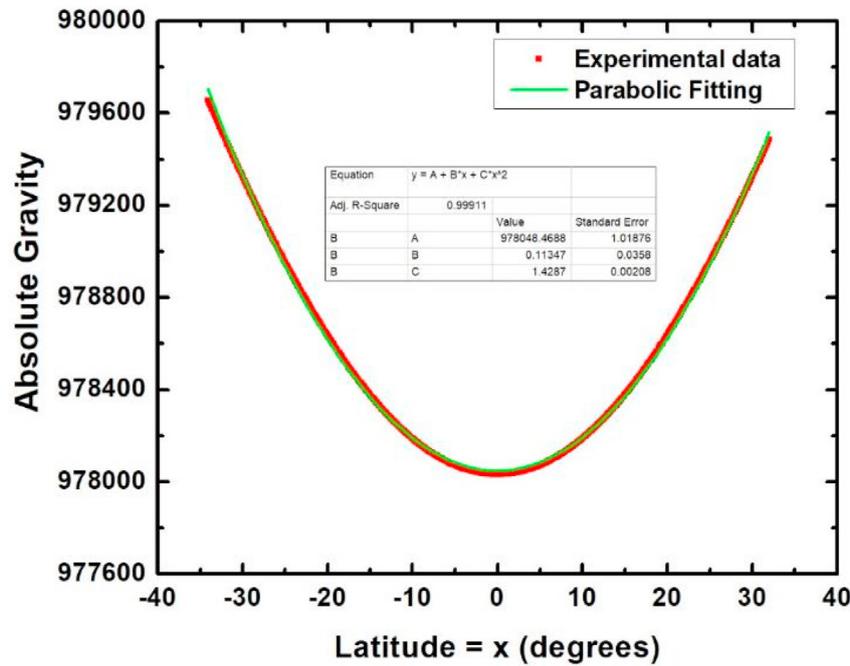
Figure 7. Experimental absolute gravity across the African continent from south to north (i.e., versus Latitude) showing excellent parabolic variation.

gravity versus latitude. Figure 7 shows the variation of the experimental absolute gravity values versus latitude within the interval of values $[-34°, +32°]$. We emphasize that the measurements have been done across the whole continent and in our modeling, we consider the data taken on the African land (i.e., in the latitude range -34° ≤ latitude ≤32°). These two limits are located near Cape Town (18°E, 33.5°S) and Benghazi (20°E, 32°N), respectively. The first observation about the mapping between the absolute gravity data and locations on the African map is that the data have no correlation with the morphology of the continent. So, likely these data do correlate strongly to the continental crust structure and size (i.e., mass density = ρ_1 and crust depth = H, respectively).

We utilize our model to calculate the gravity based on the gravitational potential. It is important to start by setting the reliable ranges of parameters such as length scales and mass densities. In our present model we consider the earth to be made of solid sphere of radius R = 6378 km with a crust of uniform thickness (H) but having two different mass densities (ρ_1 and ρ_2, averaged to a value ρ_{12}). For continental crust, the mass density should be in the range: (ρ_1 = 2.65–2.75 g/cm³). By taking the average of the experimental absolute gravity on the continent to be $<g>$ = 978525 mgal, and using an averaged crust thickness ($<H>$ = 40 km) and an averaged crust mass density ($<\rho_{12}>$ = 2.60 g/cm³), one could adjust the mass density of the earth below the crust to a value ρ_0 = 5.545 g/cm³. Of course, the crust should be of two types: (i) Oceanic crust: having mass density less than $<\rho_{12}>$ and depth smaller than $<H>$; and (ii) Continental crust: having mass density larger than $<\rho_{12}>$ and depth also larger than $<H>$. The African continental crust thickness and density will be under the scope of present modeling.

