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Bobcat Optimization Algorithm: an effective bio-inspired metaheuristic algorithm for solving supply chain optimization problems

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Supply chain efficiency is a major challenge in today's business environment, where efficient resource allocation and coordination of activities are essential for competitive advantage. Traditional efficiency strategies often struggle for resources for the complex and dynamic network. In response, bio-inspired metaheuristic algorithms have emerged as powerful tools to solve these optimization problems. Referring to the random search nature of metaheuristic algorithms and emphasizing that no metaheuristic algorithm is the best optimizer for all optimization applications, the No Free Lunch (NFL) theorem encourages researchers to design newer algorithms to be able to provide more effective solutions to optimization problems. Motivated by the NFL theorem, the innovation and novelty of this paper is in designing a new meta-heuristic algorithm called Bobcat Optimization Algorithm (BOA) that imitates the natural behavior of bobcats in the wild. The basic inspiration of BOA is derived from the hunting strategy of bobcats during the attack towards the prey and the chase process between them. The theory of BOA is stated and then mathematically modeled in two phases (i) exploration based on the simulation of the bobcat's position change while moving towards the prey and (ii) exploitation based on simulating the bobcat's position change during the chase process to catch the prey. The performance of BOA is evaluated in optimization to handle the CEC 2017 test suite for problem dimensions equal to 10, 30, 50, and 100, as well as to address CEC 2020. The optimization results show that BOA has a high ability in exploration, exploitation, and balance them during the search process in order to achieve a suitable solution for optimization problems. The results obtained from BOA are compared with the performance of twelve well-known metaheuristic algorithms. The findings show that BOA has been successful in handling the CEC 2017 test suite in 89.65, 79.31, 93.10, and 89.65% of the functions for the problem dimension equal to 10, 30, 50, and 100, respectively. Also, the findings show that in order to handle the CEC 2020 test suite, BOA has been successful in 100% of the functions of this test suite. The statistical analysis confirms that BOA has a significant statistical superiority in the competition with the compared algorithms. Also, in order to analyze the efficiency of BOA in dealing with real world applications, twenty-two constrained optimization problems from CEC 2011 test suite and four engineering design problems have been selected. The findings show that BOA has been successful in 90.90% of CEC2011 test suite optimization problems and in 100% of engineering design problems. In addition, the efficiency of BOA to handle SCM applications has been challenged to solve ten case studies in the field of sustainable lot size optimization. The findings show that BOA has successfully provided superior performance in 100% of the case studies compared to competitor algorithms.

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Optimizing supply chain operations has become essential for organizations looking to remain competitive and better meet customer needs in today's dynamic business environment. However, traditional optimization methods often fail to meet the challenges of modern supply chains, which are characterized by diverse stakeholders, overlapping processes, and ongoing market dynamics the characteristics of the solution[1].

This paper sets the stage for exploring new bio-inspired metaheuristic algorithms specifically designed to solve supply chain optimization problems. The algorithm of supply-chain management provides a promising approach for optimization in various aspects, from inventory control to production scheduling and logistics systematically[2].

Optimization problems are a type of problems that have more than one feasible solution. Optimization is the process of finding the best feasible solution among all the available solutions for an optimization problem[3]. Decision variables, constraints, and objective function are the three main parts in the mathematical model of any optimization problem. The goal in the optimization process is to determine the appropriate values for the decision variables, provided that the objective function becomes optimal (maximum or minimum) by respecting the constraints of the problem[4]. Numerous optimization problems in science, mathematics, engineering, and real-world applications need to be solved using appropriate techniques. Solving techniques for optimization problems are available in two groups: deterministic and stochastic approaches[5]. Deterministic approaches in two classes, gradient-based and non-gradient-based, are effective tools for tackling linear, convex, continuous, differentiable, and low-dimensional optimization problems[6]. Meanwhile, as the optimization problems become more complex and especially the dimensions of the problem increase, the deterministic approaches are stopped by getting stuck in the local optima[7]. On the other hand, many practical optimization problems are non-linear, non-convex, discontinuous, non-differentiable, and high-dimensional[8]. The inability and difficulties of deterministic approaches in tackling such optimization problems have led researchers to develop stochastic approaches[3].

Metaheuristic algorithms are one of the most effective stochastic approaches to solve optimization problems. Metaheuristic algorithms are able to provide suitable solutions for optimization problems without the need for a derivation process, based on random search in the problem-solving space and using trial and error processes. The optimization process in metaheuristic algorithms starts with randomly generating a certain number of solvable solutions as the population of the algorithm. Then, these initial candidate solutions are improved based on the updating steps of the algorithm and changing their position in the problem solving space. The process of updating and improving candidate solutions continues until the last iteration of the algorithm. At the end, the best candidate solution obtained is presented as the solution for the given problem[9]. This nature of random search in the performance of metaheuristic algorithms leads to there being no guarantee to achieve the global optimum. However, because the solutions obtained from metaheuristic algorithms are close to the global optimum, they are acceptable as quasi-optimal solutions for optimization problems. Achieving better quasi-optimal solutions for optimization problems has become the main motivation for researchers to develop numerous metaheuristic algorithms[10]. These metaheuristic algorithms have been used to handle optimization tasks in various fields such as: feature selection[11], engineering applications[12,13], power engineering[14], parallel implementation on the graphics processing unit (GPU)[15], data classification[16], internet of things (IOT)[17], and supply chain management[18].

The condition for a metaheuristic algorithm to have a successful performance in optimization is that the random search process mechanism is well managed at both global and local levels. Global search management with the concept of exploration expresses the ability of the metaheuristic algorithm to comprehensively scan the problem solving space in order to prevent the algorithm from getting stuck in local optima and discover the main optimal area. Local search management with the concept of exploitation expresses the ability of the metaheuristic algorithm to accurately scan the problem solving space in promising areas and around the obtained solutions. In addition to exploration and exploitation abilities, balancing them during the search process is the main key in providing an effective search process in the problem solving space in order to achieve a suitable solution for optimization problems[19].

The main research question is that despite the numerous metaheuristic algorithms developed so far, is there still a need to design newer metaheuristic algorithms? The No Free Lunch (NFL)[20] theorem answers the question that there is no unique metaheuristic algorithm that is the best optimizer for all optimization problems. In fact, the effective performance of a metaheuristic algorithm in handling a set of optimization problems is no guarantee for the similar performance of that algorithm in handling other optimization problems. According to the NFL theorem, there is no assumption about the success or failure of the result of implementing a metaheuristic algorithm on an optimization problem. By keeping the study field of metaheuristic algorithms active, the NFL theorem motivates researchers to be able to achieve better solutions for optimization problems by designing newer algorithms.

Based on the best knowledge obtained from the literature review, no metaheuristic algorithm has been designed so far inspired by the natural behavior of bobcats. Meanwhile, the bobcat hunting strategy during the stages of moving towards the prey and chasing is an intelligent process that has a special potential for designing a new optimizer. In order to address this research gap, a new biomimetics metaheuristic algorithm is introduced in this paper based on the mathematical modeling of the hunting strategy of bobcats in the wild. Therefore, the originality of the proposed approach is guaranteed and it is confirmed that this is the first time that a metaheuristic algorithm inspired by the natural behavior of the bobcat has been introduced and designed.

The innovation and novelty of this paper is in the introduction of a new metaheuristic algorithm called Bobcat Optimization Algorithm (BOA) which is used to solve optimization problems. In order to express the novelty aspects of the Bobcat Optimization Algorithm (BOA) and justify why this proposal is necessary, despite the existence of other wildcat-inspired optimizers, the unique aspects and specific advantages of BOA are highlighted:

Unique hunting strategy emulation
The BOA uniquely models the bobcat's hunting strategy, which involves distinct phases of tracking and ambushing prey, followed by a chase to catch the prey. This dual-phase approach mirrors both global exploration and local exploitation in optimization, providing a balanced search mechanism.

Adaptive predator behavior
The algorithm mimics the bobcat's adaptability to different prey and environments. This results in a dynamic balance between exploration and exploitation, where the algorithm adjusts its search behavior based on the problem landscape and current performance, leading to potentially better optimization results.

Phase-specific position updates
The BOA uses different mathematical models for position updates during the exploration and exploitation phases. The exploration phase involves extensive position changes to search broadly, while the exploitation phase makes smaller, refined adjustments to converge on the optimal solution. This phase-specific update mechanism enhances the algorithm's efficiency and accuracy.

The Bobcat Optimization Algorithm (BOA) presents a novel and justified approach to solving optimization problems by leveraging the unique hunting strategies of bobcats. Its distinct exploration and exploitation phases, adaptive behavior, and empirical validation against benchmark problems establish its novelty and necessity in the field of optimization. By addressing the limitations of existing algorithms and providing a balanced search mechanism, the BOA contributes valuable diversity to optimization techniques, aligning with the principles of the NFL theorem. Focusing on these points, the novelty and justification of BOA is clearly confirmed, and a powerful optimization tool is established to develop and use it in optimization programs. The main contributions of this paper are as follows:

- BOA is designed based on mimicking bobcat behaviors in the wild.
- The fundamental inspiration of BOA comes from the bobcat's hunting strategy.
- The BOA theory is stated and its mathematical model is presented in two phases (i) exploration based on the simulation of the bobcat's position change while moving towards the prey and (ii) exploitation based on simulating the bobcat's position change during the chase process to catch the prey.
- The performance of BOA in optimization is tested to solve CEC 2017 test suite for problem dimensions of 10, 30, 50, and 100, as well as to address CEC 2020.
- The performance of BOA is challenged in real-world applications to solve twenty-two constrained optimization problems from the CEC 2011 test suite and four engineering design problems.
- The performance of BOA is compared with the performance of twelve well-known competitor algorithms.
- The performance of BOA in handling SCM applications is challenged on sustainable lot size optimization.

The structure of the paper is as follows; literature review is presented in Sect. "Literature review". Then the proposed BOA is introduced and modeled in Section "Bobcat Optimization Algorithm". Simulation studies and results are presented in Section "Simulation studies and results". The effectiveness of BOA in solving real-world applications is investigated in Section "BOA for real-world applications". The efficiency of BOA to deal with sustainable lot size optimization is evaluated in Section "BOA for Supply Chain Management (SCM)". Analysis and discussion of the results, advantages and disadvantages of the proposed approach are presented in "Discussion". Conclusions and suggestions for future research are provided in Sect. "Conclusion and future works".

Literature review
Companies operating in local and international supply chains (SCs) are increasingly considering sustainability when making decisions[21]. Plus, the durability they are usually divided into three categories according to area of interest: social, economic, and environmental sustainability[22]. Complex supply chain management a Multi-channel supply chain. Offline and e-commerce management Supply chain companies have now become increasingly diversified chain processes[23]. Rapid supply chain management in high-demand markets it is important because supply chain management can be influenced in response to increased demand Quality of goods and services[24].

Lot size optimization plays an important role in supply chain management, as it directly affects inventory costs, product quality, and customer satisfaction Over the years, researchers have explored various optimization techniques and strategies to overcome the challenges of in lot size determination is addressed[25].

Traditional lot-sizing techniques, such as economic order quantity (EOQ) and production order quantity (POQ) have long been used for inventory management. These deterministic models aim to minimize all relative costs by a storage costs and balancing ordering/setup costs. Although EOQ and POQ provide convenience and ease of use, they often fail to account for dynamic demand patterns, timing variations, and other real-world complexities[26].

Researchers have developed stochastic lot-size models to deal with the uncertainties in supply chain operations. This model incorporates probabilistic demand forecasting, lead time variability, and other sources of uncertainty into the optimization framework. Stochastic programming, dynamic programming, and simulation-based approaches are often used to solve stochastic lot size problems. By explicitly considering uncertainty, stochastic models provide more robust and realistic lot size decisions. In recent years, bio-inspired metaheuristic algorithms have gained popularity in lot size optimization in supply chain management[27].

These metaheuristic algorithms have been developed with inspiration from natural phenomena, swarming behavior of living organisms in nature, genetic and biological sciences, concepts of physical science, human activities, and other evolutionary phenomena. Based on the main source of inspiration used in the design, metaheuristic algorithms are placed in five groups: swarm-based, evolutionary-based, physics-based, and human-based approaches.

Swarm-based metaheuristic algorithms are designed inspired by the lifestyles of insects, birds, aquatic animals, animals, plants, and other living organisms in the wild. Particle Swarm Optimization (PSO)[28], Ant Colony Optimization (ACO)[29], Artificial Bee Colony (ABC)[30], and Firefly Algorithm (FA)[31] are among the most prominent swarm-based metaheuristic algorithms. The design idea in PSO comes from the swarming behavior of birds and fish searching for food resources. The behavioral characteristic of ants in identifying the shortest communication path between the colony and food location has been employed as a source of inspiration in ACO design. The source of inspiration for the design of ABC is derived from the hierarchical activities and cooperation of colony bees with the aim of obtaining food resources. The origin of the FA design comes from the ability of fireflies to establish and exchange information through optical communication. The source of inspiration in the design of Golden Jackal Optimization (GJO) comes from the collaborative hunting behavior of the golden jackals in the wild, consisting of three steps: prey searching, enclosing, and pouncing[32]. African Vultures Optimization Algorithm (AVOA) is proposed inspired by foraging and navigation strategies among African vultures[33]. Foraging, hunting, chasing, migration, digging, and reproduction are among the most prominent activities of living organisms in the wild, which have been employed as a source of inspiration in the design of algorithms such as: Woodpecker Mating Algorithm (WMA)[34], WMA based on opposition-based learning (OWMA)[35], Hybrid Sine Cosine Algorithm-Woodpecker Mating (HSCWMA)[36], Hybrid Woodpecker Mating Algorithm-Whale Optimization Algorithm (HWMWOA)[37], Orca Predation Algorithm (OPA)[38], Electric Eel Foraging Optimization (EEFO)[39], Elk Herd Optimizer (EHO)[40], Reptile Search Algorithm (RSA)[41], White Shark Optimizer (WSO)[42], Termite Alate Optimization Algorithm (TAOA)[43], Fire Hawk Optimizer[44], Spider Wasp Optimization (SWO)[45], Marine Predator Algorithm (MPA)[46], Honey Badger Algorithm (HBA)[47], Piranha Foraging Optimization Algorithm (PFOA)[48], and Snow Ablation Optimizer (SAO)[49].

Evolutionary-based metaheuristic algorithms are designed with inspiration from the sciences of genetics, biology, concepts of natural selection, and survival of the fittest. Genetic Algorithm (GA)[50] and Differential Evolution (DE)[51] are among the most well-known evolutionary-based metaheuristic algorithms, the source of inspiration in their design comes from the reproduction process, genetic concepts, Darwin's evolutionary theory, and the evolutionary-random operators of mutation, crossover, and selection. The idea of designing Artificial Immune Systems (AISs) is derived from the confrontation of the human immune system with diseases and microbes[52]. Some other evolutionary-based metaheuristic algorithms are: Cultural Algorithm (CA)[53], Genetic programming (GP)[54], and Evolution Strategy (ES)[55].

Physics-based metaheuristic algorithms are designed with inspiration from forces, laws, transformations, transitions, processes, phenomena, laws, and other concepts in physics. Simulated Annealing (SA) is one of the most popular Physics-based metaheuristic algorithms, whose design imitates the annealing process of metals, during which metals are melted with high heat, and then, with the aim of achieving an ideal crystal, the metals are cooled slowly[56]. Physical forces have been the source of design for the development of algorithms such as: Spring Search Algorithm (SSA)[57] inspired by the implementation of Hooke's law and the tensile force of the spring between weights connected to each other with springs, Momentum Search Algorithm (MSA)[58] inspired by the force derived from the collision momentum between two balls, and Gravitational Search Algorithm (GSA)[59] inspired by from the gravitational force of attraction between masses that are located at different distances from each other. The Water Cycle Algorithm (WCA) design idea comes from the state transformations of water in its natural cycle[60]. Henry gas solubility optimization (HGSO) is developed inspired by the behavior governed by Henry's law[61]. Some other physics-based metaheuristic algorithms are: Young's Double-Slit Experiment (YDSE) optimizer[62], Prism Refraction Search (PRS)[63], Chernobyl Disaster Optimizer (CDO)[64], Kepler Optimization Algorithm (KOA)[65], Thermal Exchange Optimization (TEO)[66], Black Hole Algorithm (BHA)[67], Nuclear Reaction Optimization (NRO)[68], Electro-Magnetism Optimization (EMO)[69], Equilibrium Optimizer (EO)[70], Multi-Verse Optimizer (MVO)[71], Lichtenberg Algorithm (LA)[72], and Archimedes Optimization Algorithm (AOA)[73].

Human-based metaheuristic algorithms are designed with inspiration from decisions, thoughts, trainings, choices, interactions, collaborations, and other activities of humans in individual and social life. Teaching–Learning Based Optimization (TLBO) is one of the most widely used human-based metaheuristic algorithms, whose design idea comes from the sharing of science and knowledge between teachers and students, as well as students with each other in the classroom[74]. Mother Optimization Algorithm (MOA) is proposed based on the modeling of Eshrat's care of her children[7]. The source of inspiration in the design of Doctor and Patient Optimization (DPO) comes from the therapeutic communication between the medical staff and the patients in the hospital[75]. War Strategy Optimization (WSO) is proposed by imitating ancient war strategy and strategic movement of army troops during the war[76]. The idea of designing Election-Based Optimization Algorithm (EBOA) comes from the process of holding elections and voting from people to choose the community leader[77]. Some other human-based metaheuristic algorithms are: Great Wall Construction Algorithm (GWCA)[78], Gaining Sharing Knowledge based Algorithm (GSK)[79], Special Forces Algorithm (SFA)[80], Coronavirus Herd Immunity Optimizer (CHIO)[81], Growth Optimizer (GO)[82], and Ali Baba and the Forty Thieves (AFT)[83].

Bobcat Optimization Algorithm

In this section, the inspiration source in the design of the proposed Bobcat Optimization Algorithm (BOA) approach is stated, then it is mathematically modeled in order to use it in optimization applications.

Inspiration and main idea of BOA

The bobcat (Lynx rufus), also known as the red lynx, is one of the four extant species within the medium-sized wild cat genus Lynx. The bobcat is native to North America and ranges from southern Canada through most of the contiguous United States to Oaxaca in Mexico[84]. The coat color in bobcats varies, although it is mostly brown or grayish-brown with dark bars on the tail and forelegs and black streaks on the body. The ears are black-tipped and pointed, with short, black tufts. The ears are pointed and black-tipped, with black, short tufts. The length of an adult bobcat is about 125–47.5 cm from the head to the base of its distinctive stubby tail. The length of the tail of this animal is about 9 to 20 cm. Shoulder size for adult bobcat is around 30 to 60 cm. The weight of the male bobcat is about 6.4–18.3 kg and the female is about 4–15.3 kg[85]. An image of the bobcat is shown in Fig. 1.

The bobcat is an adaptable predator that lives in forest areas, forest edge, urban edge, semidesert areas, and swampland environments. This animal can go without food for long periods of time, but it eats more when prey is plentiful. Although the bobcat prefers hares and rabbits, it hunts geese, chickens and other birds, insects, deer, and small rodents. Prey selection depends on habitat and location, abundance, and season. The bobcat's hunting strategy is to first track the prey. Then by ambushing, it attacks the prey at the right time and hunts it after a chase process[86].

Among the natural behaviors of the bobcat in the wild, the strategy of this animal during hunting is much more prominent. This hunting strategy can be expressed in two processes: (i) tracking and moving towards the prey and (ii) chasing and catching the prey. Mathematical modeling of these intelligent processes is employed to design the proposed Bobcat Optimization Algorithm (BOA) approach, which is discussed below.

Mathematical model of BOA

In the design of the proposed BOA approach, in order to update the population of the algorithm in the problem solving space, it is inspired by the hunting strategy of bobcats in the wild. In this strategy, the bobcat first tracks the position of the prey and moves towards it. Then it ambushes and attacks the prey at the right time and finally catches it after a chasing process. According to this, changes in the position of the bobcat in its habitat during the hunting process can be considered in two parts: (i) tracking and moving towards the prey and (ii) chasing and catching the prey. Inspired by this natural strategy in the lifestyle of bobcats, in the BOA design, the position of the population members is updated in each iteration in two phases (i) exploration based on the simulation of the bobcat's position change while moving towards the prey and (ii) exploitation based on the simulation of the bobcat's position change during the chase process to catch the prey. In the following, each of these BOA update phases is described in detail.

Initialization

The proposed BOA approach is a population-based optimizer that can achieve suitable solutions for optimization problems in an iteration-based process by benefiting from the searching power of its members in the problem solving space. According to BOA's design inspiration, the wildlife habitat of the bobcats corresponds to the problem-solving space, and the location of the bobcats in this habitat corresponds to the position of the BOA members in the problem-solving space. Therefore, in BOA, each bobcat as a member of the population,

Figure 1. Photo of a Bobcat; downloaded from free media Wikimedia Commons.

according to the position it creates in the problem solving space, determines the values for the decision variables. Hence, the position of each bobcat represents a candidate solution to the problem, which can be modeled from a mathematical point of view using a vector, where each element of this vector represents a decision variable. Together, bobcats form the population of the algorithm, which can be modeled from a mathematical point of view using a matrix according to Eq. (1). The primary position of bobcats in the problem-solving space is initialized randomly using Eq. (2).

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N \times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,d} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i,1} & \cdots & x_{i,d} & \cdots & x_{i,m} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,d} & \cdots & x_{N,m} \end{bmatrix}_{N \times m}, \tag{1}$$

$$x_{i,d} = lb_d + r \cdot (ub_d - lb_d). \tag{2}$$

Here, X is the BOA population matrix, X_i is the i th bobcat (candidate solution), $x_{i,d}$ is its d th dimension in search space (decision variable), N is the number of bobcats, m is the number of decision variables, r is a random number in interval [0,1], lb_d, and ub_d are the lower bound and upper bound of the d th. decision variable, respectively.

As mentioned, the position of each bobcat represents a candidate solution for the problem, corresponding to which the objective function of the problem can be evaluated. The set of evaluated values for the objective function can be represented using a vector according to Eq. (3).

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N \times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N \times 1}. \tag{3}$$

Here, F is the vector of evaluated objective function and F_i is the evaluated objective function based on the i th bobcat.

These evaluated values for the objective function of the problem are suitable criteria to measure the quality of each BOA member in providing candidate solutions. According to this, the best evaluated value for the objective function corresponds to the best BOA member and similarly the worst evaluated value for the objective function corresponds to the worst BOA member. In BOA design, in each iteration, the position of the bobcats in the problem-solving space is updated, and accordingly, the candidate solutions and the objective function values are also updated. Therefore, in each iteration, based on the comparison of objective function values, the best BOA member should also be updated.

PHASE 1: Tracking and moving towards prey (exploration phase)

In the first phase of BOA, the position of the population members in the problem solving space is updated based on the simulation of tracking and movement of bobcats towards prey during hunting. Modeling the movement of bobcat towards the prey leads to extensive changes in the position of the population members in the problem solving space and thus increases the exploration ability of BOA in order to manage the global search.

In BOA design, for each bobcat, the position of other population members who have a better value for the objective function is considered as the prey position. The candidate prey set for each bobcat is determined using Eq. (4).

$$CP_i = \{X_k : F_k < F_i \, and \, k \neq i\}, \text{where } i = 1,2,\ldots,N \text{ and } k \in \{1,2,\ldots,N\}. \tag{4}$$

Here, CP_i is the set of candidate preys' locations for the i th bobcat, X_k is the is the population member with a better objective function value than i th bobcat, and F_k is the its objective function value.

In the BOA design, it is assumed that each bobcat randomly selects one of these preys and attacks it. Based on the modeling of the bobcat's position change while moving towards the prey in this strategy, a new position is calculated for each BOA member using Eq. (5). This new position replaces the previous position of the corresponding member if it improves the value of the objective function according to Eq. (6).

$$x_{i,j}^{P1} = x_{i,j} + \left(1 - 2r_{i,j}\right) \cdot (SP_{i,j} - I_{i,j} \cdot x_{i,j}), \tag{5}$$

$$X_i = \begin{cases} X_i^{P1}, F_i^{P1} \leq F_i, \\ X_i, else, \end{cases} \tag{6}$$

Here, SP_i is the selected prey by i th bobcat, $SP_{i,j}$ is its j th dimension, X_i^{P1} is the new position calculated for the i th bobcat based on exploration phase of the proposed BOA, $x_{i,j}^{P1}$ is its j th dimension, F_i^{P1} is its objective function value, $r_{i,j}$ are random numbers from the interval [0, 1], and $I_{i,j}$ are numbers which are randomly selected as 1 or 2.

PHASE 2: Chasing to catch prey (exploitation phase)

In the second phase of BOA, the position of the population members in the problem solving space is updated based on the chase simulation between the bobcat and the prey during hunting. This process of chasing happens near the hunting place so that finally the bobcat catches the prey. Modeling the movement of bobcat during the process of chasing and catching prey leads to small changes in the position of population members in the problem solving space and thus increases the exploitation ability of BOA in order to manage local search.

In BOA design, based on the modeling of bobcat position change during the chase process, a new position for each BOA member near the hunting place is calculated using Eq. (7). Then, this new position, if it improves the value of the objective function, replaces the previous position of the corresponding member according to Eq. (8).

$$x_{i,j}^{P2} = x_{i,j} + \frac{1 - 2r_{i,j}}{1 + t} \cdot x_{i,j} \tag{7}$$

$$X_i = \begin{cases} X_i^{P2}, F_i^{P2} \leq F_i \\ X_i, else \end{cases}. \tag{8}$$

Here, X_i^{P2} is the new position calculated for the i th bobcat based on exploitation phase of the proposed BOA, $x_{i,j}^{P2}$ is its j th dimension, F_i^{P2} is its objective function value, $r_{i,j}$ are random numbers from the interval [0, 1], and t is the iteration counter.

Repetition process, pseudo-code, and flowchart of BOA

The first iteration in the proposed BOA approach is completed after updating the position of all bobcats in the problem solving space based on exploration and exploitation phases. After that, the algorithm enters the next iteration with updated values for the position of bobcats and the objective function, and the process of updating bobcats continues until the last iteration of the algorithm based on Eqs. (4)–(8). In each iteration, the best solution is obtained until that iteration is updated and saved. After the full implementation of the algorithm, the best candidate solution obtained during the iterations of the algorithm is presented as the BOA solution for the given problem. The implementation steps of BOA are shown as a flowchart in Fig. 2 and its pseudocode is presented in Algorithm 1.

```
Start BOA.
Input problem information: variables, objective function, and constraints.
Set BOA population size (N) and iterations (T).
Generate the initial population matrix at random using Equation (2). x_{i,d} ← lb_d + r · (ub_d − lb_d)
Evaluate the objective function.
  For t = 1 to T
    For i = 1 to N
    Phase 1: Tracking and moving towards prey (exploration phase)
      Determine the preys set for the ith BOA member using Equation (4). CP_i ← {X_{k_i}: F_{k_i} < F_i and k_i ≠ i}
      Select the termite mounds for the ith BOA member at random.
      Calculate new position of ith BOA member using Equation (5). x_{i,d}^{P1} ← x_{i,j} + (1 − 2 r_{i,j}) · (SP_{i,j} − I_{i,j} · x_{i,j})
      Update ith BOA member using Equation (6). X_i ← { X_i^{P1}, F_i^{P1} < F_i ; X_i, else }
    Phase 2: Chasing to catch prey (exploitation phase)
      Calculate new position of ith BOA member using Equation (7). x_{i,d}^{P2} ← x_{i,d} + x_{i,j} + ((1−2 r_{i,j})/(1+t)) · x_{i,j}
      Update ith BOA member using Equation (8). X_i ← { X_i^{P2}, F_i^{P2} < F_i ; X_i, else }
    end
    Save the best candidate solution so far.
  end
Output the best quasi-optimal solution obtained with the BOA.
End BOA.
```

Algorithm 1. Pseudo-code of BOA.

Computational complexity of BOA

In this subsection, the computational complexity of the proposed BOA approach is analyzed.

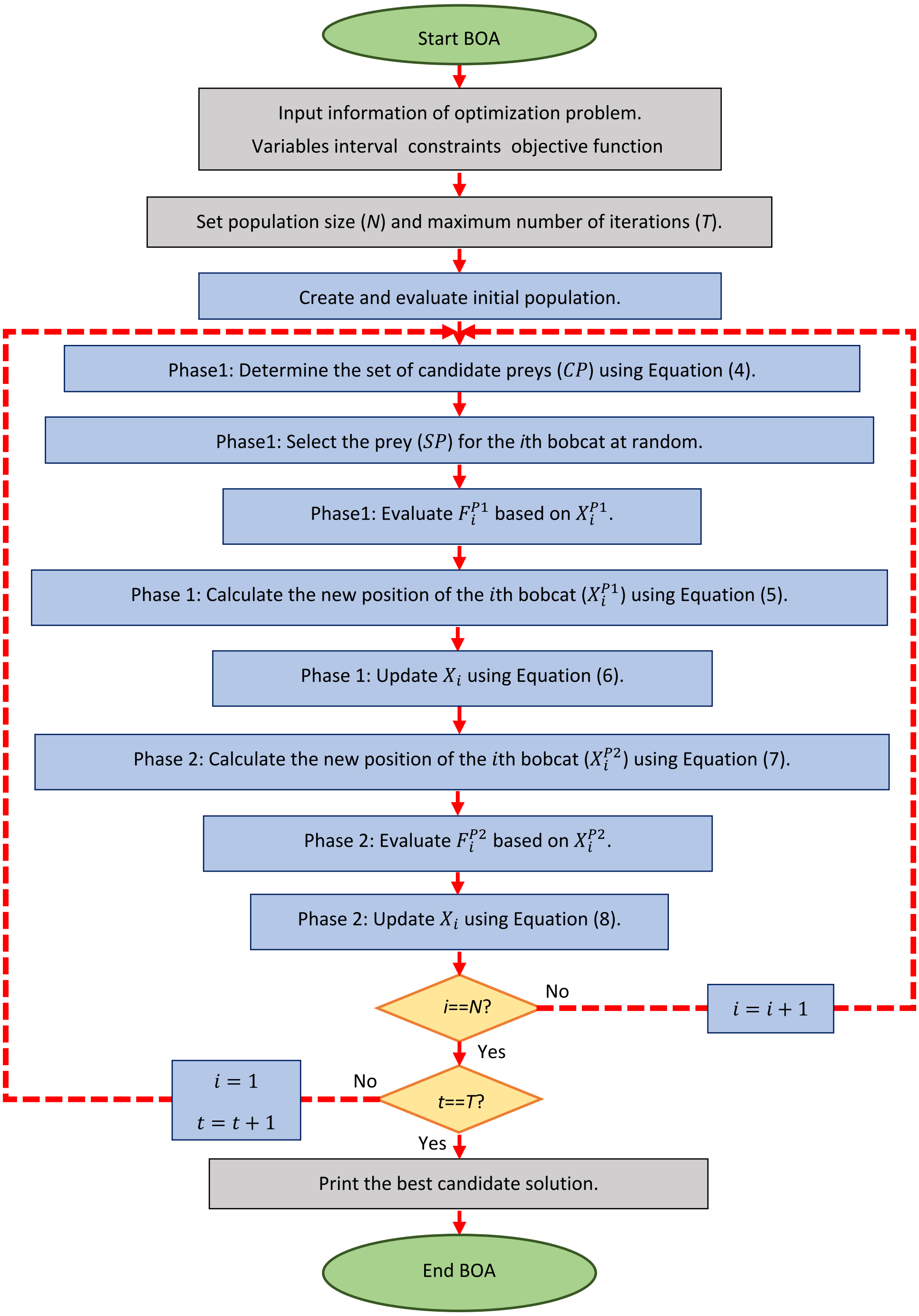

Figure 2. Flowchart of BOA.

Initialization phase

Population initialization.

- Each bobcat's position is initialized randomly within the bounds.
- Complexity: $O(N \times m)$, where N is the number of bobcats (population size) and m is the number of decision variables.

Objective function evaluation.

- Each bobcat's initial position is evaluated using the objective function.
- Complexity: $O(N)$, assuming the objective function evaluation is $O(1)$.

Iterative phases

Each iteration consists of two main phases: Exploration and Exploitation.

Phase 1: Exploration (tracking and moving towards prey). Determining candidate prey set.
- o For each bobcat, identify all other bobcats with better objective function values.
- o Complexity: $O(N^2)$, as each bobcat must compare its value with every other bobcat.

Updating positions.
- o For each bobcat, update its position based on the selected prey and the defined rules.
- o Complexity: $O(N \times m)$, as each bobcat's position is updated in the m-dimensional space.

Objective function evaluation.
- o Evaluate the new positions of the bobcats.
- o Complexity: $O(N)$.

Phase 2: Exploitation (chasing and catching prey). Updating positions.
- o For each bobcat, update its position based on the chasing process near the hunting place.
- o Complexity: $O(N \times m)$.

Objective function evaluation.
- o Evaluate the new positions of the bobcats.
- o Complexity: $O(N)$.

Total complexity per iteration

- *Exploration phase:* $O\left(N^2\right) + O(N \times m) + O(N) = O(N^2 + N \times m)$
- *Exploitation phase:* $O(N \times m) + O(N) = O(N \times m)$

Overall complexity
Assuming the algorithm runs for T iterations, the total computational complexity can be summarized as: $O(T \times (N^2 + N \times m))$

In this complexity analysis:

- N is the number of bobcats (population size).
- m is the number of decision variables.
- T is the number of iterations.

Explanation

1. *Initialization:* The initialization phase involves setting up the initial positions of the bobcats, which scales linearly with both the population size N and the number of decision variables m.
2. *Exploration phase:* During each iteration, each bobcat must compare itself with every other bobcat to determine the candidate prey set, leading to a quadratic complexity $O(N^2)$. Updating the positions of the bobcats involves linear operations with respect to the number of decision variables, resulting in $O(N \times m)$.
3. *Exploitation phase:* The exploitation phase involves updating each bobcat's position based on a local search strategy, which also scales linearly with the population size and the number of decision variables.

The quadratic term $O(N^2)$ arises from the need to compare each bobcat with every other bobcat to determine the candidate prey set. This could potentially be optimized using advanced data structures or parallel processing techniques, but in its basic form, it dominates the complexity, especially for large populations.

Population diversity, exploration, and exploitation analysis

Population diversity of BOA refers to how population members are spread out within the problem space, which is crucial for tracking the algorithm's search processes. This measure shows if the population is more geared towards exploring new possibilities or focusing on optimizing known solutions. Evaluating the diversity within the BOA population allows for assessing and adjusting the algorithm's ability to effectively explore and exploit as a group. Researchers have proposed various definitions of diversity. Pant[87] described diversity using Eqs. (9), (10).

$$Diversity = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\sum\nolimits_{d=1}^{m}\left(x_{i,d} - \overline{x}_d\right)^2}, \tag{9}$$

$$\overline{x}_d = \frac{1}{N}\sum\nolimits_{i=1}^{N} x_{i,d}. \tag{10}$$

Here, N represents the number of population members, m is the number of problem dimensions, and $\overline{x}_d$ is the mean of the population in the dth dimension. Therefore, the extent of exploration and exploitation within the population for each iteration can be defined by Eqs. (11), (12),

$$Exploration = \frac{Diversity}{Diversity_{max}}, \tag{11}$$

$$Exploitation = \frac{\left|Diversity - Diversity_{max}\right|}{Diversity_{max}}. \tag{12}$$

In this segment, the diversity, exploration, and exploitation of the population have been assessed using twenty-three standard benchmark functions, which include seven unimodal functions (F1 to F7) and sixteen multimodal functions (F8 to F23). A detailed description of these benchmark functions can be found in[88].

Figure 3 demonstrates the exploration–exploitation ratio of the BOA method over the iteration process, providing a visual tool for understanding how the algorithm balances global and local search strategies. Additionally, Table 1 presents the analysis results of population diversity, exploration, and exploitation. The simulation results indicate that the BOA maintains a favorable population diversity, with high values at the initial iteration and lower values towards the final iteration. Furthermore, the results generally show that the exploration–exploitation ratio of BOA tends to be close to 0.00%:100%. These findings validate that the proposed BOA approach, by fostering suitable population diversity throughout the algorithm's iterations, achieves effective performance in managing and balancing exploration and exploitation during the search process.

Qualitative analysis of BOA

This section delves into the qualitative analysis of the proposed Bobcat Optimization Algorithm (BOA) when tackling optimization problems. To facilitate a visual examination of BOA, four metrics are utilized: (i) search history, (ii) the trajectory of the leading bobcat in the first dimension, (iii) the average fitness of population members, and (iv) the convergence curve of BOA as it progresses towards a solution over successive iterations. Thirteen standard benchmark functions (F1 to F13) serve as the basis for this evaluation, with detailed descriptions of these functions provided in[88]. The results from the qualitative analysis of BOA on these functions are illustrated in Fig. 4.

The search history metric tracks the positional changes of bobcats within the search space throughout the algorithm's iterations. The trajectory of the leading bobcat metric maps the changes in the first dimension of the primary bobcat across iterations. The average fitness metric monitors the variations in the average fitness of the bobcat population during the algorithm's execution. Lastly, the convergence curve metric illustrates the BOA's progression towards the optimal solution during the algorithm's iterations.

Simulation studies and results

In this section, the performance of the proposed approach BOA is evaluated to handle the optimization tasks. With this aim, BOA is implemented on the CEC 2017 test suite for problem dimensions equal to 10, 30, 50, and 100, as well as CEC 2020 test suite.

Performance comparison and experimental setting

The quality of the results obtained from BOA is evaluated in comparison with the performance of twelve well-known metaheuristic algorithms: CMA-ES[89], EBOwithCMAR[90], SPS_L_SHADE_EIG[91], LSHADE_cnEpSi[92], SHADE[93], GWO[94], WOA[95], MPA[46], TSA[96], RSA[41], AVOA[33], and WSO[42]. As evident from the literature review, countless metaheuristic algorithms have been introduced and designed so far. Therefore, it is not reasonable to compare the performance of the proposed BOA approach with all these algorithms. For this reason, the authors have used these twelve metaheuristic algorithms in order to compare with the performance of BOA. The reasons and criteria for choosing these twelve algorithms are stated below:

Wide range of applications
The selected algorithms have been successfully applied to various real-world and benchmark optimization problems, showcasing their versatility and robustness.

State-of-the-art performance
The chosen algorithms are well-regarded in the optimization community for their competitive performance in terms of convergence speed, solution quality, and robustness.

Representation of different metaheuristic families
The algorithms cover a broad spectrum of metaheuristic families, including evolutionary algorithms, swarm intelligence, physics-based algorithms, and bio-inspired algorithms. This ensures a comprehensive comparison across different optimization strategies.

Popularity and citations
The selected algorithms are highly cited in academic literature, indicating their acceptance and relevance in the field of optimization.

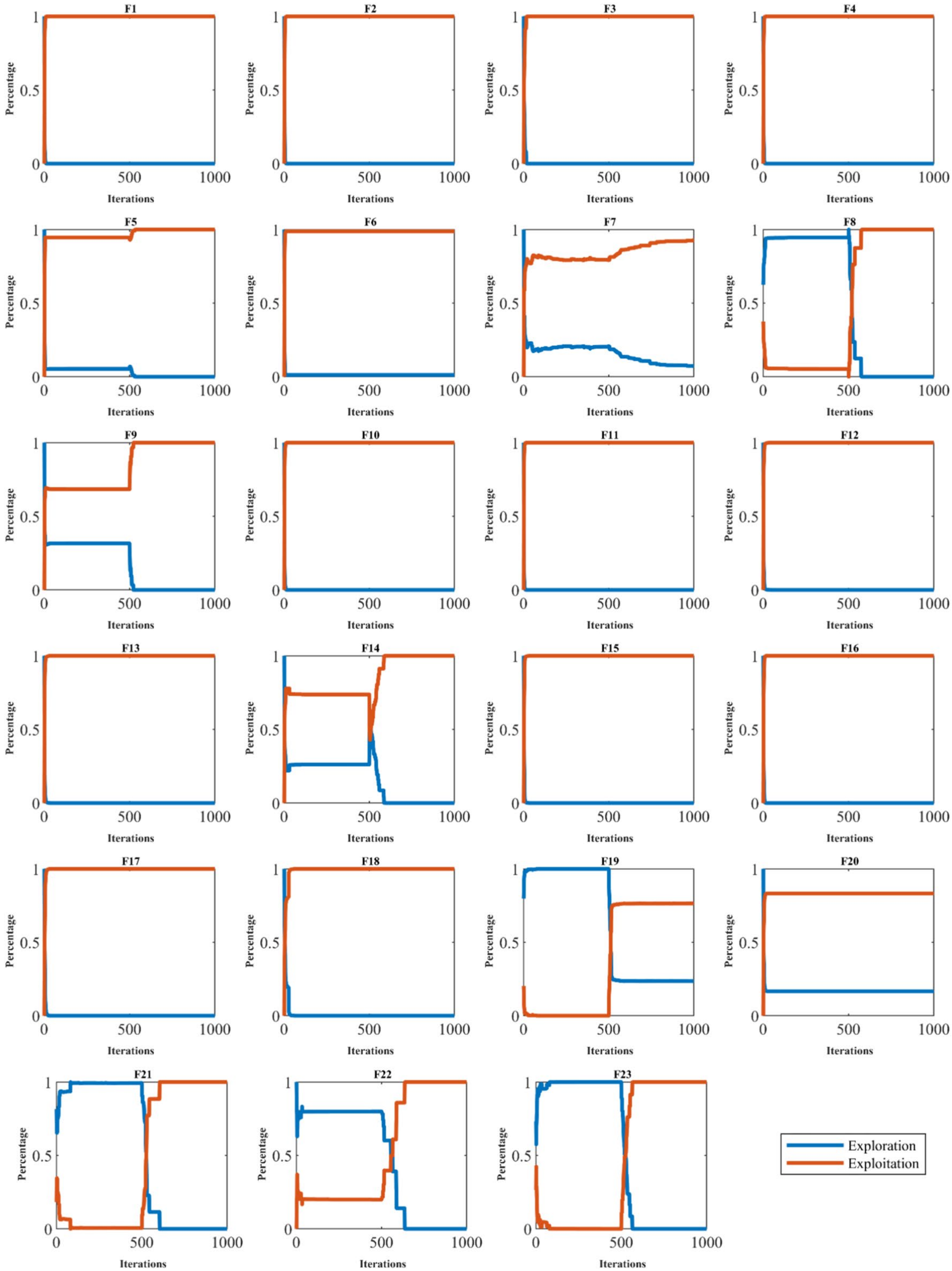

Figure 3. Exploration and exploitation of the BOA.