One further remark which one should not overlook and must take into account in the modeling, especially to keep the mass of the earth physically correct, is the proper interplay between the two densities of the crustal layer, as follows:

$$(\rho_{12}) = \rho_1 \frac{\alpha}{180°} + \rho_2 \frac{180° - \alpha}{180°} \quad (16)$$

Furthermore, it is remarkable that the variation of the experimental absolute gravity versus latitude looks like a parabola. Least-square non-linear fitting, done using a quadratic polynomial, seems to yield an

Table 1. Modeling of crust depth "H" of African continent using continental mass density ρ_1 = 2.65 g/cm³. Zones shaded in yellow correspond to better theoretical estimations of H.

Location (Latitude = x)	Experimental Absolute Gravity	H (km) If α = 10° [Average in km]	H (km) If α = 20° [Average in km]	H (km) If α = 35° [Average in km]
Cape Town (x = -34°)	979657.19	34-35 [34.50]	34.55-35.5 [35]	33.70-34.70 [34.20]
Center of Africa (x = 0)	978032.68	39,60-40,10 [39.85]	36.54-37.50 [37]	40.70-41.40 [41.05]
Benghazi (x = 32°)	979485.57	34-35 [34.50]	34.50-35.50 [35]	33.70-34.70 [34.20]
Parabolic fitting: H = A + Bx + Cx²	N/A	A = 39850 ± 250	A = 36950 ± 550	A = 41050 ± 350
		B = -9.93 ± 0.47	B = -3.62 ± 0.09	B = -12.72 ± 0.28
		C = -4.92 ± 0.24	C = -1.79 ± 0.04	C = -6.30 ± 0.14

excellent fit to the variation of g versus latitude (i.e., H = A + Bx + Cx², with A = 978048.46, B = 0.11 and C = 1.43, with B noticeably small so the function looks like having even-parity symmetry with respect to latitude variable). There are two impressions/trends one could depict from such behavior of g versus latitude: (1) Latitude zero (x = 0) lays just on the equator and the even symmetry reveals that the inner structure of earth is almost ideal spheroidal where the equatorial plane represents its symmetry mirror; (2) The gravity seems to be weakest at the center of the continent and thus should reveal that the crust might be thickest there. Of course, the mass density of continental crust remains less than that of the upper mantle. Consequently, the thinner the crust the larger the gravity would be. So, one would expect gravity to be larger at the oceanic crust. One should quote the variation of Δg = 0.016 m/s² between the south edge and the center of the African continent.

For reasonable values for continental-crust mass density we assume it is made of rocks like granite and we consider two extreme values for its mass density: ρ_1 = 2.65–2.75 g/cm³. In Table 1 and Figure 8, we took ρ_1 = 2.65 g/cm³; whereas in Table 2 and Figure 9, we took ρ_1 = 2.75 g/cm³.
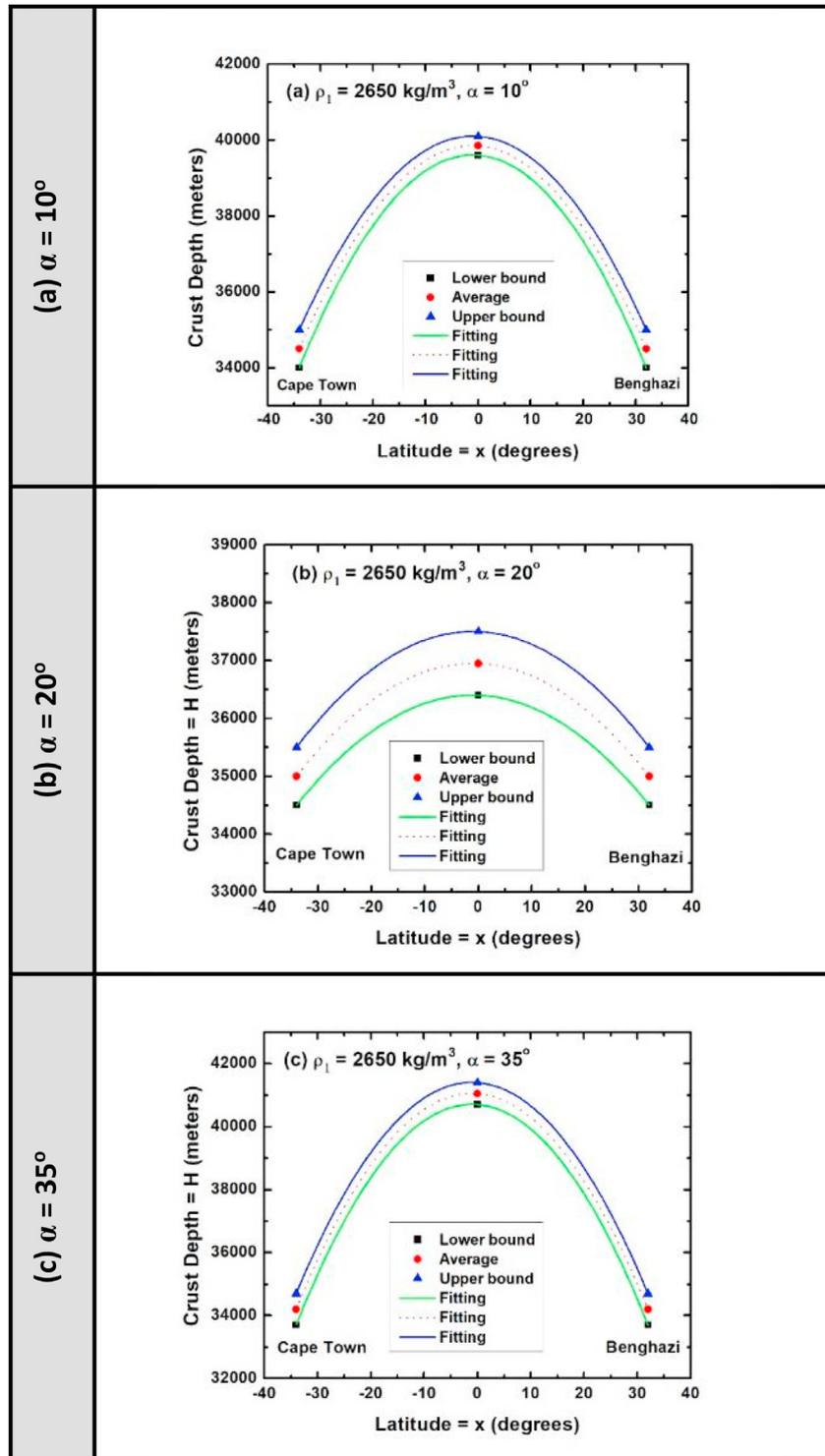
Figure 8. Variation of Crust depth of African continent versus Latitude using continental mass density $\rho_1 = 2.65$ g/cm^3 for three computational solid angles: (a) $\alpha = 10^o$; (b) $\alpha = 20^o$; (c) $\alpha = 35^o$.

Table 2. Modeling of crust depth "H" of African continent using continental mass density $\rho_1 = 2.75$ g/cm^3. Zones shaded in yellow correspond to better theoretical estimations of H.