Six statistical indicators mean, best, worst, standard deviation (std), median, and rank are used to report the simulation results. The mean index is used as a criterion for ranking metaheuristic algorithms in dealing with each of the optimization problems.

Evaluation CEC 2017 test suite

In this subsection, the ability of BOA and competitor algorithms in facing the CEC 2017 test suite has been analyzed for problem dimensions equal to 10, 30, 50, and 100. CEC 2017 test suite has thirty standard benchmark

Function name	Exploration	Exploitation	Diversity	
			First iteration	Last iteration
F1	9.39E−165	1	129.60658	1.22E−162
F2	0	1	17.253897	0
F3	0	1	264.31188	0
F4	0	1	211.89513	0
F5	0	1	39.256078	0
F6	0.0123597	0.9876403	117.86504	1.4567729
F7	0.072464	0.927536	1.386376	0.1004623
F8	5.96E−10	1	1290.0798	1.23E−06
F9	4.04E−10	1	10.449092	4.23E−09
F10	1.70E−17	1	46.046617	7.82E−16
F11	3.61E−11	1	730.84902	2.64E−08
F12	0	1	78.178087	0
F13	0	1	83.89034	0
F14	2.32E−09	1	32.439964	7.54E−08
F15	4.37E−11	1	2.8757232	1.26E−10
F16	0	1	1.6292818	0
F17	1.28E−09	1	3.9168571	5.01E−09
F18	2.902E−10	1	0.9188602	2.667E−10
F19	0.2354498	0.7645502	0.3785558	0.1117081
F20	0.1670655	0.8329345	0.4281158	0.0715234
F21	8.75E−11	1	3.661421	3.92E−10
F22	2.79E−10	1	2.7633739	7.70E−10
F23	5.79E−11	1	3.3034174	2.61E−10

Table 1. Population diversity, exploration, and exploitation percentage results.

functions consisting of: (i) three unimodal functions of C17-F1 to C17-F3, (ii) seven multimodal functions of C17-F4 to C17-F10, (iii) ten hybrid functions of C17-F11 to C17-F20, and (iv) ten composition functions of C17-F21 to C17-F30. Among these benchmark functions, C17-F2 is excluded from simulation studies due to its unstable behavior. Full description and details of CEC 2017 test suite are available at[97].

Tables 2–5 report the implementation results of BOA and competitor algorithms on CEC 2017 test suite for problem dimensions equal to 10, 30, 50, and 100. Figures 5–8 show the convergence curves resulting from the performance of metaheuristic algorithms in this implementation. Analysis of simulation results shows that BOA in handling the CEC 2017 test suite, for problem dimension equal to 10 ($m = 10$), is the first best optimizer for functions C17-F1, C17-F3 to C17-F17, C17-F19 to C17-F23, and C17-F26 to C17-F30 (26 functions from 29 functions). For problem dimension equal to 30 ($m = 30$), proposed GAO approach is the first best optimizer for functions C17-F1, C17-F3 to C17-F6, C17-F9, C17-F10, C17-F12 to C17-F22, C17-F24, C17-F25, C17-F27, C17-F28, and C17-F30 (23 functions from 29 functions). For problem dimension equal to 50 ($m = 50$), proposed GAO approach is the first best optimizer for functions C17-F1, C17-F3, C17-F5 to C17-F25, and C17-F27 to C17-F30 (27 functions from 29 functions). For problem dimension equal to 100 ($m = 100$), proposed GAO approach is the first best optimizer for functions C17-F1, C17-F3 to C17-F22, and C17-F24 to C17-F28 (26 functions from 29 functions).

The findings obtained from the optimization results are that BOA is able to find suitable solutions for optimization problems with its high quality in exploration, exploitation, and balancing them during the search process in the problem solving space. The findings obtained from the analysis of the simulation results are that BOA is presented superior performance in order to solve the CEC 2017 test suite for problem dimensions of 10, 30, 50, and 100, by providing better results for most of the benchmark functions and ranking as the first best optimizer overall.

Evaluation of CEC 2020 test suite

This subsection provides an in-depth assessment of the proposed methodology's effectiveness in solving the CEC 2020 benchmark problems. This particular test suite encompasses ten numerical optimization functions, each with specific boundary constraints. The ten functions are categorized as follows: C20-F1 is unimodal, which means it has a single global optimum. C20-F2 to C20-F4 are basic functions, which are relatively simpler and often used to test the fundamental capabilities of optimization algorithms. C20-F5 to C20-F7 are hybrid functions, combining multiple basic functions to create a more complex landscape. Finally, C20-F8 to C20-F10 are composition functions, which are even more intricate, blending several different types of functions to challenge the optimization process further. For a comprehensive understanding, detailed descriptions and formulations of these benchmark functions are thoroughly documented in source[98].

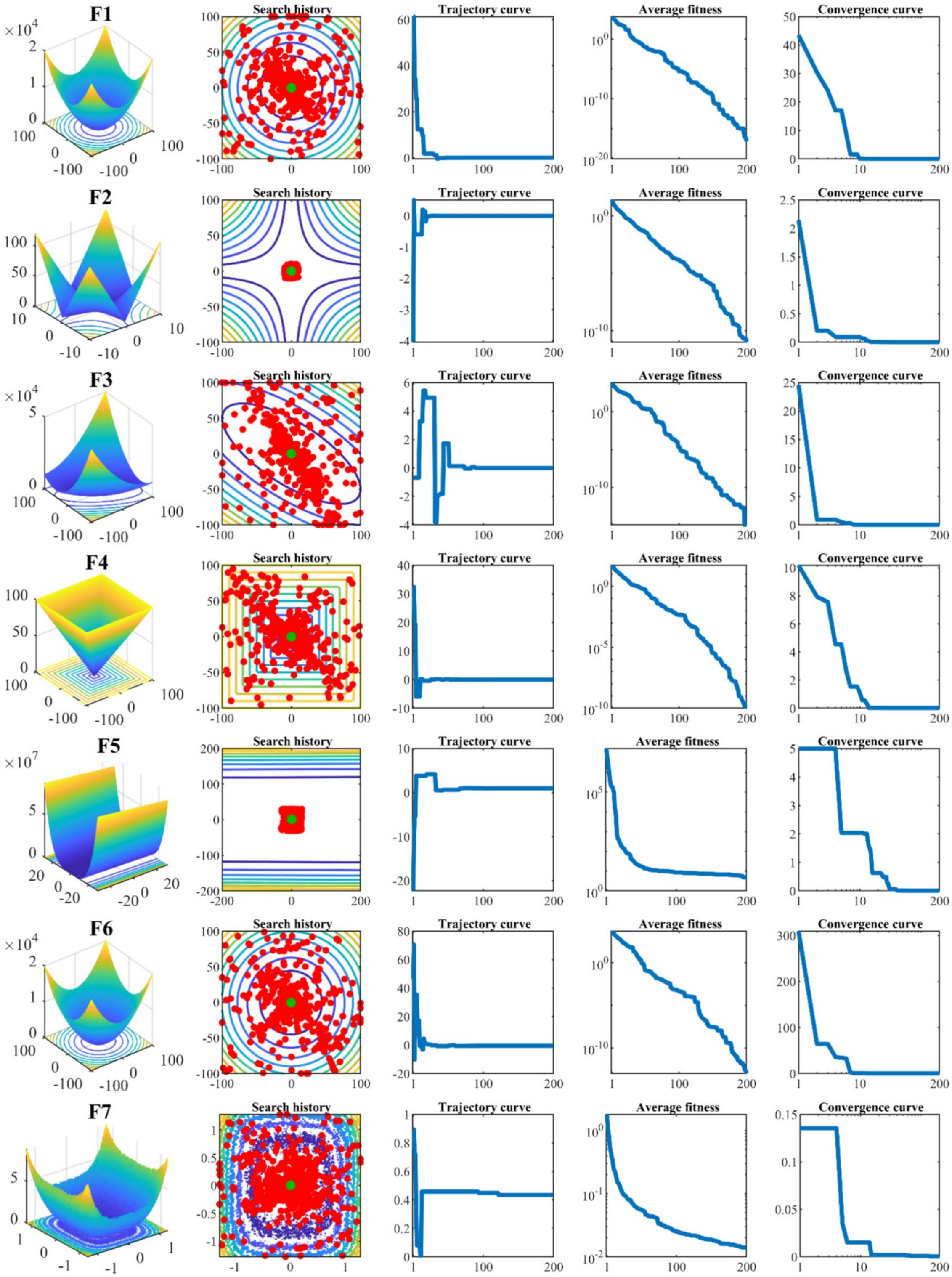

Figure 4. Qualitative analysis of BOA.

The results of the experiments, where the proposed BOA approach was tested against other competing algorithms, are meticulously detailed in Table 6. This table includes various performance metrics that showcase how each algorithm performed across the different benchmark functions. Additionally, boxplot diagrams, which visually represent the statistical performance data of these metaheuristic algorithms, are presented in Fig. 9. These diagrams help in comparing the consistency and reliability of the algorithms' performances.

From the collected data and analysis, it is evident that the BOA method stands out as the most effective optimizer among the tested algorithms. It achieved the best performance across all ten functions, C20-F1 through

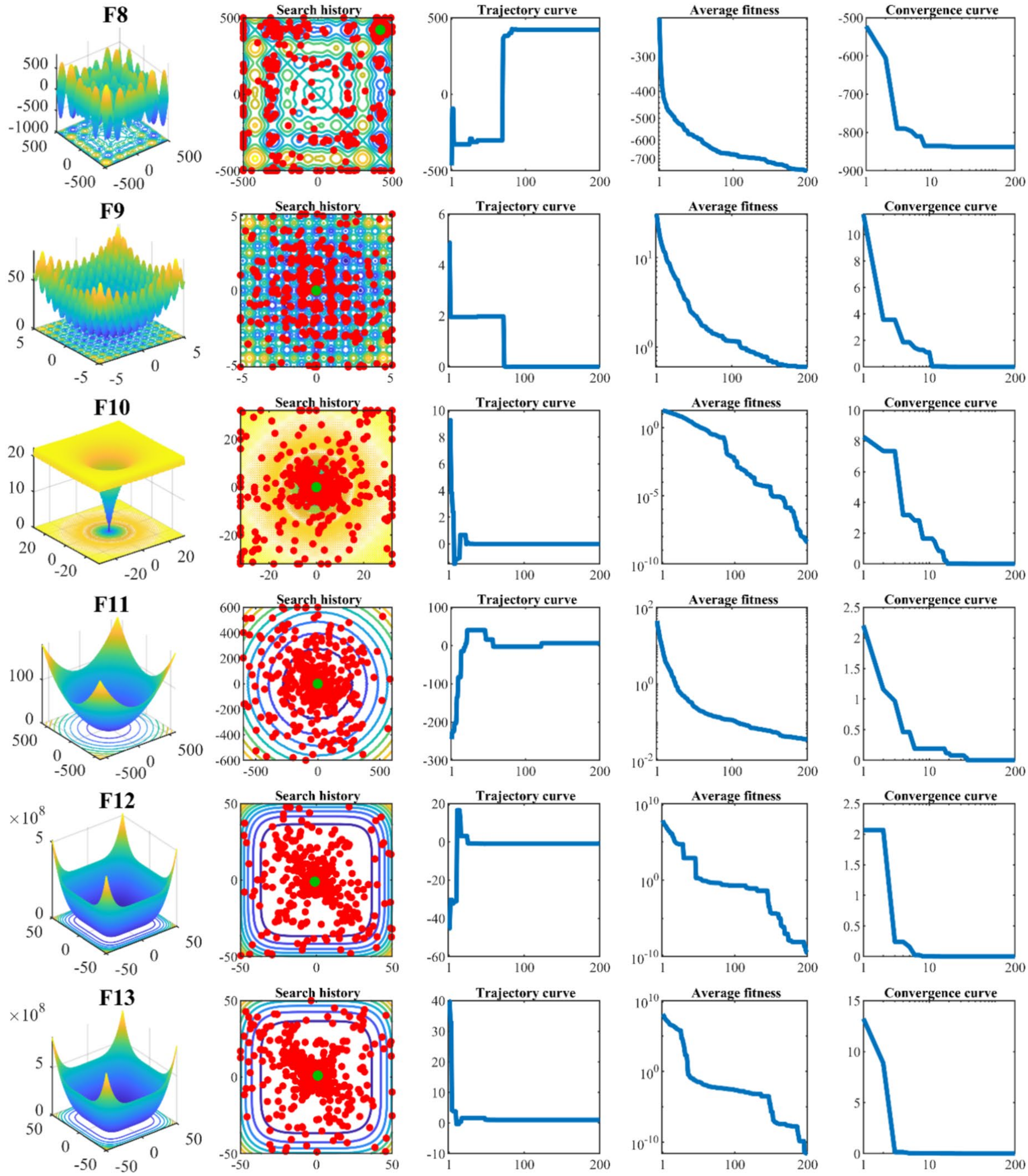

Figure 4. (continued)

C20-F10. The simulation findings clearly demonstrate that the BOA method provided significantly better results in handling the CEC 2020 benchmarks when compared to its competitors. This superior performance is indicative of the BOA's robustness and efficiency in navigating and solving complex optimization problems, thereby validating the proposed approach's capability in a comprehensive range of optimization scenarios.

Statistical analysis

In this subsection, by applying statistical analysis on the obtained results, it has been checked whether the superiority of BOA against competitor algorithms is significant from a statistical point of view. In this regard, the Wilcoxon rank sum test[99] is employed, which is a non-parametric test and has application to determine the existence of a significant difference between the means of two data samples. In the Wilcoxon rank sum test, using a measure called p-value, the presence or absence of a significant difference between the performance of two metaheuristic algorithms is determined.

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F1	Mean	100	100	49,446,990	1.04E+09	4.47E+08	2.19E+08	5.37E+09	68,759,580	9.99E+09	9.49E+08	1.76E+09	75,024,396	1.54E+08
	Best	100	100	10,201,910	8.88E+08	3.73E+08	46,467,314	4.49E+09	14,185,822	8.62E+09	1.96E+08	3.76E+08	22,439,029	15,105,169
	Worst	100	100	1.04E+08	1.29E+09	5.78E+08	4.73E+08	6.79E+09	1.45E+08	1.2E+10	2E+09	3.83E+09	1.52E+08	3.53E+08
	Std	0	2.27E-06	44,219,185	1.9E+08	97,771,736	2.05E+08	1.09E+09	61,492,032	1.63E+09	8.48E+08	1.67E+09	61,250,216	1.61E+08
	Median	100	100	41,669,794	9.98E+08	4.18E+08	1.78E+08	5.1E+09	57,943,503	9.69E+09	7.99E+08	1.42E+09	63,056,602	1.25E+08
	Rank	1	2	3	10	8	7	12	4	13	9	11	5	6
C17-F3	Mean	300	300	658.3707	1567.415	1035.785	1718.612	7875.002	799.9016	9881.012	7170.877	11,391.43	2187.368	3488.392
	Best	300	300	530.7419	1007.154	739.3996	916.096	4321.901	624.2364	5383.462	4718.626	4473.82	1099.255	2120.726
	Worst	300	300	751.1155	1888.548	1235.565	2262.432	10,511.52	927.2803	13,039.93	8953.478	16,024.92	3871.904	6049.213
	Std	0	0	102.5367	452.4927	235.0364	621.7373	2935.289	140.8449	3829.993	1970.775	5337.375	1381.135	2004.609
	Median	300	300	675.8127	1686.98	1084.089	1847.961	8333.295	824.0448	10,550.33	7505.702	12,533.49	1889.156	2891.815
	Rank	1	1	2	5	4	6	10	3	11	9	12	7	8
C17-F4	Mean	400	400	405.6514	497.7583	442.4615	422.3624	911.2356	411.8357	1331.959	499.5358	578.774	431.6715	418.6298
	Best	400	400	402.4407	450.4407	424.5178	409.8974	674.4453	404.4327	842.5651	444.4999	478.9232	411.1547	409.2564
	Worst	400	400	408.7351	543.1399	458.7272	436.4779	1124.121	417.6045	1809.98	555.3846	694.7475	482.7955	437.4958
	Std	0	0	3.25651	43.9835	17.08205	14.19227	221.1607	6.164928	450.5825	59.5619	115.1473	37.18891	14.19873
	Median	400	400	405.715	498.7264	443.3005	421.5372	923.1881	412.6528	1337.645	499.1293	570.7126	416.3679	413.8836
	Rank	1	1	2	8	7	5	11	3	12	9	10	6	4
C17-F5	Mean	501.2464	505.4435	507.6042	510.4319	507.0206	509.6011	563.4823	546.2748	574.5227	542.4813	566.2232	543.2544	515.8181
	Best	500.9951	502.5889	505.9747	508.2874	505.4592	506.8417	548.2794	529.7658	560.1478	531.5449	544.8278	525.4061	510.6061
	Worst	501.9917	507.1773	508.7698	511.8189	507.9541	514.1539	574.1879	564.0895	588.6143	557.2533	598.7156	579.5145	523.3654
	Std	0.540776	2.34263	1.309248	1.680442	1.234469	3.609747	12.60787	19.32928	17.42253	13.28413	26.10928	27.61185	5.86738
	Median	500.9993	506.0039	507.8362	510.8107	507.3345	508.7045	565.7309	545.6219	574.6643	540.5634	560.6747	534.0485	514.6505
	Rank	1	2	4	6	3	5	11	10	13	8	12	9	7
C17-F6	Mean	600	600.0001	602.4637	604.772	602.59	603.205	632.8669	618.1262	641.1858	614.4673	625.5317	623.892	602.16
	Best	600	600	602.0988	604.3908	602.3065	601.945	628.0002	617.0353	637.929	608.7684	615.4994	608.0567	601.5473
	Worst	600	600.0001	602.7903	604.979	602.9029	605.1553	637.3933	620.5529	644.9667	622.3602	641.4764	645.5276	603.3158
	Std	0	3.64E–05	0.342664	0.285084	0.268124	1.494745	4.524349	1.807928	3.322621	6.195272	12.17762	17.11693	0.885974
	Median	600	600.0001	602.4829	604.8592	602.5753	602.8599	633.037	617.4583	640.9238	613.3703	622.5755	620.9919	601.8884
	Rank	1	2	4	7	5	6	12	9	13	8	11	10	3
C17-F7	Mean	711.1267	716.9658	720.202	724.0323	719.5458	726.4131	800.465	769.9512	808.2156	782.1408	831.9985	766.525	730.956
	Best	710.6726	714.6235	717.0243	721.6864	717.7082	721.4155	782.7288	747.5659	794.1395	767.6955	791.4329	755.9759	722.8068
	Worst	711.7995	719.6695	723.2522	726.0619	720.7905	731.2867	813.3535	797.615	822.3515	804.4877	874.5463	794.6544	747.207
	Std	0.557384	2.351387	3.114441	2.112622	1.422938	4.622409	13.98072	25.35077	14.33531	18.48587	39.33738	20.42937	12.36744
	Median	711.0174	716.7852	720.2657	724.1905	719.8423	726.475	802.8889	767.312	808.1857	778.19	831.0073	757.7349	726.9051
	Rank	1	2	4	5	3	6	11	9	12	10	13	8	7