Location (Latitude = x)	Experimental Absolute Gravity	H (km) If $\alpha = 10^o$ [Average in km]	H (km) If $\alpha = 20^o$ [Average in km]	H (km) If $\alpha = 35^o$ [Average in km]
Cape Town (x = -34°)	979657.19	35.50-36.50 [36]	35.50-36.50 [36]	34.60-35.60 [35.10]
Center of Africa (x = 0)	978032.68	40.50-42.50 [41.50]	40.50-42.70 [41,60]	43.8-45.00 [44.40]
Benghazi (x = 32°)	979485.57	35.50-36.50 [36]	35.50-36.50 [36]	34.60-35.60 [35.10]
Parabolic fitting: $H = A + Bx + Cx^2$	N/A	$A = 41500 \pm 1000$	$A = 41600 \pm 1100$	$A = 44400 \pm 600$
		$B = -10.21 \pm 0.93$	$B = -10.40 \pm 4.71$	$B = -17.27 \pm 0.19$
		$C = -5.065 \pm 0.46$	$C = -5.15 \pm 0.33$	$C = -8.55 \pm 0.09$
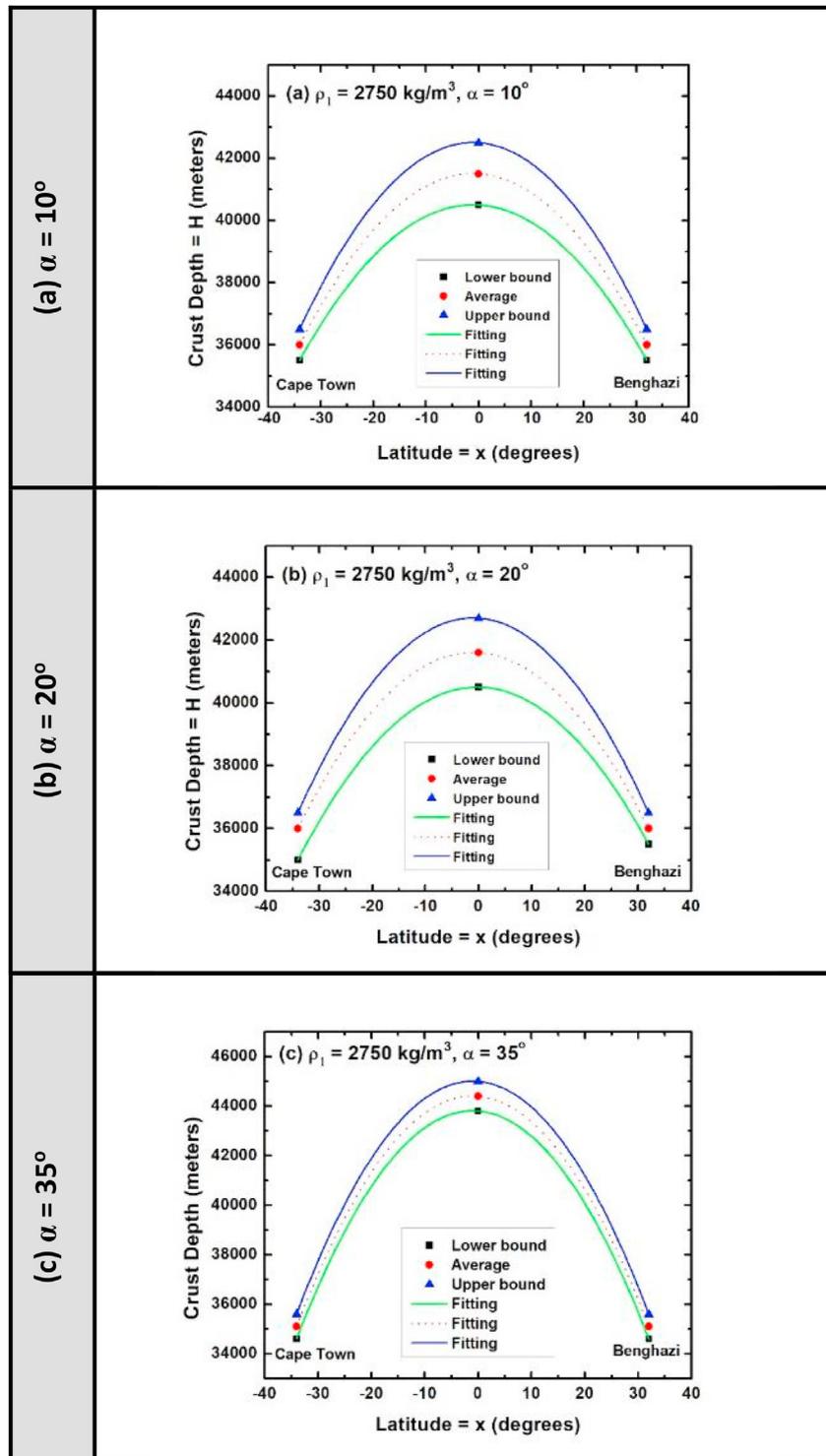
Figure 9. Variation of Crust depth of African continent versus Latitude using continental mass density $\rho_1 = 2.75$ g/cm^3 for three computational solid angles: (a) $\alpha = 10^o$; (b) $\alpha = 20^o$; (c) $\alpha = 35^o$.