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F8	Mean	801.4928	805.4485	806.1538	808.3831	805.9604	807.8488	849.215	833.1422	855.4124	834.7571	850.0756	838.3122	818.0727
	Best	800.995	803.9824	805.0556	807.2537	805.4009	806.228	843.4496	823.0302	843.6964	826.2281	833.4507	820.5555	813.3967
	Worst	801.9912	806.4723	807.1183	809.2297	806.8402	809.7271	856.1535	848.5307	861.164	842.3286	869.6964	849.7042	822.7817
	Std	0.625636	1.296798	0.96697	0.947766	0.696812	1.970332	7.061327	11.81215	8.64082	7.942529	17.53729	13.76801	4.400448
	Median	801.4926	805.6697	806.2205	808.5245	805.8003	807.7201	848.6284	830.504	858.3947	835.2358	848.5777	841.4945	818.0563
	Rank	1	2	4	6	3	5	11	8	13	9	12	10	7
C17-F9	Mean	900	901.2521	942.0683	969.6386	940.7185	961.0878	1434.275	1203.167	1478.588	1161.179	1393.168	1387.824	930.7069
	Best	900	900.5006	918.4961	956.9936	930.3285	934.2048	1275.819	982.674	1379.65	1048.394	1175.173	1082.538	915.4595
	Worst	900	902	990.128	984.5583	951.1056	997.2761	1583.7	1671.729	1616.089	1309.298	1688.28	1675.867	953.4869
	Std	0	0.703323	35.46292	14.49815	10.73259	29.76249	144.8417	351.5674	110.495	124.1712	241.7145	270.62	17.78838
	Median	900	901.2538	929.8244	968.5013	940.7199	956.4352	1438.79	1079.134	1459.307	1143.511	1354.609	1396.445	926.9407
	Rank	1	2	5	7	4	6	12	9	13	8	11	10	3
C17-F10	Mean	1006.179	1146.638	1130.759	1209.087	1139.37	1155.695	2341.748	1828.185	2610.664	1948.679	2077.29	2069.862	1776.969
	Best	1000.284	1069.573	1103.307	1190.005	1125.305	1124.567	2040.879	1541.462	2434.88	1832.441	1809.319	1498.574	1595.509
	Worst	1012.668	1250.627	1192.443	1244.088	1159.569	1185.309	2483.037	2441.083	2969.284	2040.999	2324.49	2583.363	2038.687
	Std	7.244311	82.20178	45.46066	27.30946	17.63192	32.94717	225.5596	458.264	268.4555	93.88141	300.3383	568.0875	204.4007
	Median	1005.882	1133.176	1113.642	1201.127	1136.303	1156.451	2421.539	1665.098	2519.245	1960.638	2087.676	2098.755	1736.839
	Rank	1	4	2	6	3	5	12	8	13	9	11	10	7
C17-F11	Mean	1100	1100.626	1225.77	1502.827	1334.304	1646.975	3541.763	1315.635	4083.361	3421.658	5523.368	1318.03	1322.241
	Best	1100	1100.497	1220.651	1257.107	1236.846	1627.727	2326.475	1288.062	1619.752	3330.239	5372.066	1284.07	1287.229
	Worst	1100	1100.995	1232.453	1741.785	1426.984	1656.399	4722.591	1369.695	6511.473	3464.798	5604.802	1341.09	1396.718
	Std	0	0.267993	5.335282	237.7396	93.61907	14.17297	1171.585	39.99991	2393.554	67.14437	112.9346	29.57295	55.47797
	Median	1100	1100.505	1224.987	1506.207	1336.694	1651.888	3558.994	1302.392	4101.11	3445.798	5558.302	1323.48	1302.509
	Rank	1	2	3	8	7	9	11	4	12	10	13	5	6
C17-F12	Mean	1352.959	1380.763	168,741.7	69,162,662	27,724,741	162,774.6	3.46E + 08	1,160,119	6.9E + 08	1,146,605	1,100,510	2,386,786	1,468,135
	Best	1318.646	1378.411	112,456.4	15,368,066	6,192,597	69,834.46	77,855,883	454,651.5	1.53E + 08	304,671.2	549,643.9	263,329.9	66,481.64
	Worst	1438.176	1386.621	223,542.2	1.21E + 08	48,433,365	202,448	6.04E + 08	1,975,277	1.21E + 09	1,500,101	1,353,644	3,930,492	2,276,736
	Std	62.35801	4.261811	62,456.89	58,096,395	23,234,203	67,926.27	2.9E + 08	784,997.6	5.8E + 08	617,111.9	411,981.5	1,833,095	1,062,733
	Median	1327.506	1379.009	169,484.1	70,179,348	28,136,501	189,408	3.51E + 08	1,105,275	7E + 08	1,390,825	1,249,376	2,676,661	1,764,661
	Rank	1	2	4	11	10	3	12	7	13	6	5	9	8
C17-F13	Mean	1305.324	1306.916	3514.683	3,369,375	1,348,920	2966.654	16,826,486	18,719.91	33,642,756	11,692.23	13,245.25	8197.21	10,858.99
	Best	1303.114	1305.811	1981.114	281,419.2	113,804.1	2378.463	1,403,686	3439.815	2,793,979	10,093.92	8090.041	3875.71	7302.447
	Worst	1308.508	1308.508	4904.51	11,181,707	4,473,914	3804.379	55,850,375	31,672.49	1.12E + 08	13,812.05	20,682.49	15,577.08	14,854.02
	Std	2.473462	1.236731	1599.599	5,680,404	2,272,202	663.0794	28,374,453	15,819.4	56,745,869	1691.819	5904.875	5785.284	3372.484
	Median	1304.837	1306.673	3586.554	1,007,187	403,980	2841.886	5,025,942	19,883.67	10,045,519	11,431.46	12,104.23	6668.024	10,639.76
	Rank	1	2	4	11	10	3	12	9	13	7	8	5	6

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F14	Mean	1400.746	1402.488	1545.936	1872.004	1641.137	1679.768	3874.683	2126.398	5383.728	3017.461	3463.344	1633.835	2444.369
	Best	1400	1401.492	1514.792	1741.313	1541.232	1414.556	3125.548	1793.724	4757.38	1497.578	1493.17	1494.08	1492.644
	Worst	1400.995	1403.483	1572.547	2056.049	1734.419	1927.2	5121.683	2806.077	6948.083	3868.096	5660.909	1718.198	5011.255
	Std	0.541408	0.988645	25.89337	143.8941	86.91836	282.4258	972.0209	502.3075	1143.027	1163.221	2389.636	105.506	1863.835
	Median	1400.995	1402.487	1548.202	1845.326	1644.448	1688.658	3625.752	1952.896	4914.725	3352.086	3349.648	1661.531	1636.788
	Rank	1	2	3	7	5	6	12	8	13	10	11	4	9
C17-F15	Mean	1500.331	1500.608	2160.495	3001.481	2319.736	2327.641	10,425.71	5620.691	14,021.92	7020.214	7290.439	6522.206	6126.41
	Best	1500.001	1500.306	1800.563	1781.121	1739.971	1740.473	3429.69	2615.513	2930.934	4563.031	2524.873	2433.986	4082.632
	Worst	1500.5	1500.795	2900.785	4639.605	2946.744	2982.359	17,517.08	12,829.87	30,200.22	9152.519	12,875.06	13,597.2	7219.698
	Std	0.256213	0.229968	544.2832	1323.59	548.7565	572.9175	6568.928	5262.624	12,904.94	2061.25	4828.659	5304.984	1546.511
	Median	1500.413	1500.665	1970.315	2792.599	2296.114	2293.866	10,378.03	3518.692	11,478.26	7182.652	6880.908	5028.818	6601.654
	Rank	1	2	3	6	4	5	12	7	13	10	11	9	8
C17-F16	Mean	1600.76	1600.805	1638.113	1658.393	1637.379	1661.42	2018.011	1828.746	2031.336	1924.572	2061.578	1966.756	1749.34
	Best	1600.356	1600.578	1625.816	1643.177	1631.209	1636.177	1947.982	1670.624	1844.059	1782.202	1870.153	1792.071	1628.65
	Worst	1601.12	1601.25	1644.714	1689.828	1650.88	1684.102	2140.669	1932.666	2306.41	2019.15	2249.215	2083.07	1850.057
	Std	0.343807	0.328359	9.22092	23.09767	9.935448	23.72145	92.57894	121.3034	213.9591	120.0359	187.1323	154.6238	100.056
	Median	1600.781	1600.696	1640.961	1650.283	1633.713	1662.7	1991.696	1855.847	1987.438	1948.468	2063.473	1995.942	1759.326
	Rank	1	2	4	5	3	6	11	8	12	9	13	10	7
C17-F17	Mean	1700.099	1703.369	1709.696	1716.306	1711.035	1714.719	1826.132	1756.36	1822.388	1788.461	1806.535	1845.435	1773.554
	Best	1700.02	1700.908	1706.842	1715.599	1709.867	1712.001	1808.897	1738.123	1808.963	1761.101	1789.658	1778.074	1728.379
	Worst	1700.332	1710.327	1713.449	1716.839	1712.807	1716.827	1834.772	1798.802	1830.733	1831.784	1816.75	1895.071	1877.683
	Std	0.168864	5.050248	3.14449	0.604745	1.36759	2.202366	12.72187	30.95368	10.20348	32.99872	12.77625	54.60981	75.98779
	Median	1700.022	1701.121	1709.247	1716.393	1710.732	1715.024	1830.429	1744.258	1824.929	1780.479	1809.865	1854.297	1744.078
	Rank	1	2	3	6	4	5	12	7	11	9	10	13	8
C17-F18	Mean	1805.36	1803.022	3577.806	559,679.4	225,638.7	3597.611	2,793,772	12,723.82	5,568,029	16,947.01	12,921.67	23,910.81	20,587.35
	Best	1800.003	1800.238	2958.001	30,070.89	13,763.1	3214.393	145,169	5970.34	276,820.7	11,915.66	8531.628	7313.488	6953.641
	Worst	1820.451	1810.861	4222.316	1,620,395	650,198.7	3814.269	8,093,972	16,771.18	16,161,899	22,373.79	16,688.7	36,542.06	34,054.56
	Std	10.95197	5.689074	581.9884	802,004.7	320,966.7	291.8325	4,006,668	5150.476	8,010,157	4851.084	3676.492	15,477.83	15,004.41
	Median	1800.492	1800.495	3565.454	294,125.9	119,296.5	3680.892	1,467,973	14,076.87	2,916,698	16,749.29	13,233.17	25,893.84	20,670.6
	Rank	2	1	3	11	10	4	12	5	13	7	6	9	8
C17-F19	Mean	1900.445	1903.036	5967.925	74,154.86	33,282.38	17,587.81	383,635.9	11,600.06	692,769.1	70,904.02	127,679.6	39,051.49	10,305.92
	Best	1900.039	1902.175	2640.901	6241.901	3721.315	1956.575	25,125.23	8897.414	44,870.44	2798.478	2061.188	7592.564	3751.434
	Worst	1901.559	1903.534	9196.539	156,366.1	68,084.21	33,481.02	807,204.7	13,447.6	1,486,811	140,867.8	255,075.1	62,761.65	13,999.45
	Std	0.810364	0.656221	3759.288	71,946.34	30,005.71	19,469.62	370,114.3	2130.035	705,349.2	82,264.86	157,649	25,073.18	4901.535
	Median	1900.09	1903.217	6017.131	67,005.7	30,661.99	17,456.82	351,106.8	12,027.62	619,697.3	69,974.91	126,791	42,925.87	11,736.4
	Rank	1	2	3	10	7	6	12	5	13	9	11	8	4

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F20	Mean	2000.312	2008.144	2027.299	2032.43	2022.093	2030.9	2224.4	2181.015	2232.273	2198.594	2216.988	2216.221	2180.384
	Best	2000.312	2000.468	2014.278	2025.61	2017.122	2017.827	2170.276	2045.819	2175.39	2136.706	2114.142	2111.276	2145.668
	Worst	2000.312	2012.302	2041.855	2038.76	2026.181	2044.438	2293.247	2305.49	2287.25	2245.308	2331.288	2299.026	2255.552
	Std	0	5.740471	12.83227	7.779869	4.273473	12.04358	55.48822	125.8927	61.81132	49.3031	99.7429	97.9501	55.54853
	Median	2000.312	2009.903	2026.531	2032.675	2022.535	2030.667	2217.04	2186.375	2233.226	2206.182	2211.26	2227.292	2160.158
	Rank	1	2	4	6	3	5	12	8	13	9	11	10	7
C17-F21	Mean	2200	2200	2207.934	2213.152	2210.285	2218.833	2298.664	2222.652	2274.779	2326.263	2331.531	2316.56	2319.92
	Best	2200	2200	2204.163	2210.253	2206.662	2205.765	2249.567	2209.874	2234.419	2270.735	2225.884	2223.11	2315.379
	Worst	2200	2200	2211.727	2215.878	2212.332	2224.755	2327.358	2249.14	2299.227	2351.711	2379.287	2360.219	2326.508
	Std	0	0	3.371978	3.383607	2.830273	9.663023	38.80061	19.47232	31.26281	41.38876	77.99781	68.48564	5.621791
	Median	2200	2200	2207.922	2213.237	2211.073	2222.405	2308.865	2215.797	2282.735	2341.304	2360.476	2341.456	2318.896
	Rank	1	1	2	4	3	5	8	6	7	11	12	9	10
C17-F22	Mean	2300.073	2300.404	2312.582	2372.017	2336.085	2352.235	2705.414	2324.972	2918.709	2523.303	2721.094	2339.469	2324.598
	Best	2300	2300.289	2305.224	2349.814	2326.117	2318.739	2586.295	2316.651	2711.635	2379.262	2451.661	2333.611	2306.985
	Worst	2300.29	2300.464	2318.036	2392.939	2347.977	2378.5	2829.463	2331.276	3077.007	2637.785	2932.766	2348.021	2346.411
	Std	0.157893	0.089609	6.260226	20.38738	11.29786	28.97467	118.7708	7.064274	167.4238	125.1193	233.6481	6.775369	17.7289
	Median	2300	2300.432	2313.535	2372.658	2335.123	2355.852	2702.949	2325.98	2943.098	2538.082	2749.974	2338.121	2322.498
	Rank	1	2	3	9	6	8	11	5	13	10	12	7	4
C17-F23	Mean	2600.919	2606.152	2608.858	2614.598	2609.584	2616.838	2693.282	2646.715	2704.055	2675.667	2726.431	2653.229	2618.905
	Best	2600.003	2604.966	2605.194	2609.648	2605.794	2605.449	2655.942	2633.351	2676.062	2630.598	2635.896	2632.804	2613.498
	Worst	2602.87	2606.943	2610.827	2618.806	2611.283	2621.595	2713.942	2665.274	2744.99	2698.241	2771.661	2673.43	2626.635
	Std	1.436922	0.983904	2.719853	4.229174	2.771703	8.31615	29.89263	16.3158	36.21729	33.41629	66.74684	23.2471	6.349692
	Median	2600.403	2606.351	2609.705	2614.968	2610.63	2620.153	2701.622	2644.118	2697.584	2686.914	2749.084	2653.34	2617.744
	Rank	1	2	3	5	4	6	11	8	12	10	13	9	7
C17-F24	Mean	2630.488	2559.993	2648.332	2656.405	2638.815	2638.588	2789.781	2770.984	2851.628	2714.732	2673.643	2764.127	2752.53
	Best	2516.677	2530.043	2550.262	2554.148	2537.092	2526.346	2738.408	2749.06	2816.023	2637.321	2522.92	2739.37	2729.749
	Worst	2732.32	2636.477	2737.996	2750.371	2729.79	2740.594	2857.35	2794.112	2912.165	2789.122	2814.49	2794.567	2777.558
	Std	126.7883	55.91368	108.3019	110.339	109.5195	110.7933	61.01757	25.72389	45.83245	91.78961	169.8203	25.928	21.31351
	Median	2636.477	2536.726	2652.535	2660.55	2644.189	2643.706	2781.682	2770.381	2839.163	2716.242	2678.58	2761.286	2751.407
	Rank	2	1	5	6	4	3	12	11	13	8	7	10	9
C17-F25	Mean	2932.639	2939.767	2936.375	2971.989	2949.91	2957.955	3140.944	2921.691	3277.465	3040.315	3137.262	2915.865	2946.376
	Best	2898.047	2921.92	2920.063	2964.54	2938.377	2938.692	3066.64	2903.115	3201.328	2924.851	2905.286	2770.954	2924.273
	Worst	2945.793	2947.293	2943.392	2978.624	2956.026	2994.427	3317.913	2947.583	3373.857	3318.394	3672.413	2988.339	2973.187
	Std	25.12878	13.0092	12.01691	7.245209	8.698791	27.62378	130.3285	22.71768	78.10141	203.3647	392.4887	106.803	21.93672
	Median	2943.359	2944.927	2941.023	2972.397	2952.618	2949.35	3089.612	2918.033	3267.337	2959.007	2985.674	2952.083	2944.023
	Rank	3	5	4	9	7	8	12	2	13	10	11	1	6

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F26	Mean	2900	2900.875	2935.749	3011.907	2960.868	2998.614	3582.778	3017.06	3777.855	3435.518	3645.066	3216.125	3296.765
	Best	2900	2900.5	2922.133	2981.218	2940.045	2932.804	3265.578	2852.338	3462.729	3069.786	3151.594	2967.687	2980.137
	Worst	2900	2902	2954.69	3048.304	2976.602	3076.399	3800.732	3192.653	4112.752	3706.845	4300.477	3592.426	3927.881
	Std	4.04e−13	0.816228	14.84774	38.61269	20.06232	71.46853	263.4737	198.3441	312.2096	289.578	603.8627	293.3009	465.2699
	Median	2900	2900.5	2933.087	3009.053	2963.414	2992.627	3632.401	3011.624	3767.97	3482.722	3564.096	3152.193	3139.521
	Rank	1	2	3	6	4	5	11	7	13	10	12	8	9
C17-F27	Mean	3089.518	3089.765	3095.829	3106.728	3099.417	3101.667	3212.135	3124.006	3232.887	3153.218	3182.331	3197.416	3120.192
	Best	3089.518	3089.518	3090.92	3095.918	3094.848	3091.619	3164.348	3096.35	3130.211	3105.523	3103.329	3185.296	3098.096
	Worst	3089.518	3090.262	3104.287	3123.081	3104.478	3107.306	3288.369	3187.152	3417.633	3200.848	3224.601	3208.082	3183.092
	Std	2.86e−13	0.381583	6.367342	12.60222	4.370133	7.838288	57.93929	46.10078	137.7529	43.65257	60.44605	10.2978	45.64548
	Median	3089.518	3089.64	3094.054	3103.957	3099.17	3103.872	3197.911	3106.262	3191.851	3153.25	3200.698	3198.144	3099.79
	Rank	1	2	3	6	4	5	12	8	13	9	10	11	7
C17-F28	Mean	3100	3100.225	3133.229	3186.435	3148.341	3167.533	3610.046	3260.867	3792.381	3481.624	3603.56	3310.481	3367.376
	Best	3100	3100	3125.955	3183.565	3143.771	3143.91	3560.702	3136.091	3720.256	3339.773	3428.464	3174.202	3210.052
	Worst	3100	3100.5	3140.943	3189.103	3152.195	3194.088	3649.691	3401.518	3845.497	3597.882	3816.724	3420.728	3439.919
	Std	0	0.286219	6.798818	3.092518	4.596807	28.12012	40.2973	127.0334	61.23039	136.8979	221.1373	130.6775	115.6128
	Median	3100	3100.2	3133.008	3186.535	3148.698	3166.067	3614.896	3252.929	3801.885	3494.42	3584.526	3323.496	3409.767
	Rank	1	2	3	6	4	5	12	7	13	10	11	8	9
C17-F29	Mean	3132.241	3136.88	3153.714	3162.608	3150.855	3148.977	3344.468	3290.587	3379.439	3257.532	3243.263	3353.655	3271.63
	Best	3130.076	3131.922	3144.547	3153.713	3146.209	3140.356	3323.833	3217.871	3309.433	3197.317	3170.076	3244.351	3193.392
	Worst	3134.841	3142.083	3163.604	3172.434	3156.996	3153.955	3362.958	3365.312	3446.896	3299.371	3311.853	3493.211	3386.464
	Std	2.701544	4.543489	10.2828	10.04304	4.990471	6.722454	17.46574	83.82292	78.74978	47.91013	63.16795	113.2373	99.06063
	Median	3132.023	3136.758	3153.353	3162.143	3150.108	3150.798	3345.541	3289.584	3380.712	3266.72	3245.562	3338.528	3253.332
	Rank	1	2	5	6	4	3	11	10	13	8	7	12	9
C17-F30	Mean	3418.734	3447.108	71,180.58	401,393.3	195,850.6	102,411.7	2,258,187	342,295.3	3,641,031	757,502.6	654,285.9	1,022,712	967,757.2
	Best	3394.682	3399.627	53,873.97	130,920.5	119,170.4	50,398.61	1,689,464	167,548.9	872,854.2	698,176.9	160,223.7	56,822.57	83,217.92
	Worst	3442.907	3544.769	115,763.3	606,313.4	268,252.6	166,182	3,195,913	801,556.2	5,714,927	904,774.6	1,318,134	3,719,592	1,386,777
	Std	30.22288	71.83739	32,494.72	216,243	66,795.71	53,801.23	713,285	333,657.1	2,205,869	107,717.7	535,486	1,959,071	662,749.2
	Median	3418.673	3422.019	57,542.52	434,169.7	197,989.7	96,533.15	2,073,685	200,038	3,988,172	713,529.5	569,393	157,217.6	1,200,517
	Rank	1	2	3	7	5	4	12	6	13	9	8	11	10
Sum rank		33	58	98	205	148	155	332	201	362	260	305	243	203
Mean rank		1.137931	2	3.37931	7.068966	5.103448	5.344828	11.44828	6.931034	12.48276	8.965517	10.51724	8.37931	7
Total rank		1	2	3	8	4	5	12	6	13	10	11	9	7

Table 2. Optimization results of CEC 2017 test suite (dimension = 10).

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F1	Mean	100	100	525.2343	3.56E+09	2572.895	1.55E+09	2.55E+10	5050.118	3.99E+10	28,024.38	1.74E+10	1.65E+09	1.62E+09
	Best	100	100	125.3709	3.17E+09	1183.076	9.74E+08	2.2E+10	1479.384	3.56E+10	12,894.3	1.09E+10	1.3E+09	2.67E+08
	Worst	100	100	1166.022	4.38E+09	3624.058	2.11E+09	3.19E+10	8797.164	4.91E+10	42,610.55	2.37E+10	2.05E+09	4.88E+09
	Std	0	0	530.8751	6.04E+08	1258.109	5.8E+08	5.05E+09	3552.356	6.77E+09	15,629.24	6.5E+09	4.17E+08	2.38E+09
	Median	100	100	404.772	3.34E+09	2742.222	1.56E+09	2.41E+10	4961.963	3.75E+10	28,296.33	1.75E+10	1.62E+09	6.68E+08
	Rank	1	1	2	9	3	6	11	4	12	5	10	8	7
C17-F3	Mean	300	300	6565.254	9066.368	2792.424	6781.348	94,582.04	43,470.29	71,532.19	1140.041	45,894.82	225,237.5	40,523.43
	Best	300	300	3682.329	7749.21	1699.559	5509.773	86,360.46	23,653.06	55,391.71	876.8499	43,477.63	186,348	35,418.86
	Worst	300	300	8414.531	10,153.34	3497.417	7281.88	103,859.3	56,185.7	77,718.1	1406.853	48,363.57	258,766.2	45,240.16
	Std	0	0	2208.226	1235.88	839.4788	929.9679	9397.629	15,181.32	11,749.21	259.0144	2651.236	32,782.69	4383.614
	Median	300	300	7082.078	9181.462	2986.361	7166.87	94,054.22	47,021.2	76,509.47	1138.231	45,869.04	227,917.9	40,717.34
	Rank	1	1	4	6	3	5	11	8	10	2	9	12	7
C17-F4	Mean	458.5616	476.9909	466.4815	1272.3	464.6179	815.3504	6276.985	515.6812	9556.771	494.7727	4429.907	848.1344	571.2331
	Best	458.5616	471.4064	463.2555	967.9651	462.6823	513.5814	3532.178	492.8805	6130.829	483.7999	1032.751	784.6896	518.8363
	Worst	458.5616	488.587	469.0458	1610.649	466.7225	1074.719	8490.844	533.0723	13,351.78	517.5573	7353.352	926.2325	600.8925
	Std	0	8.586591	2.614747	290.4992	1.929612	257.9621	2236.698	18.35678	3262.154	16.87146	2904.745	69.82834	39.19413
	Median	458.5616	473.9852	466.8124	1255.293	464.5335	836.5508	6542.459	518.386	9372.239	488.8668	4666.763	840.8078	582.6019
	Rank	1	4	3	10	2	8	12	6	13	5	11	9	7
C17-F5	Mean	502.4874	545.6045	534.0158	547.775	521.684	539.9613	841.8028	725.5667	879.9412	587.2064	792.2731	820.7513	624.8613
	Best	500.995	533.3368	530.0703	544.7685	519.6226	536.7361	821.8094	689.1086	854.0182	563.5821	763.8969	790.8107	587.325
	Worst	503.9798	557.2101	541.0506	550.8726	525.6169	542.7751	860.8991	784.613	911.4654	611.4499	823.3475	836.065	652.434
	Std	1.397909	10.86063	5.52813	3.741157	2.922131	2.695546	17.883	46.76575	30.17356	21.837	30.17184	22.10747	35.1804
	Median	502.4874	545.9355	532.4712	547.7295	520.7483	540.167	842.2513	714.2726	877.1406	586.8968	790.924	828.0648	629.843
	Rank	1	5	3	6	2	4	12	9	13	7	10	11	8
C17-F6	Mean	600	601.7394	606.5797	609.7179	602.8271	609.1976	677.6632	645.5208	680.7309	603.4177	674.8927	674.1497	611.7754
	Best	600	601.0628	606.2999	609.207	602.6652	607.7642	676.4087	643.5545	675.486	602.0882	659.9829	663.6036	604.7434
	Worst	600	602.4864	607.0172	610.1858	603.032	610.1459	679.0473	648.5625	687.1535	604.8855	683.665	679.3651	618.5571
	Std	7.14E-14	0.673215	0.337389	0.461705	0.203512	1.168364	1.183057	2.364833	5.861909	1.322774	12.07787	7.850088	6.193764
	Median	600	601.7042	606.5008	609.7394	602.8057	609.4401	677.5984	644.9831	680.1422	603.3485	677.9614	676.815	611.9005
	Rank	1	2	5	7	3	6	12	9	13	4	11	10	8
C17-F7	Mean	733.478	733.2604	791.7949	808.4296	765.8664	798.5026	1290.414	1143.364	1330.001	852.4524	1218.623	1299.135	890.3038
	Best	732.8186	732.9343	776.3185	804.1516	760.7962	786.6768	1242.791	1034.76	1314.646	824.0866	1076.036	1259.98	820.7428
	Worst	734.5199	733.785	814.7096	816.1302	772.4074	807.2898	1325.459	1298.707	1356.707	908.8367	1363.835	1375.725	927.4041
	Std	0.820605	0.413302	18.63559	5.936736	5.241062	9.630095	39.9868	127.5866	20.36431	41.6995	136.7575	59.12413	51.59223
	Median	733.2867	733.1612	788.0757	806.7182	765.131	800.0219	1296.704	1119.995	1324.325	838.4431	1217.31	1280.417	906.5341
	Rank	2	1	4	6	3	5	11	9	13	7	10	12	8

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F8	Mean	803.3298	803.0269	824.0433	838.9714	818.9394	833.4912	1083.715	952.5163	1120.006	895.2526	1058.52	1030.778	896.6205
	Best	801.2023	802.5876	820.4724	836.2391	818.1973	828.6911	1069.43	923.6371	1100.2	888.1522	1015.85	975.2302	889.7656
	Worst	804.1574	803.9798	826.7746	840.6936	820.3179	843.912	1103.387	973.6607	1146.513	903.7711	1159.698	1072.034	904.5969
	Std	1.546288	0.716304	2.942021	2.216686	1.026	7.696347	17.18396	24.77438	25.42245	7.006999	74.06327	45.01877	6.867113
	Median	803.9798	802.7701	824.463	839.4764	818.6213	830.6808	1081.021	956.3837	1116.656	894.5436	1029.266	1037.924	896.0598
	Rank	2	1	4	6	3	5	12	9	13	7	11	10	8
C17-F9	Mean	900	998.5146	1453.833	1955.999	1130.187	2030.735	10,688.49	4724.795	10,358.98	1093.567	11,197.49	10,748.74	2057.9
	Best	900	915.8802	1276.092	1892.146	1054.468	1651.156	9126.623	3495.69	10,110.94	931.2024	6825.771	8219.176	1540.903
	Worst	900	1079.99	1548.803	2004.203	1180.149	2402.618	12,154.32	5389.86	10,499.34	1253.656	15,118.25	12,816.86	2810.953
	Std	7.14E-14	82.55345	132.8288	51.18418	60.13245	333.9188	1372.207	920.8116	185.415	162.206	3727.276	2520.837	668.8422
	Median	900	999.094	1495.218	1963.824	1143.066	2034.582	10,736.51	5006.814	10,412.82	1094.706	11,422.98	10,979.47	1939.871
	Rank	1	2	5	6	4	7	11	9	10	3	13	12	8
C17-F10	Mean	2293.267	3200.385	2740.379	2953.034	2613.523	2836.471	7218.581	5498.916	7884.852	4075.628	6577.052	6515.196	4852.939
	Best	1851.756	3055.93	2429.445	2525.896	2299.871	2362.87	6709.981	4748.18	7105.86	3694.441	5276.749	5655.646	4358.23
	Worst	2525.027	3334.852	2894.499	3127.903	2750.447	3024.194	7533.338	6023.704	8504.416	4569.92	7166.67	7870.134	5231.683
	Std	326.8979	126.8707	229.3471	311.0372	229.0032	344.4844	388.0349	649.4315	632.7996	439.4308	952.4196	1076.306	398.3631
	Median	2398.142	3205.379	2818.786	3079.168	2701.887	2979.411	7315.502	5611.891	7964.567	4019.075	6932.394	6267.502	4910.922
	Rank	1	6	3	5	2	4	12	9	13	7	11	10	8
C17-F11	Mean	1102.987	1102.453	1124.931	1779.487	1117.277	1460.906	7338.619	1259.072	8603.028	1173.189	5028.623	7642.592	2171.296
	Best	1100.995	1100.852	1117.992	1641.009	1116.394	1330.546	6041.218	1196.067	7006.32	1123.197	3580.88	5504.074	1387.209
	Worst	1105.977	1103.983	1134.014	1874.216	1119.083	1685.434	8402.15	1317.952	9680.719	1208.309	7575.706	11,297.32	4258.462
	Std	2.342568	1.391847	7.535193	115.9931	1.335511	172.5001	1129.092	55.30804	1333.321	40.07236	1954.297	2748.882	1516.321
	Median	1102.487	1102.489	1123.86	1801.361	1116.815	1413.821	7455.554	1261.134	8862.536	1180.624	4478.952	6884.489	1519.756
	Rank	2	1	4	8	3	7	11	6	13	5	10	12	9
C17-F12	Mean	1744.553	12,376.36	2,954,018	9.48E+08	1,145,213	4.41E+08	6.84E+09	20,316,999	1.06E+10	22,634.56	4.94E+09	2.41E+08	51,191,976
	Best	1721.81	9080.69	416,946.4	8.44E+08	163,883.7	2.29E+08	5.66E+09	2,860,148	9.47E+09	16,139.37	2.54E+09	61,702,028	4,970,733
	Worst	1764.937	15,574.63	7,212,400	1.19E+09	2,792,231	5.76E+08	8.69E+09	49,617,900	1.34E+10	28,906.11	6.46E+09	4.82E+08	1.07E+08
	Std	21.9323	3012.084	3,255,377	1.79E+08	1,259,174	1.63E+08	1.41E+09	22,399,508	2.02E+09	5920.301	1.84E+09	2.1E+08	48,575,835
	Median	1745.733	12,425.05	2,093,363	8.78E+08	812,369.1	4.8E+08	6.51E+09	14,394,974	9.83E+09	22,746.39	5.38E+09	2.11E+08	46,223,549
	Rank	1	2	5	10	4	9	12	6	13	3	11	8	7
C17-F13	Mean	1315.791	1622.421	22,305.52	9.16E+08	9483.018	1.27E+08	5.56E+09	145,783.7	1.03E+10	1918.277	1.43E+09	880,916.1	735,290.4
	Best	1314.587	1475.071	12,852.07	4.81E+08	5873.136	1,718,103	2.71E+09	80,785.36	5.39E+09	1629.916	19,213,290	415,679.5	88,884.41
	Worst	1318.646	1909.465	34,624.88	1.12E+09	14,232.97	4.41E+08	7.79E+09	230,518.8	1.26E+10	2483.44	4.95E+09	1,302,430	2,281,519
	Std	2.107258	213.3208	9840.238	3.2E+08	3782.956	2.3E+08	2.29E+09	67,675.33	3.59E+09	419.9068	2.58E+09	503,148.1	1,136,393
	Median	1314.967	1552.575	20,872.57	1.03E+09	8912.986	32,742,550	5.88E+09	135,915.4	1.15E+10	1779.876	3.67E+08	902,777.4	285,379.2
	Rank	1	2	5	10	4	9	12	6	13	3	11	8	7
Continued														

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F14	Mean	1423.017	1432.303	39,561.7	206,450.1	16,174.01	117,873.2	1,843,450	263,845.6	2,136,293	1441.264	1,142,482	2,162,793	518,558.1
	Best	1422.014	1430.045	6583.36	131,247.1	3420.301	80,958.13	1,136,802	36,931.22	1,073,742	1437.793	817,519.6	34,963.54	33,464.42
	Worst	1423.993	1434.867	89,985.45	286,821.6	35,674.96	179,400.1	2,333,569	610,797	3,181,138	1446.28	1,614,008	6,607,149	1,111,364
	Std	0.87954	2.253062	40,048.59	84,178.27	15,488.46	47,035.75	609,618.5	275,561.5	1,103,276	4.261836	397,812.4	3,284,549	595,537.1
	Median	1423.03	1432.151	30,839	203,865.8	12,800.39	105,567.2	1,951,714	203,827.1	2,145,146	1440.491	1,069,199	1,004,530	464,701.9
	Rank	1	2	5	7	4	6	11	8	12	3	10	13	9
C17-F15	Mean	1503.129	1564.908	6581.689	51,792,231	3477.21	1,248,755	2.96E+08	36,456.03	5.81E+08	1624.516	13,973,340	4,903,708	15,384,815
	Best	1502.462	1544.579	2834.353	44,695,515	2024.389	493,698.2	2.56E+08	10,672.81	5.01E+08	1585.216	5,504,642	225,930.2	95,560.67
	Worst	1504.265	1573.58	9883.623	57,162,982	4755.255	2,898,983	3.28E+08	59,175.45	6.41E+08	1642.187	32,504,767	15,921,706	57,603,068
	Std	0.931104	14.82345	3240.077	6,662,048	1255.224	1,207,967	38,631,543	22,292.2	74,746,382	28.74484	13,563,865	8,103,049	30,645,906
	Median	1502.893	1570.736	6804.39	52,655,213	3564.598	801,168.8	3E+08	37,987.93	5.91E+08	1635.331	8,941,976	1,733,597	1,920,315
	Rank	1	2	5	11	4	7	12	6	13	3	9	8	10
C17-F16	Mean	1663.469	1857.831	1852.503	2023.613	1768.145	1875.986	4272.52	2991.842	4911.65	2045.364	3255.307	4200.515	2547.718
	Best	1614.72	1677.853	1747.672	1906.911	1697.511	1794.312	3932.289	2580.34	4136.918	1738.623	2824.714	3471.588	2382.922
	Worst	1744.118	1971.482	1940.571	2127.109	1827.477	1957.979	4549.606	3509.299	5602.214	2315.707	3513.939	5051.555	2689.735
	Std	67.44425	144.4366	100.9172	98.33669	73.0482	92.95225	298.2819	418.7419	835.7595	282.9957	328.2705	710.3294	168.5539
	Median	1647.519	1890.995	1860.886	2030.216	1773.797	1875.826	4304.093	2938.865	4953.734	2063.563	3341.288	4139.458	2559.108
	Rank	1	4	3	6	2	5	12	9	13	7	10	11	8
C17-F17	Mean	1728.099	1801.246	1834.012	1941.411	1780.94	1903.333	3376.041	2467.276	3672.267	1871.824	3245.041	2831.977	1940.809
	Best	1718.761	1737.687	1813.076	1914.413	1769.667	1818.448	2790.312	2327.516	3301.761	1755.948	2221.276	2365.243	1806.82
	Worst	1733.659	1836.162	1847.43	1991.708	1790.59	2140.869	4094.551	2572.482	4331.726	1936.476	5920.747	3141.741	2090.536
	Std	7.30039	47.35308	16.50838	39.17219	10.92373	172.5059	603.8741	117.4547	509.5366	86.67033	1944.166	362.7341	145.6458
	Median	1729.987	1815.568	1837.771	1929.761	1781.751	1827.007	3309.651	2484.553	3527.791	1897.436	2419.071	2910.462	1932.94
	Rank	1	3	4	8	2	6	12	9	13	5	11	10	7
C17-F18	Mean	1825.696	1863.714	375,777.5	2,977,286	146,451.3	3,294,413	27,625,198	2,574,885	31,763,190	1900.397	35,321,279	5,736,096	407,812.5
	Best	1822.524	1852.861	41,419.2	932,245.8	17,143.45	405,753.7	7,957,929	274,244.5	10,269,107	1876.442	1,295,250	1,933,050	76,286.17
	Worst	1828.42	1871.5	748,201.7	5,777,074	290,479.4	6,181,130	53,649,581	5,137,435	62,402,143	1914.061	66,935,558	11,839,212	1,047,754
	Std	2.940513	8.699882	360,493.4	2,235,534	139,414	3,522,502	21,982,787	2,480,466	24,058,987	18.46345	39,669,613	4,632,814	497,561.9
	Median	1825.92	1865.249	356,744.5	2,599,912	139,091.2	3,295,384	24,446,641	2,443,929	27,190,756	1905.543	36,527,153	4,586,061	253,604.9
	Rank	1	2	5	8	4	9	11	7	12	3	13	10	6
C17-F19	Mean	1910.989	1917.851	11,203.58	84,826,569	5505.855	25,516,438	5.64E+08	65,851.59	9.52E+08	1924.472	2.86E+08	13,926,092	3,920,288
	Best	1908.84	1915.996	3678.613	61,245,633	2594.943	322,075.6	4.22E+08	14,080.32	6.87E+08	1921.806	3,554,244	1,812,174	68,877.17
	Worst	1913.095	1919.496	22,943.44	1.29E+08	10,046.47	70,655,452	7.35E+08	146,623.8	1.44E+09	1929.778	7.93E+08	24,046,472	12,641,461
	Std	2.10261	1.611527	9133.068	32,480,789	3532.638	35,338,862	1.71E+08	62,834.57	3.64E+08	3.925742	3.96E+08	11,034,648	6,370,660
	Median	1911.01	1917.956	9096.134	74,748,609	4691.005	15,544,112	5.51E+08	51,351.15	8.39E+08	1923.152	1.74E+08	14,922,860	1,485,408
	Rank	1	2	5	10	4	9	12	6	13	3	11	8	7