For both cases, we started with measured gravity values of absolute gravity at three different locations: (a) Near Cape Town (South Africa), of latitude x = -34°, with gravity 979657.19; (b) Center of Continent at equator, of latitude x = 0, with gravity 978032.68; and (c) Near Benghazi (Libya), of latitude x = 32°, with gravity 979485.57. For each of these three locations, we modeled the gravity (i.e., we performed computation to obtain close to the same experimental value of gravity at that location by varying/adjusting the crust depth "H"). In our computation we

considered three different solid angles of solid angle values: $\alpha = 10^o$, 20° and 35°. Crust within the solid angle is assumed to have continental-crust mass density ρ_1 and the rest of the crust around the globe is supposed to have and averaged oceanic-like crust mass density ρ_2. (We emphasize here that $\alpha_{max} = 35^o$) is set based on the size of the African continent.

The center of the continent at the equator of latitude = 0, where gravity was measured, has also a longitude of 20.46 ^{o}E. So, from this center to the Senegal west coast, of longitude 14.5 ^{o}W, therefore a
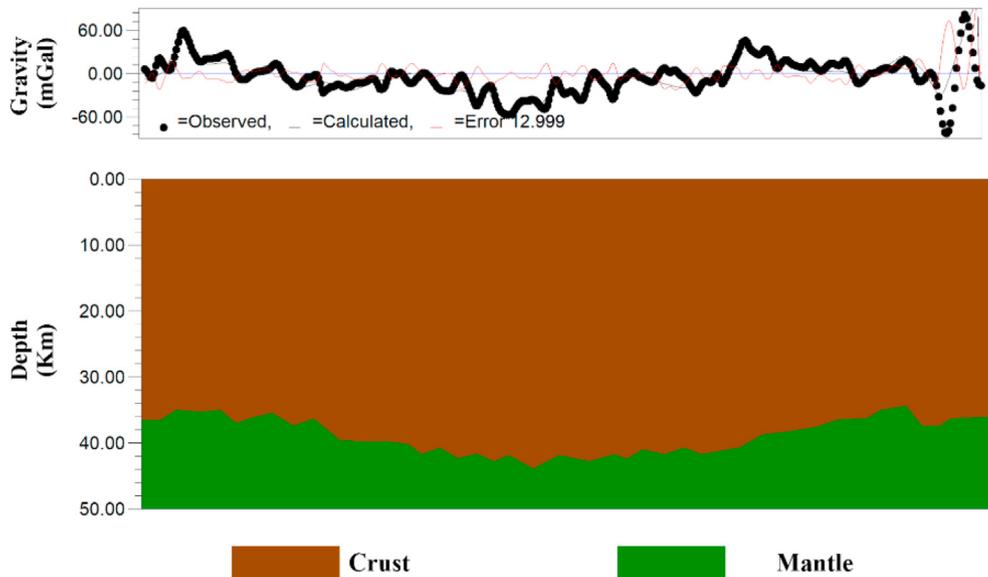
Figure 10. Forward model of Bouguer anomaly from Cape Town (South Africa) left of the figure to Benghazi (Libya in North Africa) right of the figure showing the center of Africa to have a thick crust of about 44 km and south and north African continent to have a crust of about 36 km thickness.

Table 3. Comparison between forward model results and current developed method.

Location	Thickness of Crust (km)		
	Cap Town (South Africa)	Center of Africa	Benghazi (North Africa)
Forward model	36	44	37
Current developed method	36	44.4	36

distance of a polar-coordinate angle of 35° is justified. In addition to this, Cape-Town in South Africa is located at latitude of -34°, which further corroborates and justifies the use of ($\alpha_{max} = 35°$). Furthermore, pursuing our discussion above, the use of the gravity measurements at three different locations with three different hypothetical computational solid angles enabled us to obtain estimates of crust depths with three variances (see Figures 8 and 9). Thereafter, we made a parabolic fit to the three gravity values versus latitude for each solid angle. In each location, the simulation of experimental gravity allowed us to predict two extreme values for H with their average and we made three curves which are presented in each panel. The results are summarized in Table 1 and Table 2 and plotted in Figures 8 and 9 for the mass density values $\rho_1 = 2.65$ g/cm^3 and 2.75 g/cm^3, respectively.

At fixed continental-crust mass density, one can notice two trends versus the solid angle (α). At the continent edges, H increases with decreasing angle α. As the local mass density at the edges should be lower, then one can deduce that H could be as low as $H_{min} = 34.5$ km or even smaller at both edges, especially at the latitude of Cape Town. The second trend is that at the continent center: H increases with the increase of solid angle α. Of course, reliable calculation recommends that α be as large as 35°. Meanwhile the crust density at the center of continent should also be large. So, one can predict that the crust should be as great as $H_{max} = 44.4$ km at equator in the center of continent.

5. Forward modeling of gravity profile from Cape Town (South Africa) to Benghazi (Libya)

In order to check the results of the developed method, we pursued modeling the Bouguer anomaly data along the profile passing from Cape Town in South Africa to Benghazi in Libya (north Africa) using a crust density of 2.75 g/cm^3 and a mantle density of 5.55 g/cm^3. The forward modeling was obtained by using the GM-SYS code of Geosoft Montaj software (Seequent, 2021). The forward model result (Figure 10) shows a crust thickness ranging from 36 km to 44 km (Table 3).

Benghazi region (Libya, North Africa) is part of the East African Metacraton and tectonically it belongs to East Sahara Phanerozoic cover. The Cape Town region (South Africa) is tectonically part of Saldania.
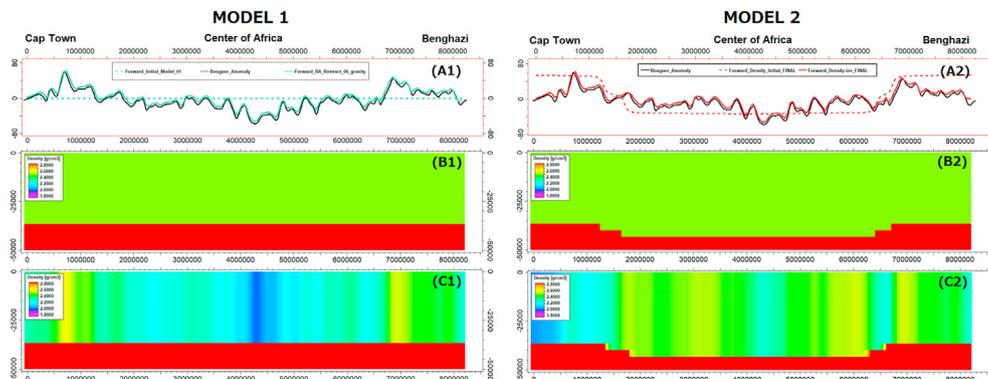
Figure 11. Bouguer anomaly inversion modeling for two different configurations of crustal structure. (A1, A2): Bouguer anomaly (solid black line) with inverted gravity (solid colored lines) and initial model gravity (dashed colored lines). (B1, B2): initial density models. (C1, C2): Inverted density models after 100 iterations.

Central Africa is part of Congo tectonic region. Begg et al. (2009) estimated the crustal thickness of different tectonic regions in Africa. They estimated a value of 39 km for the three regions (Cape Town, Central Africa, and Benghazi). The differences between results of Begg et al. (2009) and our results are +/- 3 to +/- 5 km.

Our proposed method is different with Waltkins et al. (2015) method because they used spherical coordinates but we used just solid angle of one geometrical variable (alpha) which is the solid angle. The developed method is also different with the method presented by Younis et al. (2013) because their cap has azimuthal angle phi-zero and polar angle lamda zero and the point P on the cap is moving with two variables solid angle theta and another polar angle around the axis of the solid angle called alpha. Our method is simpler and the point P is at the center of the cap (centered at gravity stations observed at surface) and it extends as far as the land of the continental crust.