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F20	Mean	2065.787	2125.376	2146.902	2174.568	2106.836	2165.597	2890.063	2632.403	2942.806	2182.872	2842.157	2830.062	2377.345
	Best	2029.521	2048.993	2093.212	2135.418	2069.275	2113.229	2802.156	2481.021	2769.972	2062.483	2705.603	2631.318	2199.982
	Worst	2161.126	2217.014	2215.975	2256.373	2191.416	2239.678	2995.452	2856.032	3044.153	2270.939	2984.038	3012.109	2546.293
	Std	69.26656	75.38854	56.94018	62.04962	61.7878	58.5921	87.17717	178.2672	131.1936	95.56318	126.0693	175.7902	153.9625
	Median	2036.25	2117.749	2139.21	2153.24	2083.326	2154.74	2881.321	2596.28	2978.549	2199.032	2839.494	2838.412	2381.553
	Rank	1	3	4	6	2	5	12	9	13	7	11	10	8
C17-F21	Mean	2308.456	2339.671	2327.357	2348.22	2320.836	2335.569	2620.827	2442.956	2677.029	2369.789	2535.087	2608.931	2392.958
	Best	2304.034	2332.425	2298.158	2338.771	2312.563	2313.15	2528.153	2224.968	2599.665	2359.818	2312.208	2534.236	2359.415
	Worst	2312.987	2349.732	2353.606	2362.734	2333.306	2359.802	2681.412	2596.515	2769.268	2386.592	2666.034	2673.291	2407.94
	Std	4.852783	8.359126	24.78077	11.14647	9.659995	20.81423	79.10045	170.2729	80.28238	12.82255	170.2267	74.64918	24.82267
	Median	2308.402	2338.264	2328.833	2345.688	2318.738	2334.662	2636.872	2475.17	2669.592	2366.373	2581.053	2614.099	2402.238
	Rank	1	5	3	6	2	4	12	9	13	7	10	11	8
C17-F22	Mean	2300	2301.574	2794.537	2966.531	2491.509	3056.746	7870.673	5702.987	7632.724	2303.093	8644.909	7304.193	2696.166
	Best	2300	2301.027	2300.435	2804.912	2300.36	2845.472	7538.031	2303.163	6622.976	2302.018	8418.351	6366.415	2569.377
	Worst	2300	2302.498	2985.438	3116.796	2565.339	3132.034	8389.649	7016.524	8647.5	2304.909	8751.768	8140.766	2956.62
	Std	0	0.718287	359.0646	140.1046	138.9066	153.3364	396.0803	2470.658	946.9328	1.411334	170.5654	802.4926	192.7069
	Median	2300	2301.385	2946.137	2972.208	2550.169	3124.738	7777.507	6746.131	7630.21	2302.722	8704.759	7354.796	2629.334
	Rank	1	2	6	7	4	8	12	9	11	3	13	10	5
C17-F23	Mean	2655.081	2650.208	2694.043	2722.12	2669.357	2717.727	3183.402	2922.477	3237.49	2645.506	3188.203	3040.981	2749.264
	Best	2653.745	2555.313	2677.25	2718.152	2646.847	2710.003	3103.897	2801.455	3187.366	2460.34	3076.202	2871.462	2728.717
	Worst	2657.377	2686.212	2718.895	2729.19	2684.863	2730.646	3268.768	3097.982	3301.766	2716.149	3387.524	3144.208	2773.143
	Std	1.79918	68.98752	19.89966	5.397303	17.60135	9.869591	81.0102	138.7769	53.74706	134.6965	150.3745	128.7383	22.97704
	Median	2654.6	2679.653	2690.014	2720.569	2672.86	2715.129	3180.471	2895.236	3230.414	2702.767	3144.543	3074.127	2747.597
	Rank	3	2	5	7	4	6	11	9	13	1	12	10	8
C17-F24	Mean	2831.409	2859.663	2880.103	2901.592	2854.83	2889.783	3312.868	3170.48	3411.585	2886.924	3279.087	3116.748	2923.827
	Best	2829.992	2851.603	2858.973	2885.8	2846.437	2877.342	3274.152	3033.994	3322.112	2870.214	3171.247	3053.857	2912.005
	Worst	2832.366	2863.179	2903.158	2917.599	2862.844	2900.223	3391.053	3322.522	3565.474	2894.261	3330.362	3142.705	2931.413
	Std	1.246718	5.888718	20.79996	14.20343	7.693601	11.30971	57.56735	138.092	122.0065	12.29561	80.68756	45.93786	9.413149
	Median	2831.64	2861.935	2879.14	2901.486	2855.02	2890.783	3293.134	3162.702	3379.377	2891.611	3307.371	3135.215	2925.946
	Rank	1	3	4	7	2	6	12	10	13	5	11	9	8
C17-F25	Mean	2886.698	2889.178	2889.85	3035.421	2888.32	2939.12	3925.344	2908.733	4542.005	2891.571	3461.538	3079.402	2992.135
	Best	2886.691	2885.523	2887.804	2981.654	2886.932	2905.091	3554.202	2894.184	3946.314	2884.397	3088.135	3042.886	2954.082
	Worst	2886.707	2892.528	2895.466	3105.422	2890.253	2975.804	4202.919	2947.116	5335.804	2898.158	3848.489	3098.225	3062.277
	Std	0.008278	3.444675	4.078142	55.93674	1.516575	37.15413	294.774	27.88481	629.7821	6.767994	404.4581	28.37109	54.54305
	Median	2886.698	2889.331	2888.064	3027.303	2888.047	2937.793	3972.128	2896.817	4442.951	2891.864	3454.765	3088.249	2976.09
	Rank	1	3	4	9	2	7	12	6	13	5	11	10	8

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F26	Mean	3578.65	3239.506	4115.01	4328.87	3730.991	4238.231	9041.53	7220.882	9620.345	2912.279	8603.399	8251.686	4501.039
	Best	3559.841	3230.016	3955.603	4208.781	3684.801	4203.907	8620.16	5951.238	8791.84	2911.781	7950.602	7528.815	4112.973
	Worst	3607.686	3254.083	4229.537	4448.636	3779.17	4283.824	9798.111	7965.604	11,084.01	2912.905	9016.403	9092.969	5103.866
	Std	24.78775	12.47715	126.7818	108.3784	42.82998	36.416	597.9834	964.7783	1170.831	0.607902	497.0778	699.2433	460.484
	Median	3573.536	3236.962	4137.449	4329.032	3729.995	4232.597	8873.924	7483.344	9302.764	2912.215	8723.295	8192.481	4393.659
	Rank	3	2	5	7	4	6	12	9	13	1	11	10	8
C17-F27	Mean	3207.018	3211.128	3228.291	3264.375	3215.913	3238.685	3605.623	3353.971	3758.826	3215.093	3470.591	3425.163	3249.84
	Best	3200.749	3205.011	3218.797	3243.205	3212.801	3225.476	3545.779	3267.224	3480.903	3199.952	3340.499	3257.742	3243.047
	Worst	3210.656	3219.268	3238.488	3288.239	3218.51	3264.168	3707.029	3427.98	4044.69	3237.137	3716.104	3548.884	3263.306
	Std	5.058229	7.8499	9.882968	21.5598	2.558314	18.99515	78.25414	92.51505	263.1273	18.43796	182.9847	135.711	9.930066
	Median	3208.335	3210.116	3227.938	3263.027	3216.169	3232.547	3584.842	3360.341	3754.856	3211.641	3412.881	3447.012	3246.504
	Rank	1	2	5	8	4	6	12	9	13	3	11	10	7
C17-F28	Mean	3100	3161.663	3123.773	3336.938	3119.21	3202.245	4766.414	3272.357	5664.019	3221.159	4152.795	3442.83	3599.399
	Best	3100	3152.676	3119.335	3309.551	3116.033	3153.141	4532.424	3240.539	5357.193	3203.5	3601.124	3382.285	3402.213
	Worst	3100	3178.058	3128.388	3367.289	3123.658	3250.668	5021.477	3306.449	5986.855	3253.373	4714.066	3496.511	4079.318
	Std	2.86e-13	12.34473	4.028214	30.20473	3.500736	49.75237	227.8643	29.3586	327.1858	24.25568	561.6129	54.75774	350.4573
	Median	3100	3157.959	3123.684	3335.456	3118.574	3202.586	4755.877	3271.22	5656.013	3213.881	4147.995	3446.261	3458.032
	Rank	1	4	3	8	2	5	12	7	13	6	11	9	10
C17-F29	Mean	3353.75	3339.568	3493.96	3607.255	3434.697	3573.977	5395.961	4341.922	5613.069	3677.005	5239.693	5087.593	3808.31
	Best	3325.385	3325.385	3418.955	3558.832	3386.211	3482.241	4954.524	3990.818	4993.81	3513.346	4700.865	4808.271	3714.22
	Worst	3370.797	3348.091	3530.777	3660.852	3469.206	3674.251	5877.986	4558.502	6468.21	3824.072	6124.628	5267.252	3925.266
	Std	21.42746	10.71373	56.82622	55.11367	40.73257	87.51387	486.8584	275.039	795.2219	148.9627	722.8365	213.6429	99.47889
	Median	3359.41	3342.398	3513.053	3604.668	3441.685	3569.708	5375.667	4409.183	5495.129	3685.302	5066.64	5137.425	3796.876
	Rank	2	1	4	6	3	5	12	9	13	7	11	10	8
C17-F30	Mean	5007.854	6444.225	206,887	2.46E+08	83,314.48	3,430,490	1.4E+09	1,394,284	2.76E+09	7830.123	37,562,345	38,330,184	6,235,146
	Best	4955.449	5743.386	75,689.22	1.77E+08	32,438.52	1,212,204	1.03E+09	491,766.3	1.98E+09	6449.159	12,842,104	7,644,163	1,391,096
	Worst	5086.396	7852.294	363,018.5	2.72E+08	143,677.2	7,910,538	1.54E+09	2,468,803	3.05E+09	10,521	87,766,766	61,420,483	16,836,721
	Std	64.18196	1081.06	130,736.4	50,369,952	50,629.97	3,301,747	2.68E+08	899,587.1	5.65E+08	2075.181	37,013,086	24,395,807	7,763,531
	Median	4994.785	6090.61	194,420.1	2.68E+08	78,571.08	2,299,609	1.51E+09	1,308,284	3E+09	7175.167	24,820,254	42,128,045	3,356,384
	Rank	1	2	5	11	4	7	12	6	13	3	9	10	8
Sum rank		37	72	122	221	89	182	340	227	366	130	313	291	225
Mean rank		1.275862	2.482759	4.206897	7.62069	3.068966	6.275862	11.72414	7.827586	12.62069	4.482759	10.7931	10.03448	7.758621
Total rank		1	2	4	7	3	6	12	9	13	5	11	10	8

Table 3. Optimization results of CEC 2017 test suite (dimension = 30).

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F1	Mean	100	100	1,728,328	1.21E+10	1,262,718	4.91E+09	5.79E+10	9,383,880	9.08E+10	5,883,740	3.69E+10	7.46E+09	9.06E+09
	Best	100	100	365,868	1.06E+10	487,389.5	4.52E+09	5.17E+10	1,244,365	7.94E+10	2,270,806	3.4E+10	4.4E+09	6.53E+09
	Worst	100	100	3,508,112	1.32E+10	3,201,611	5.29E+09	6.2E+10	23,985,403	9.92E+10	14,918,729	3.97E+10	1.12E+10	1.24E+10
	Std	0	0	1,495,618	1.25E+09	1,416,860	3.43E+08	4.94E+09	10,889,335	9.4E+09	6,602,386	2.58E+09	3.48E+09	2.66E+09
	Median	100	100	1,519,665	1.23E+10	680,936.6	4.92E+09	5.9E+10	6,152,875	9.22E+10	3,172,711	3.7E+10	7.14E+09	8.66E+09
	Rank	1	1	3	9	2	6	11	5	12	4	10	7	8
C17-F3	Mean	300	300	20,816.92	22,273.9	4250.119	15,993.19	154,765.8	143,245.2	154,197.8	18,707.05	106,983.7	227,858.9	127,167.9
	Best	300	300	16,211.97	20,459.68	3703.14	14,360.71	132,990.7	110,191.2	140,063.7	16,158.2	94,215.61	172,059.9	111,942.1
	Worst	300	300	24,758.2	23,872.95	4974.16	17,230.88	178,033.9	173,875.2	167,781.4	22,080.99	114,218.8	347,128.6	142,826.2
	Std	0	0	4167.541	1603.357	620.917	1366.083	20,635.53	31,270.07	13,350.27	2893.394	9981.826	89,698.21	13,735.77
	Median	300	300	21,148.75	22,381.48	4161.589	16,190.6	154,019.3	144,457.2	154,473.1	18,294.51	109,750.1	196,123.5	126,951.6
	Rank	1	1	5	6	2	3	11	9	10	4	7	12	8
C17-F4	Mean	470.3679	454.1516	505.4095	3475.109	483.9694	1486.007	14,309.01	694.922	23,019.15	533.749	8066.4	1897.604	1409.741
	Best	428.5127	427.3579	468.3653	2457.862	443.6215	1237.5	11,117.03	678.966	15,197.41	498.9177	6466.766	1208.486	1053.559
	Worst	525.7252	477.1189	553.5483	4117.71	538.5948	1821.362	16,290.19	721.2109	27,491.1	585.6961	10,412.64	2270.34	1722.278
	Std	53.9489	23.22949	45.19507	808.0173	51.65667	267.373	2518.511	21.20138	6107.346	43.98938	1814.879	517.2535	327.7629
	Median	463.6168	456.0648	499.8623	3662.433	476.8306	1442.583	14,914.42	689.7555	24,694.04	525.1912	7693.098	2055.795	1431.564
	Rank	2	1	4	10	3	8	12	6	13	5	11	9	7
C17-F5	Mean	504.7261	627.4162	572.1139	606.9869	556.459	609.2853	1097.359	863.4781	1125.63	745.7952	1142.908	958.3595	735.4237
	Best	503.9798	583.7917	559.8676	600.3399	537.6329	590.8618	1065.531	827.9444	1112.078	660.799	1004.885	920.7484	702.8731
	Worst	505.9698	661.9043	584.4721	611.5908	571.7204	622.8597	1130.321	908.2177	1135.138	812.3595	1246.962	982.483	767.5635
	Std	1.036717	35.58634	11.07283	5.163424	15.44164	14.9463	33.72082	37.25954	11.2993	69.20695	128.8821	31.43439	35.66723
	Median	504.4773	631.9845	572.058	608.0084	558.2414	611.7098	1096.792	858.8752	1127.651	755.0111	1159.892	965.1034	735.6291
	Rank	1	6	3	4	2	5	11	9	12	8	13	10	7
C17-F6	Mean	600	606.0008	608.7898	613.5406	602.5303	612.6	692.5622	658.8454	694.5593	611.7907	687.4881	695.1809	622.8304
	Best	600	604.5287	608.4385	612.9849	601.9096	610.0235	689.6993	654.4824	692.1551	608.8983	667.8911	689.7955	617.3844
	Worst	600	607.9465	609.3708	614.0251	603.3507	615.0881	697.625	664.0122	697.4107	615.6138	703.8476	703.1332	632.1326
	Std	0	1.593136	0.459398	0.571056	0.671755	2.386215	4.000128	4.794587	2.712736	3.130291	17.32511	6.231282	7.14612
	Median	600	605.764	608.6749	613.5763	602.4304	612.6442	691.4623	658.4435	694.3357	611.3255	689.1067	693.8974	620.9022
	Rank	1	3	4	7	2	6	11	9	12	5	10	13	8
C17-F7	Mean	756.7298	758.34	899.261	928.3168	817.4625	901.4014	1776.794	1654.643	1873.065	1039.736	1670.733	1695.209	1075.365
	Best	754.7543	707.4727	893.3654	916.7166	803.2437	885.0934	1756.169	1588.785	1798.422	980.7088	1526.601	1637.339	1048.793
	Worst	758.3522	780.353	912.9335	944.8564	828.7823	917.4084	1803.394	1722.359	1975.86	1089.766	1811.056	1776.621	1097.276
	Std	1.69049	37.44172	9.975088	13.11921	12.89696	17.96458	21.31842	61.45655	83.85688	57.55841	146.5103	69.92579	24.87366
	Median	756.9065	772.7672	895.3725	925.847	818.9119	901.5518	1773.806	1653.714	1858.989	1044.235	1672.637	1683.438	1077.695
	Rank	1	2	4	6	3	5	12	9	13	7	10	11	8

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F8	Mean	805.721	807.0118	866.08	907.5884	851.4696	906.275	1414.098	1128.735	1440.767	1018.904	1430.894	1322.231	1041.505
	Best	802.9849	801.6807	855.0874	905.1832	842.4735	888.4689	1361.422	1081.82	1412.677	986.9969	1332.759	1195.684	1005.978
	Worst	810.9445	810.9445	878.5152	914.4937	860.9036	920.5888	1454.64	1176.14	1459.906	1051.991	1558.859	1429.853	1081.191
	Std	3.891615	4.214412	12.69294	5.010839	9.751803	15.49269	46.71159	58.40734	21.758	36.67694	105.4953	104.8771	36.7792
	Median	804.4773	807.7109	865.3587	905.3383	851.2508	908.0211	1420.165	1128.489	1445.243	1018.313	1415.978	1331.694	1039.426
	Rank	1	2	4	6	3	5	11	9	13	7	12	10	8
C17-F9	Mean	900	906.5083	2717.002	5673.45	1440.498	5875.782	35,053.43	13,014.56	35,239.17	3418.651	36,760.16	32,028.99	6837.551
	Best	900	902.8101	2607.995	5368.443	1161.641	5376.775	33,657.02	12,413.41	33,102.33	2119.215	33,805.79	29,824.26	5877.653
	Worst	900	910.5513	2818.132	6045.145	1774.203	6578.817	38,375.71	13,831.64	37,079.77	4973.672	41,091.56	37,541.77	7740.074
	Std	1.01E−13	3.601126	119.6364	313.164	275.3931	554.0872	2439.892	649.003	2006.603	1283.296	3407.392	4009.678	1067.369
	Median	900	906.3359	2720.94	5640.105	1413.074	5773.767	34,090.5	12,906.59	35,387.28	3290.858	36,071.65	30,374.97	6866.238
	Rank	1	2	4	6	3	7	11	9	12	5	13	10	8
C17-F10	Mean	4347.157	5031.389	5069.465	5825.77	4839.501	5512.11	12,878.09	8370.132	14,055.53	6641.414	11,697.65	11,705.13	8698.195
	Best	3555.132	4635.376	4581.528	5232.089	4358.582	4907.108	12,396.33	7834.038	13,835.82	5694.772	10,736.36	10,576.81	6626.385
	Worst	5099.795	5407.708	5693.648	6516.752	5559.483	6290.174	13,603.07	8975.069	14,438	7299.106	12,734.73	12,818.27	13,703.26
	Std	701.6898	350.8449	516.779	583.5159	556.7897	643.9142	611.0757	514.4093	302.2446	839.7788	913.7876	1065.163	3657.422
	Median	4366.851	5041.235	5001.341	5777.119	4719.969	5425.579	12,756.47	8335.711	13,974.16	6785.889	11,659.75	11,712.73	7231.57
	Rank	1	3	4	6	2	5	12	8	13	7	10	11	9
C17-F11	Mean	1128.435	1195.781	1201.957	3696.413	1156.832	2659.485	14,938.23	1599.983	20,351.65	1260.761	12,556.71	4976.95	5976.752
	Best	1121.25	1166.763	1189.814	3401.302	1140.441	2427.613	13,765.7	1483.275	18,107.77	1210.678	10,800.91	4392.506	3605.466
	Worst	1133.132	1211.714	1213.451	3927.65	1164.326	2983.189	15,679.15	1744.926	22,054.96	1293.24	15,063.23	6219.993	10,347.9
	Std	5.923599	22.27869	13.32997	238.9926	12.07493	259.9143	921.5994	131.2327	1795.055	39.90806	1999.666	915.3222	3389.918
	Median	1129.678	1202.324	1202.282	3728.351	1161.28	2613.57	15,154.03	1585.866	20,621.93	1269.564	12,181.35	4647.65	4976.823
	Rank	1	3	4	8	2	7	12	6	13	5	11	9	10
C17-F12	Mean	2905.102	3036.736	10,835,520	9.18E+09	3,231,144	3.34E+09	4.23E+10	72,210,721	6.9E+10	15,046,086	2.51E+10	1.28E+09	9.29E+08
	Best	2527.376	2847.873	5,302,690	6.69E+09	3,044,014	1.41E+09	3.55E+10	31,152,337	5.03E+10	14,173,120	1.06E+10	1.06E+09	1.47E+08
	Worst	3168.37	3168.37	15,962,790	1.26E+10	3,382,637	5.61E+09	5.07E+10	1.11E+08	9.46E+10	15,751,606	4.22E+10	1.74E+09	1.73E+09
	Std	297.8769	148.9384	6,210,566	2.95E+09	171,961.4	1.89E+09	7.46E+09	46,523,307	2.22E+10	801,932.9	1.42E+10	3.43E+08	8.57E+08
	Median	2962.331	3065.351	11,038,301	8.72E+09	3,248,963	3.16E+09	4.14E+10	73,364,701	6.55E+10	15,129,808	2.38E+10	1.16E+09	9.22E+08
	Rank	1	2	4	10	3	9	12	6	13	5	11	8	7
C17-F13	Mean	1340.1	1341.558	21,812.21	5.56E+09	4701.675	1.3E+09	2.38E+10	145,630.2	4.18E+10	17,004.61	9.77E+09	92,005,033	3.46E+08
	Best	1333.781	1338.398	7285.236	2.81E+09	2978	6.91E+08	1.37E+10	34,608.78	2.11E+10	8968.589	5.19E+09	69,170,466	1.57E+08
	Worst	1343.015	1343.015	44,208.19	7.99E+09	5345.758	2.02E+09	3.25E+10	318,909.7	6.01E+10	20,029.08	1.52E+10	1.04E+08	8.71E+08
	Std	4.660414	2.330207	17,139.03	2.36E+09	1251.73	6.14E+08	8.97E+09	132,272.1	1.78E+10	5836.482	4.61E+09	16,990,756	3.81E+08
	Median	1341.801	1342.408	17,877.7	5.72E+09	5241.47	1.24E+09	2.46E+10	114,501.1	4.3E+10	19,510.38	9.35E+09	97,187,627	1.79E+08
	Rank	1	2	5	10	3	9	12	6	13	4	11	7	8

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F14	Mean	1429.458	1430.699	159,056.5	6,246,426	1459.394	347,985.6	25,180,496	1,186,279	46,947,075	1568.956	2,606,522	4,625,011	1,117,114
	Best	1425.995	1428.967	50,133.52	1,916,665	1453.756	92,851.54	8,224,920	367,478.6	14,398,783	1555.358	688,564	4,094,799	87,027.4
	Worst	1431.939	1431.939	377,111.3	12,645,778	1466.815	551,194.5	49,294,337	2,825,440	95,053,148	1594.456	4,134,141	5,496,105	2,155,652
	Std	2.852761	1.42638	161,755.2	4,966,828	6.160042	206,827	18,862,139	1,215,944	37,337,317	19.52985	1,554,815	658,568.4	919,113.3
	Median	1429.95	1430.944	104,490.6	5,211,631	1458.503	373,948.1	21,601,363	776,098.8	39,168,184	1563.004	2,801,692	4,454,570	1,112,888
	Rank	1	2	5	11	3	6	12	8	13	4	9	10	7
C17-F15	Mean	1530.66	1541.806	6306.17	5.4E+08	1694.536	2.2E+08	2.53E+09	36,961.49	4.06E+09	2294.303	1.65E+09	9,625,454	5,771,669
	Best	1526.359	1534.656	4441.728	4.22E+08	1665.53	75,585,731	1.79E+09	22,862.95	3.17E+09	2154.886	5.68E+08	887,275.2	41,155.58
	Worst	1532.953	1547.387	10,421.45	6.4E+08	1724.282	4.79E+08	3.31E+09	67,970.93	4.81E+09	2448.657	3.6E+09	17,972,455	15,201,618
	Std	3.193106	7.068247	3020.312	1.05E+08	35.12765	2.04E+08	7.79E+08	22,757.73	7.91E+08	169.1411	1.53E+09	8,173,029	7,199,361
	Median	1531.664	1542.591	5180.753	5.49E+08	1694.166	1.63E+08	2.51E+09	28,506.04	4.13E+09	2286.833	1.22E+09	9,821,042	3,921,950
	Rank	1	2	5	10	3	9	12	6	13	4	11	8	7
C17-F16	Mean	2062.891	2430.47	2424.6	2841.58	2217.882	2461.717	6210.967	4339.096	7473.678	2785.133	4618.121	5443.461	3340.042
	Best	1728.6	2199.536	2110.157	2379.318	1927.173	2117.055	5416.686	4029.476	5615.348	2605.815	4081.328	4496.541	2928.585
	Worst	2242.663	2583.257	2586.19	3367.88	2330.345	2624.391	7929.194	4737.26	11,143.9	3066.803	4889.552	6082.335	3901.676
	Std	253.4793	181.3098	240.2846	451.1051	211.6042	253.4277	1291.414	360.1978	2745.716	226.1482	407.586	765.4826	506.6587
	Median	2140.15	2469.543	2501.026	2809.562	2307.006	2552.712	5748.994	4294.825	6567.732	2733.956	4750.802	5597.483	3264.954
	Rank	1	4	3	7	2	5	12	9	13	6	10	11	8
C17-F17	Mean	2021.151	2307.398	2270.896	3238.363	2141.849	2322.412	7495.543	3553.678	10,826.45	2583.586	3940.937	4500.193	2981.041
	Best	1900.43	2214.205	2113.38	2752.617	2032.735	2121.041	5727.661	3123.189	7928.552	2516.955	3180.781	4045.755	2823.708
	Worst	2138.267	2390.625	2430.248	3579.007	2244.675	2477.367	9176.459	4061.752	14,045.85	2634.116	4383.337	4713.962	3244.121
	Std	146.0805	94.63085	148.7026	380.7458	123.9587	177.0502	1546.272	490.5933	2742.804	53.38413	572.154	340.4218	199.0557
	Median	2022.954	2312.381	2269.979	3310.913	2144.993	2345.619	7539.026	3514.886	10,665.69	2591.637	4099.815	4620.528	2928.167
	Rank	1	4	3	8	2	5	12	9	13	6	10	11	7
C17-F18	Mean	1830.62	14,834.48	315,307.7	14,506,540	7313.769	4,530,757	73,493,269	2,342,674	1.09E+08	27,381.39	34,031,618	43,861,630	5,558,878
	Best	1822.239	2845.365	44,791.03	6,524,976	2253.646	408,674.2	58,811,415	305,555.5	49,019,271	3832.54	3,059,032	11,878,385	1,062,392
	Worst	1841.673	21,765.95	575,515.1	20,128,473	10,242.85	12,937,564	86,662,453	4,290,330	1.51E+08	40,990.1	97,224,842	79,395,296	11,087,130
	Std	8.863799	8994.812	291,836.7	7,275,826	3797.064	6,265,654	13,092,616	2,196,734	54,701,433	17,666.3	47,097,266	36,340,770	5,693,575
	Median	1829.285	17,363.3	320,462.3	15,686,357	8379.288	2,388,395	74,249,604	2,387,406	1.18E+08	32,351.46	17,921,299	42,086,419	5,042,995
	Rank	1	3	5	9	2	7	12	6	13	4	10	11	8
C17-F19	Mean	1925.185	2008.683	35,232.79	4.96E+08	1960.392	3.45E+08	2.64E+09	252,209	3.73E+09	2089.248	2.6E+09	6,642,240	1,129,103
	Best	1924.437	1976.451	13,465.94	3.34E+08	1947.343	1,264,528	1.26E+09	88,565.24	2.51E+09	2025.014	9,493,223	999,084.6	552,694
	Worst	1926.121	2024.478	70,862.98	6.13E+08	1966.62	1.01E+09	4.41E+09	520,086.9	4.61E+09	2121.004	7.58E+09	15,655,246	1,736,020
	Std	0.861219	23.90603	27,108.58	1.35E+08	9.63636	4.9E+08	1.45E+09	203,808.7	1.01E+09	47.718	3.69E+09	6,854,172	538,429.4
	Median	1925.091	2016.902	28,301.13	5.18E+08	1963.803	1.86E+08	2.45E+09	200,092	3.89E+09	2105.486	1.4E+09	4,957,314	1,113,849
	Rank	1	3	5	10	2	9	12	6	13	4	11	8	7
Continued														

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F20	Mean	2160.172	2426.094	2350.572	2457.966	2272.299	2372.457	3818.294	3271.072	4078.394	2682.672	3435.592	3742.097	2647.932
	Best	2104.423	2249.499	2307.361	2351.727	2165.86	2243.8	3472.19	2686.731	3786.257	2389.035	2974.936	3432.905	2432.775
	Worst	2323.891	2553.017	2406.868	2613.295	2415.205	2533.392	3991.693	3788.533	4238.511	2982.887	3637.853	4310.485	2845.809
	Std	118.7931	159.6473	50.4003	121.677	118.9274	130.8615	257.9001	508.6783	219.5092	271.1155	336.6562	427.477	239.5512
	Median	2106.186	2450.93	2344.029	2433.422	2254.066	2356.318	3904.646	3304.513	4144.404	2679.383	3564.789	3612.498	2656.572
	Rank	1	5	3	6	2	4	12	9	13	8	10	11	7
C17-F21	Mean	2314.895	2386.58	2384.842	2420.569	2345.122	2411.048	2985.352	2754.343	3022.914	2455.747	2951.339	2942.466	2525.838
	Best	2309.045	2374.749	2364.134	2402.04	2337.798	2394.824	2951.46	2634.377	2919.324	2433.424	2848.81	2829.596	2471.377
	Worst	2329.683	2397.029	2418.682	2438.489	2351.946	2428.142	3017.152	2935.103	3109.597	2481.921	3115.544	3037.254	2569.128
	Std	10.75977	10.81005	25.76744	17.97491	6.331155	16.25502	37.33181	140.6224	97.19509	26.6435	125.037	96.15197	45.57984
	Median	2310.426	2387.272	2378.277	2420.874	2345.371	2410.613	2986.398	2723.945	3031.366	2453.821	2920.502	2951.507	2531.423
	Rank	1	4	3	6	2	5	12	9	13	7	11	10	8
C17-F22	Mean	3095.169	4301.842	4353.954	5015.909	3603.969	4690.836	14,844.02	11,104	16,080.14	5466.112	13,636.45	13,565.09	8891.015
	Best	2300	2310.803	3352.594	4086.112	2304.555	3864.987	14,624.7	9072.719	15,713.78	2321.225	13,266.32	13,214.35	8150.214
	Worst	5480.678	5635.11	6104.487	6619.811	4815.22	6263.418	15,118.06	13,072.02	16,551.97	8853.027	14,051.51	13,956.63	9344.081
	Std	1730.769	1664.648	1334.929	1203.636	1118.054	1164.356	221.8845	1962.285	385.2036	3922.801	363.7488	369.0986	564.0953
	Median	2300	4630.727	3979.368	4678.858	3648.051	4317.47	14,816.67	11,135.63	16,027.4	5345.098	13,613.99	13,544.69	9034.883
	Rank	1	3	4	6	2	5	12	9	13	7	11	10	8
C17-F23	Mean	2743.354	2822.142	2828.983	2907.905	2776.576	2887.957	3810.928	3292.127	3885.408	2898.161	3735.453	3737.942	3025.767
	Best	2729.988	2815.735	2812.453	2902.971	2766.144	2850.117	3734.249	3208.488	3836.639	2883.383	3529.727	3559.038	2946.351
	Worst	2752.657	2835.525	2845.976	2915.833	2785.54	2938.101	3905.441	3373.231	3927.046	2918.916	4066.542	3835.795	3161.506
	Std	10.90099	9.836059	18.7029	6.096415	9.178608	41.74159	81.5168	84.20105	41.27055	16.4899	278.4858	135.2446	102.0861
	Median	2745.387	2818.654	2828.753	2906.408	2777.31	2881.805	3802.012	3293.395	3888.973	2895.173	3672.772	3778.467	2997.605
	Rank	1	3	4	7	2	5	12	9	13	6	10	11	8
C17-F24	Mean	2919.043	2998.106	3010.999	3138.401	2952.38	3075.414	4201.538	3515.045	4472.769	3074.39	3999.275	3826.852	3205.763
	Best	2909.046	2982.923	2995.071	3067.828	2947.07	3059.067	3951.712	3403.073	3994.009	3042.645	3906.996	3714.602	3111.032
	Worst	2924.412	3019.88	3041.078	3300.706	2963.092	3096.057	4766.892	3700.867	5652.579	3114.62	4135.221	3881.308	3335.355
	Std	7.426653	17.38225	22.5738	119.0299	8.206583	17.18515	413.4996	140.2912	864.449	35.43005	113.8106	83.32412	102.4439
	Median	2921.358	2994.81	3003.923	3092.535	2949.68	3073.266	4043.773	3478.119	4122.245	3070.147	3977.441	3855.75	3188.332
	Rank	1	3	4	7	2	6	12	9	13	5	11	10	8
C17-F25	Mean	2983.145	3028.885	3016.819	4160.354	3002.432	3386.076	8503.528	3181.172	11,777.51	3073.018	5956.998	4138.375	4020.254
	Best	2980.235	3016.48	3012.703	3852.722	2995.539	3238.822	7014.089	3155.181	9469.883	3051.416	4854.992	3737.456	3826.264
	Worst	2991.831	3037.569	3022.084	4347.142	3008.724	3523.827	9447.758	3225.324	13,196.26	3092.881	7007.127	4440.102	4221.149
	Std	6.301777	9.711209	4.748516	251.5774	6.05772	134.9921	1173.621	33.20242	1905.155	18.69219	1006.72	325.1956	224.362
	Median	2980.257	3030.746	3016.245	4220.776	3002.732	3390.827	8776.132	3172.092	12,221.94	3073.889	5982.936	4187.971	4016.802
	Rank	1	4	3	10	2	7	12	6	13	5	11	9	8

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F26	Mean	3776.432	3534.365	4674.871	5210.504	3674.363	4890.658	13,880.59	10,821.96	14,848.49	3300.805	12,444.1	13,615.56	6435.27
	Best	3748.807	3426.015	4589.765	5111.326	3629.797	4605.908	13,634.91	10,302.44	14,223.19	3086.043	10,348.61	12,722.68	6034.118
	Worst	3793.643	3698.566	4734.417	5327.503	3753.553	5097.837	14,089.8	11,314.65	15,773.08	3606.831	13,712.26	15,297.94	6804.599
	Std	21.16788	132.7729	69.67746	106.1743	59.70874	224.3401	220.1548	450.5373	728.6106	257.4706	1585.665	1247.093	423.9868
	Median	3781.639	3506.44	4687.651	5201.593	3657.051	4929.443	13,898.82	10,835.37	14,698.84	3255.173	12,857.77	13,220.81	6451.181
	Rank	4	2	5	7	3	6	12	9	13	1	10	11	8
C17-F27	Mean	3251.26	3267.38	3342.424	3491.207	3281.391	3454.721	4782.803	3850.474	4968.92	3391.669	4694.644	4449.75	3643.809
	Best	3227.701	3226.201	3310.144	3429.118	3239.343	3333.439	4470.203	3805.153	4599.936	3278.521	3985.868	3884.686	3599.679
	Worst	3313.631	3357.439	3393.576	3575.439	3332.103	3568.694	4992.065	3924.173	5228.946	3500.31	5178.239	5026.176	3702.527
	Std	45.39257	66.63495	39.84239	67.62137	42.35711	106.662	253.329	60.24189	324.8883	98.71482	567.1225	582.992	56.94769
	Median	3231.854	3242.94	3332.989	3480.135	3277.06	3458.375	4834.472	3836.286	5023.4	3393.922	4807.234	4444.069	3636.514
	Rank	1	2	4	7	3	6	12	9	13	5	11	10	8
C17-F28	Mean	3258.849	3309.289	3311.635	4302.859	3280.117	3790.18	8641.227	3594.89	11,046.25	3357.956	7192.27	4802.1	4390.731
	Best	3258.849	3289.437	3297.286	4131.732	3271.746	3606.765	7804.812	3510.94	9783.748	3318.95	5833.522	4200.081	4121.383
	Worst	3258.849	3333.666	3325.582	4750.089	3290.396	3978.033	10,765.69	3685.137	14,393.63	3405.855	8589.827	5034.901	4729.271
	Std	0	23.60971	16.09386	325.066	9.955164	201.7974	1553.677	94.20029	2433.087	46.38978	1518.346	438.1466	307.0653
	Median	3258.849	3307.026	3311.837	4164.808	3279.163	3787.961	7997.203	3591.742	10,003.81	3353.509	7172.866	4986.709	4356.135
	Rank	1	3	4	8	2	7	12	6	13	5	11	10	9
C17-F29	Mean	3263.038	3372.426	3636.557	5465.747	3452.348	3819.38	13,513.61	5529.962	19,280.59	4145.197	6904.299	9010.133	4913.192
	Best	3247.132	3262.959	3609.542	4292.42	3360.144	3777.299	8965.018	5399.1	10,261.58	3767.857	6476.157	6083.289	4730.925
	Worst	3278.787	3684.843	3647.438	6902.735	3503.384	3894.382	18,487.13	5642.667	30,398.7	4406.173	7417.179	11,763.79	5220.795
	Std	18.99818	226.7964	19.67192	1268.705	71.65467	57.35829	4767.813	108.9735	9758.886	313.7848	423.1056	2546.369	254.6502
	Median	3263.116	3270.951	3644.624	5333.916	3472.931	3802.919	13,301.15	5539.04	18,231.04	4203.38	6861.93	9096.728	4850.523
	Rank	1	2	4	8	3	5	12	9	13	6	10	11	7
C17-F30	Mean	623,575.2	639,606.3	3,468,185	7.11E+08	856,324.5	2.15E+08	3.18E+09	21,342,386	5.34E+09	1,708,157	1.61E+09	1.54E+08	1.36E+08
	Best	582,411.6	619,024.5	2,358,942	4.36E+08	737,866.2	26,911,264	2.46E+09	13,054,541	3.28E+09	1,282,383	1.98E+08	1.04E+08	65,642,799
	Worst	655,637.4	655,637.4	4,501,728	1.11E+09	1,083,484	4.35E+08	4.32E+09	29,183,659	8.38E+09	2,804,158	3.27E+09	2.13E+08	2.01E+08
	Std	35,550.35	17,775.18	1,187,995	3.18E+08	168,390.8	2.29E+08	8.85E+08	8,652,493	2.39E+09	798,717.4	1.72E+09	59,268,049	74,512,121
	Median	628,125.9	641,881.7	3,506,035	6.45E+08	801,973.7	1.99E+08	2.98E+09	21,565,672	4.85E+09	1,373,043	1.49E+09	1.5E+08	1.38E+08
	Rank	1	2	5	10	3	9	12	6	13	4	11	8	7
Sum rank		33	79	117	225	70	181	342	225	370	153	307	287	226
Mean rank		1.137931	2.724138	4.034483	7.758621	2.413793	6.241379	11.7931	7.758621	12.75862	5.275862	10.58621	9.896552	7.793103
Total rank		1	3	4	7	2	6	11	7	12	5	10	9	8

Table 4. Optimization results of CEC 2017 test suite (dimension = 50).

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F1	Mean	100	351.0861	3.64E + 08	2.02E + 10	78,245,465	1.09E + 10	1.62E + 11	3.75E + 09	2.26E + 11	5.44E + 08	1.22E + 11	6.09E + 10	5.55E + 10
	Best	100	346.0338	1.89E + 08	1.98E + 10	59,260,468	9.64E + 09	1.58E + 11	1.84E + 09	2.22E + 11	4.12E + 08	1.08E + 11	5.75E + 10	4.81E + 10
	Worst	100	356.1384	5.18E + 08	2.04E + 10	98,826,437	1.22E + 10	1.66E + 11	5.39E + 09	2.28E + 11	6.87E + 08	1.37E + 11	6.82E + 10	6.28E + 10
	Std	1.26E-14	6.346648	1.47E + 08	2.57E + 08	20,761,805	1.17E + 09	3.61E + 09	1.59E + 09	2.84E + 09	1.44E + 08	1.31E + 10	5.33E + 09	7.59E + 09
	Median	100	351.0861	3.75E + 08	2.03E + 10	77,447,477	1.09E + 10	1.62E + 11	3.89E + 09	2.27E + 11	5.38E + 08	1.22E + 11	5.9E + 10	5.55E + 10
	Rank	1	2	4	8	3	7	12	6	13	5	11	10	9
C17-F3	Mean	300	469.3157	38,582.69	38,251.11	24,007.89	41,792.1	427,300.8	328,597.9	324,877.7	165,072.2	364,606.8	777,399.7	369,098.6
	Best	300	467.33	36,054.43	36,461.39	18,437.12	33,178.02	387,632.5	322,286.8	316,318.6	126,354.8	291,744.1	679,089.5	337,369.8
	Worst	300	472.678	41,337.36	39,386.66	28,996.52	47,177.5	448,871.8	338,205	330,100.9	199,743.6	415,783.1	900,908.2	402,265.2
	Std	0	2.757906	2374.072	1423.624	4989.972	6563.477	30,834.37	7813.752	6718.086	34,680.8	57,089.19	103,974.9	38,387.77
	Median	300	468.6275	38,469.48	38,578.19	24,298.95	43,406.44	436,349.5	326,950	326,545.6	167,095.1	375,450	764,800.5	368,379.8
	Rank	1	2	5	4	3	6	12	9	8	7	10	13	11
C17-F4	Mean	602.1722	651.0861	711.13	7067.642	664.8263	1960.079	43,240.38	1557.141	72,875.69	1037.624	15,570.05	10,657.62	4381.014
	Best	592.0676	646.0338	695.2462	6473.441	639.3376	1468.582	39,802.25	1323.068	66,067.05	920.5989	10,200.25	9090.213	3380.446
	Worst	612.2769	656.1384	734.6212	7793.314	690.9789	2432.115	47,396.17	1703.024	81,186.5	1159.264	20,694.4	11,689.69	6568.461
	Std	12.6933	6.346648	18.19496	594.2757	28.72931	434.8693	3553.518	190.5223	6819.691	125.3281	4718.411	1203.701	1600.204
	Median	602.1722	651.0861	707.3263	7001.907	664.4943	1969.81	42,881.56	1601.235	72,124.6	1035.317	15,692.78	10,925.29	3787.575
	Rank	1	2	4	9	3	7	12	6	13	5	11	10	8
C17-F5	Mean	512.9345	556.4672	623.8912	678.8884	616.3228	694.5348	1962.646	1316.41	1933.464	1231.494	2109.013	1816.05	1191.072
	Best	510.9445	555.4723	615.811	672.2167	596.6042	688.1597	1950.652	1311.134	1896.535	1100.367	2081.887	1713.742	1142.995
	Worst	514.9244	557.4622	630.9675	685.2879	630.7342	702.6485	1980.063	1322.432	1972.947	1319.814	2127.599	1958.233	1234.456
	Std	1.976192	0.988096	6.970171	7.213227	16.95655	7.028909	14.08997	5.041924	37.01288	115.2647	24.13385	113.1366	41.7828
	Median	512.9345	556.4672	624.3932	679.0245	618.9765	693.6654	1959.935	1316.036	1932.187	1252.897	2113.284	1796.112	1193.419
	Rank	1	2	4	5	3	6	12	9	11	8	13	10	7
C17-F6	Mean	600	619.3157	607.5226	611.1272	605.4608	611.6577	703.1688	661.0867	701.5294	637.9528	707.4813	700.7985	640.6603
	Best	600	617.33	606.9412	610.4788	604.8994	610.3852	700.334	656.9595	696.5844	634.0511	695.5344	691.2534	636.4012
	Worst	600	622.678	607.9663	611.5488	606.4113	612.7789	706.0436	665.1952	704.3773	644.559	716.0021	716.9074	646.372
	Std	0	2.757906	0.561469	0.526955	0.779687	1.11214	2.681655	3.745924	3.718362	5.418902	10.53378	12.44468	4.822993
	Median	600	618.6275	607.5916	611.2407	605.2662	611.7334	703.1489	661.096	702.578	636.6005	709.1944	697.5166	639.9339
	Rank	1	6	3	4	2	5	12	9	11	7	13	10	8
C17-F7	Mean	811.392	855.696	1065.837	1120.314	961.5571	1095.83	3526.427	3025.046	3636.269	1855.054	3361.569	3499.109	2025.421
	Best	810.0205	855.0103	1050.15	1113.525	954.4338	1084.605	3443.917	2868.861	3550.136	1794.951	3197.587	3382.085	1841.101
	Worst	813.1726	856.5863	1076.511	1130.809	972.3757	1107.99	3619.086	3150.248	3715.992	1938.405	3517.813	3661.86	2158.569
	Std	1.589565	0.794782	13.2943	8.31067	8.81897	11.3537	78.14919	151.7011	76.61161	67.41705	157.7082	140.1255	145.2443
	Median	811.1874	855.5937	1068.343	1118.461	959.7094	1095.364	3521.352	3040.537	3639.473	1843.43	3365.438	3476.245	2051.007
	Rank	1	2	4	6	3	5	12	9	13	7	10	11	8