5.1. Inversion modeling

In order to verify the validity of the crustal model estimated from our developed method (Tables 1 and 2), we estimated the density model of this Bouguer anomaly using the inversion modeling algorithm proposed by Priezzhev and Pfutzner (2011).

We simulated two different scenarios of the crustal and mantle structure: the first model represents a flat crust–mantle interface at 36 km depth (Figure 11-B1) and the second model represents an irregular interface with a maximum depth of 45 km and a minimum depth of 36 km (Figure 11-B2). The objective of the experiment is to prove which models best satisfy the gravity data at the surface. We created initial models for inversion using a fixed mantle density of 5.5 g/cm^3 and a crustal density of 2.3 g/cm^3, with gravity search limits between 1.8 g/cm^3 and 2.8 g/cm^3. The algorithm then estimates the crustal densities by inversion, finding the best match to the Bouguer anomaly values measured at the surface over 100 inversion iterations. The crustal density inversion results (Figure 11) for both models show overfitting of the inversion gravity results compared to the measured Bouguer anomaly data (Figure 11-A1 and 11-A2). The curves match, from very basic initial models, which means that the inversion modeling scheme properly reproduced the best density models to fit our gravity solution.

The inverted density model 1 (Figure 11-C1) displays a significant lateral variation at the center of Africa, which gives a nonrealistic representation of the constant density distribution assumed in our crustal model. In contrast, the irregular model 2 (Figure 11-C2) shows a more homogeneous crustal density model, that resolves the lateral density variation at the surface and matches the model described in Figure 10. Results from this inversion modeling of the density show that the surface Bouguer anomaly variation between Cape Town and Benghazi can be interpreted by a thickening of the African crust from 36 km to 45 km at the center of Africa, as suggested in our previous experiment.

6. Summary and conclusions

A combination of experimental and computational efforts was focused on estimating the crust depth of the African continent. The computational method explores the concept of using gravitational potential theory to derive the gravity. It keeps as constraints: the mass and radius of the earth and both the averaged crust density as $<\rho_{12}> = 2.6$ g/cm^3 and the gravity measured at the surface of continent. The developed code uses as variables: (i) continental density $\rho_1 = 2.65$–2.75 g/cm^3; (ii) crust depth H < 100 km; and (iii) solid angle α_o (three justifies values were used: $\alpha_o = 10^o$, 20°, 35°). The obtained results are as follows:

(1) The measured absolute gravity versus latitude follows a perfect parabolic profile across the continental bisecting line going from south (i.e., Cape Town city, latitude -34°) to north (i.e., Benghazi city, latitude 32°). This behavior originates from crust structure, is

independent of surface morphology and, consequently made our theoretical task easier.

(2) Using the code, the fitting of gravity near continental edges (i.e., Cape Town and Benghazi) yielded crust thickness of about 36 km. Whereas at the center of continent the crust thickness is estimated to be about 44 km.

(3) The results of our calculated crust depths are in excellent agreement with those existing in literature; for instance, Tedla and coworkers (2011) who reported H = 39 ± 5 km using an Euler deconvolution method and also in good agreement with the developed forward model. The results of our simulations are corroborated by a numerical forward model making our code more reliable for further geoscience explorations.

(4) The results from the inversion of the Bouguer anomaly from Cape Town (South) to Benghazi (North) suggest a thickening of the crust in the center of Africa, agreeing with the results of the proposed method, and also lateral changes of density within the African crust, which agrees with the complexity of the geology and mineralogical composition of the African crust from south to north.

(5) The proposed method is easy, computationally fast and need three density values only (Oceanic Crust, Continental Crust and Mantle).

(6) One of the disadvantage of the method is that it assumes basic assumptions of uniform continental crustal and mantle density, which may vary locally and it does not take into account the lateral variations in rock types from sedimentary rocks to granites or to gabbros/pyroxinites which may produce significant gravity variations that significantly affect crustal thickness estimates.

Declarations

Author contribution statement

Hakim Saibi: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Nacir Tit: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Mohamed Abdel Zaher, Jean d'Amour Uwiduhaye, Mohamed Amrouche & Walid Farhi: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Data availability statement

Data will be made available on request.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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