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F8	Mean	812.437	856.2185	928.2307	990.1627	902.5278	983.6496	2370.186	1726.958	2421.819	1438.577	2348.744	2273.267	1518.859
	Best	808.9546	854.4773	918.3788	981.1985	877.8001	976.5229	2326.263	1678.118	2402.548	1263.757	2292.204	2070.435	1422.92
	Worst	816.9143	858.4572	934.3019	995.7426	915.4648	989.9358	2427.702	1761.643	2432.573	1547.446	2431.964	2426.969	1651.997
	Std	3.697116	1.848558	7.518489	7.086819	19.26191	6.373157	51.32183	40.4049	14.50231	135.3321	71.80609	194.3419	117.0481
	Median	811.9395	855.9698	930.1211	991.8548	908.4231	984.0699	2363.39	1734.036	2426.079	1471.552	2335.404	2297.832	1500.26
	Rank	1	2	4	6	3	5	12	9	13	7	11	10	8
C17-F9	Mean	900	1201.699	4362.986	8602.506	4024.423	12,209.83	86,030.86	26,413.64	73,980.05	22,615.05	114,453.3	73,524.21	35,279.47
	Best	900	1152.54	4017.956	8291.176	3798.493	10,423.6	77,001.9	22,283.48	71,451.65	21,044.81	94,143.36	57,575.77	22,319.02
	Worst	900	1313.445	4684.454	8804.553	4126.692	14,724.04	99,171.73	29,583.79	75,987.95	23,325.83	142,403.1	92,284.05	47,438.86
	Std	1.01E–s13	81.64302	303.0827	247.5	165.1683	1967.173	10,400.92	3301.909	2152.314	1147.936	22,045.48	18,825.5	13,324.57
	Median	900	1170.406	4374.767	8657.147	4086.254	11,845.83	83,974.9	26,893.65	74,240.29	23,044.78	110,633.4	72,118.51	35,679.99
	Rank	1	2	4	5	3	6	12	8	11	7	13	10	9
C17-F10	Mean	11,023.04	12,493.16	11,614.63	12,933.65	11,438.65	12,745.17	29,430.82	15,885.97	30,685.08	13,911.62	28,570.46	27,585.7	15,149.97
	Best	9625.608	11,729.92	10,355.04	11,735.27	10,220.52	11,510.65	29,086.08	13,390.21	29,771.54	13,173.51	27,893.54	26,747.8	14,154.84
	Worst	11,858.81	13,317.06	12,152.18	13,735.09	12,271.07	13,444.84	29,729.19	17,981.11	31,150.04	14,724.07	29,394.95	28,930.53	15,642.59
	Std	1054.018	736.5392	918.2688	929.7812	947.3872	923.2409	291.1719	2178.714	687.6687	697.6376	712.1526	1087.37	744.8405
	Median	11,303.87	12,462.83	11,975.64	13,132.11	11,631.52	13,012.6	29,454	16,086.27	30,909.37	13,874.46	28,496.68	27,332.24	15,401.23
	Rank	1	4	3	6	2	5	12	9	13	7	11	10	8
C17-F11	Mean	1162.329	1191.495	6726.109	18,792.84	1697.682	6831.688	156,791.9	61,300.84	196,686.6	4883.085	62,485.41	198,515	83,236.51
	Best	1139.568	1180.115	6179.49	14,685.55	1527.969	3785.401	121,782.4	55,137.79	150,573.8	3838.995	28,625.66	115,569.8	69,218.3
	Worst	1220.662	1220.662	7702.809	26,146.5	1885.45	9326.822	182,359.3	73,119.23	280,053.1	5841.006	89,331.42	319,876.5	93,681.48
	Std	42.46663	21.23332	743.4373	5594.678	164.1205	2492.432	28,335.65	9042.123	63,453.11	937.9812	27,372.42	103,382.3	11,405.92
	Median	1144.542	1182.602	6511.068	17,169.64	1688.654	7107.264	161,513.1	58,473.17	178,059.8	4926.169	65,992.28	179,306.8	85,023.13
	Rank	1	2	5	7	3	6	11	8	12	4	9	13	10
C17-F12	Mean	5974.805	6272.502	72,203,454	1.46E+10	38,479,586	4.84E+09	1E+11	6.46E+08	1.63E+11	2.67E+08	5.41E+10	1.26E+10	1.09E+10
	Best	5383.905	5977.052	38,755,502	1.09E+10	21,467,896	2.49E+09	7.13E+10	3.43E+08	1.22E+11	1.49E+08	2.78E+10	1.02E+10	7.56E+09
	Worst	6570.199	6570.199	1.09E+08	1.69E+10	46,126,066	8.01E+09	1.12E+11	1.02E+09	1.9E+11	3.21E+08	8.97E+10	1.44E+10	1.3E+10
	Std	537.9317	268.9658	32,094,074	3E+09	12,435,330	2.51E+09	2.11E+10	3.18E+08	3.37E+10	86,427,414	2.82E+10	1.9E+09	2.54E+09
	Median	5972.559	6271.379	70,628,527	1.52E+10	43,162,191	4.42E+09	1.09E+11	6.08E+08	1.71E+11	3E+08	4.94E+10	1.29E+10	1.15E+10
	Rank	1	2	4	10	3	7	12	6	13	5	11	9	8
C17-F13	Mean	1407.28	1448.673	15,619.83	3.62E+09	15,505.35	1.81E+09	2.65E+10	100,674.7	4.06E+10	99,390.26	2.03E+10	4.98E+08	9.02E+08
	Best	1371.145	1430.606	9671.949	2.8E+09	7319.591	1.29E+09	2.31E+10	69,106.32	3.14E+10	42,537.66	1.44E+10	3.54E+08	77,763,101
	Worst	1439.935	1465	23,692.42	4.1E+09	36,715.84	2.17E+09	2.94E+10	132,189.4	4.6E+10	246,943.4	2.43E+10	6.73E+08	2.38E+09
	Std	37.80433	18.90216	6618.923	6.55E+08	15,475.72	4.06E+08	3.59E+09	28,017.23	7.35E+09	107,647.9	4.56E+09	1.79E+08	1.16E+09
	Median	1409.02	1449.543	14,557.47	3.79E+09	8992.981	1.9E+09	2.68E+10	100,701.5	4.25E+10	54,040	2.13E+10	4.82E+08	5.73E+08
	Rank	1	2	4	10	3	9	12	6	13	5	11	7	8
Continued														

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F14	Mean	1467.509	1469.788	574,173.4	6,777,962	14,690.74	763,036.3	43,310,991	6,370,632	75,975,662	93,370.28	8,489,629	13,881,357	9,178,449
	Best	1458.803	1464.276	357,363.4	6,179,602	5087.83	347,523.9	37,400,460	3,873,473	69,290,959	26,615.77	3,856,435	7,990,253	5,808,607
	Worst	1472.733	1474.385	949,069.8	7,419,590	29,814.15	1,482,654	49,475,783	10,571,739	83,169,437	198,448.4	16,558,429	18,967,904	13,751,690
	Std	6.576739	4.611801	285,362.8	644,999.7	12,045.56	543,952.8	5,767,938	3,212,131	7,249,018	83,710.3	6,088,234	4,910,582	3,786,075
	Median	1469.25	1470.246	495,130.2	6,756,328	11,930.49	610,983.6	43,183,860	5,518,658	75,721,126	74,208.46	6,771,827	14,283,635	8,576,749
	Rank	1	2	5	8	3	6	12	7	13	4	9	11	10
C17-F15	Mean	1609.893	1638.423	12,062.95	2E+09	9653.356	1.03E+09	1.47E+10	84,547.79	2.24E+10	57,512.76	1.15E+10	66,853,509	4.78E+08
	Best	1551.154	1616.383	10,945.56	1.43E+09	3795.138	21,290,538	1.36E+10	71,754.84	1.6E+10	16,545.27	2.39E+08	37,236,985	31,364,902
	Worst	1652.294	1666.953	13,280.01	2.49E+09	13,969.09	1.92E+09	1.65E+10	102,122.2	2.8E+10	87,367.8	2.15E+10	1.28E+08	1.43E+09
	Std	48.04352	23.81868	1163.663	5.74E+08	4663.037	8.97E+08	1.38E+09	16,478.28	6.44E+09	32,527.42	1.01E+10	45,306,903	7.06E+08
	Median	1618.063	1635.179	12,013.11	2.04E+09	10,424.6	1.08E+09	1.43E+10	82,157.08	2.29E+10	63,068.98	1.21E+10	50,863,904	2.24E+08
	Rank	1	2	4	10	3	9	12	6	13	5	11	7	8
C17-F16	Mean	2711.795	3080.184	3265.654	4584.441	3128.8	3897.385	18,406.61	7145.499	21,941.97	5610.023	14,233.37	15,828.06	6161.007
	Best	2171.69	2784.508	2839.84	4354.137	2665.312	3220.067	17,095.26	5959.997	17,246.08	5508.525	11,826.64	12,943.32	5609.789
	Worst	3397.326	3499.82	3744.344	4750.255	3709.011	4724.039	19,006.46	7872.512	24,550.63	5765.584	16,951.94	17,519.97	6858.166
	Std	554.7769	328.2854	403.3499	214.2929	470.3519	676.7922	960.8126	911.955	3595.736	120.3522	2290.018	2212.928	686.8413
	Median	2639.081	3018.203	3239.217	4616.686	3070.438	3822.717	18,762.36	7374.743	22,985.59	5582.991	14,077.45	16,424.47	6088.036
	Rank	1	2	4	6	3	5	12	9	13	7	10	11	8
C17-F17	Mean	2716.564	3269.055	3103.946	708,934.1	3002.147	21,216.13	4,028,568	5843.555	7,925,089	4701.396	209,058	16,529.33	5519.63
	Best	2275.021	2852.074	2698.345	193,671.4	2595.455	3490.096	1,180,841	5656.47	2,148,329	4502.067	9987.966	10,286.6	4477.483
	Worst	3429.127	3707.353	3712.396	1,628,502	3627.237	51,664.28	9,165,296	6275.762	18,235,508	4953.425	555,042.6	27,763.26	7041.103
	Std	559.6669	409.2269	488.3048	734,261.6	488.5605	22,850.57	4,094,711	317.2476	8,233,806	234.1216	259,094.4	8559.722	1229.754
	Median	2581.054	3258.397	3002.522	506,781.6	2892.949	14,855.08	2,884,067	5720.994	5,658,260	4675.047	135,600.7	14,033.72	5279.966
	Rank	1	4	3	11	2	9	12	7	13	5	10	8	6
C17-F18	Mean	1903.746	1911.833	255,922.3	8,785,988	35,992.38	1,284,367	55,774,794	2,706,384	98,411,684	238,823.2	14,245,291	11,476,821	10,481,403
	Best	1881.15	1900.536	131,150.9	3,417,092	25,575.29	486,796.7	25,273,897	1,349,424	38,204,364	166,481.1	5,341,276	8,537,374	3,326,346
	Worst	1919.921	1919.921	406,934.7	16,063,643	63,579.96	2,619,382	1.01E+08	4,282,821	1.8E+08	430,463.3	29,105,956	13,587,540	16,921,842
	Std	21.08244	10.54122	134,726.5	5,800,126	20,071.07	1,045,547	35,129,013	1,444,338	65,000,813	139,418.9	11,649,071	2,501,835	6,085,190
	Median	1906.955	1913.438	242,801.7	7,831,608	27,407.14	1,015,645	48,480,952	2,596,645	87,747,510	179,174.1	11,266,967	11,891,185	10,838,712
	Rank	1	2	5	8	3	6	12	7	13	4	11	10	9
C17-F19	Mean	1972.839	1975.354	264,433	1.9E+09	43,173.45	4.3E+08	1.22E+10	2,770,801	2.14E+10	288,321.2	4.82E+09	1.28E+08	3.44E+08
	Best	1967.139	1972.504	125,417.1	1.39E+09	10,410.2	1.9E+08	1.07E+10	1,088,144	1.56E+10	60,603.23	2.14E+09	50,828,601	2,758,147
	Worst	1977.869	1977.869	473,452.7	2.37E+09	71,966.78	8.54E+08	1.43E+10	5,086,604	2.66E+10	488,443.8	9.58E+09	2.16E+08	1.04E+09
	Std	4.935585	2.467793	161,865.5	4.4E+08	27,794.91	3.19E+08	1.76E+09	1,840,286	4.94E+09	193,189.2	3.58E+09	83,134,983	5.25E+08
	Median	1973.174	1975.522	229,431.2	1.93E+09	45,158.41	3.38E+08	1.18E+10	2,454,228	2.16E+10	302,118.8	3.79E+09	1.23E+08	1.7E+08
	Rank	1	2	4	10	3	9	12	6	13	5	11	7	8
Continued														

		BOA	CMA-ES	EBOwithCMAR	SPS_L_ SHADE_EIG	LSHADE_ cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F20	Mean	3192.04	3643.508	3530.064	3646.613	3386.316	3602.895	7219.906	6155.102	7462.748	4542.275	6972.239	6984.301	6057.491
	Best	2806.762	3234.441	3207.448	3326.024	3070.22	3238.171	7054.703	5768.121	7318.98	4496.939	6391.091	6520.245	4834.003
	Worst	3662.121	4101.245	3897.068	4051.989	3786.265	4067.972	7362.879	6465.885	7517.838	4637.818	7685.619	7328.663	6921.05
	Std	477.9749	509.2964	392.5952	403.0636	403.8726	456.9505	138.4994	339.6087	104.4166	70.40518	600.8826	390.8434	1122.176
	Median	3149.639	3619.173	3507.871	3604.22	3344.389	3552.718	7231.02	6193.2	7507.086	4517.171	6906.123	7044.147	6237.455
	Rank	1	5	3	6	2	4	12	9	13	7	10	11	8
C17-F21	Mean	2342.155	2344.085	2485.784	2549.034	2414.857	2524.447	4234.604	3643.219	4352.875	2847.438	4077.012	4175.314	2983.843
	Best	2338.689	2342.352	2468.101	2545.452	2409.606	2510.01	4185.98	3442.4	4278.28	2800.93	3939.451	3882.818	2906.078
	Worst	2346.015	2346.015	2500.635	2552.497	2418.316	2535.851	4305.384	3775.438	4404.135	2883.83	4173.547	4401.45	3035.272
	Std	3.664912	1.832456	14.68361	3.1551	4.329621	13.32008	60.7186	156.372	60.07205	38.28203	125.2249	251.8069	59.87561
	Median	2341.959	2343.987	2487.199	2549.094	2415.753	2525.963	4223.526	3677.518	4364.542	2852.497	4097.524	4208.495	2997.011
	Rank	1	2	4	6	3	5	12	9	13	7	10	11	8
C17-F22	Mean	11,739	12,170.3	12,904.13	14,025.07	12,771.32	13,797.53	31,390.76	20,403.84	32,980.49	18,913.72	30,427.53	28,861.27	23,346.87
	Best	11,119.08	11,860.34	12,550.2	13,480.97	12,270.15	13,181.86	30,647.75	19,025.14	32,475.92	17,445.71	29,348.3	27,455.66	18,580.65
	Worst	12,601.6	12,601.6	13,582.4	14,638.2	13,298.58	14,536.46	31,801.72	22,261.23	33,530.74	20,683.73	31,334.39	29,870.22	34,134.14
	Std	710.0872	355.0436	512.4113	535.4523	458.8858	620.0113	555.515	1525.472	520.5315	1484.7	889.2408	1157.959	7978.063
	Median	11,617.67	12,109.63	12,741.97	13,990.56	12,758.28	13,735.9	31,556.78	20,164.48	32,957.66	18,762.72	30,513.72	29,059.6	20,336.34
	Rank	1	2	4	6	3	5	12	8	13	7	11	10	9
C17-F23	Mean	2877.697	2875.589	3010.146	3115.324	2940.315	3125.509	5274.368	4096.383	5276.454	3312.901	5390.732	5094.604	3622.467
	Best	2872.107	2870.985	3000.431	3086.934	2934.469	3057.028	5029.962	4017.613	5016.156	3295.872	4652.619	4957.67	3590.872
	Worst	2884.013	2879.463	3021.132	3130.818	2946.629	3218.246	5562.974	4181.407	5480.526	3345.498	6392.98	5234.637	3666.726
	Std	5.674312	3.986124	9.244106	21.10773	6.106137	79.64641	259.1477	84.1507	209.2536	24.11426	846.953	144.1403	37.07665
	Median	2877.334	2875.954	3009.51	3121.771	2940.082	3113.381	5252.268	4093.256	5304.568	3305.118	5258.665	5093.055	3616.135
	Rank	2	1	4	5	3	6	11	9	12	7	13	10	8
C17-F24	Mean	3327.407	3438.574	3528.773	3971.524	3385.756	3640.892	8393.821	5337.554	10,305.12	3732.942	6595.5	6313.067	4293.436
	Best	3295.518	3326.754	3507.888	3641.6	3352.647	3568.718	6566.955	5118.13	6934.558	3679.971	6116.84	5905.299	4055.058
	Worst	3357.991	3577.621	3543.623	4184.713	3411.828	3671.245	9637.757	5518.404	12,551.07	3800.638	6915.197	6941.397	4502.341
	Std	32.22323	141.8265	16.3297	276.5615	29.81515	52.85822	1599.567	192.1685	2958.584	61.78451	369.7674	492.3058	250.2186
	Median	3328.059	3424.96	3531.79	4029.892	3389.275	3661.802	8685.287	5356.841	10,867.42	3725.58	6674.981	6202.786	4308.173
	Rank	1	3	4	7	2	5	12	9	13	6	11	10	8
C17-F25	Mean	3185.232	3451.441	3299.668	4782.963	3260.492	3846.463	14,922.29	4147.848	20,790.06	3708.296	10,282.76	7212.153	6367.811
	Best	3137.371	3335.503	3266.295	4618.836	3193.385	3737.93	14,198.28	3779.429	19,286.75	3526.674	9636.341	6613.177	6216.329
	Worst	3261.571	3552.557	3368.718	5100.777	3343.836	3955.388	16,634.41	4484.541	24,153.12	3833.318	10,694.79	7576.751	6751.439
	Std	65.17161	97.99757	50.78221	235.1846	72.26648	97.04669	1253.78	317.0745	2491.988	140.7546	524.2721	475.1564	279.8204
	Median	3170.992	3458.852	3281.83	4706.119	3252.373	3846.268	14,428.23	4163.711	19,860.18	3736.596	10,399.95	7329.341	6251.739
	Rank	1	4	3	8	2	6	12	7	13	5	11	10	9

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C17-F26	Mean	5757.621	5764.504	7761.864	9556.954	6640.165	8509.996	38,865.2	24,476.58	44,617.06	11,891.39	32,870.45	33,477.66	16,957.71
	Best	5645.905	5698.132	7539.821	9208.285	6467.739	8450.885	38,298.81	21,742.13	42,106	11,127.29	31,702.11	30,013.03	15,159.37
	Worst	5844.642	5817.111	8041.965	9720.971	6780.955	8563.205	39,389.69	27,252.78	46,177.38	12,679.73	33,608.15	36,412.16	18,447.33
	Std	91.29453	58.35293	276.581	262.6612	147.7125	59.63827	486.9257	2579.893	2100.645	830.7982	897.1648	3403.273	1521.967
	Median	5769.969	5771.387	7732.834	9649.28	6655.983	8512.948	38,886.14	24,455.7	45,092.44	11,879.26	33,085.76	33,742.72	17,112.07
	Rank	1	2	4	6	3	5	12	9	13	7	10	11	8
C17-F27	Mean	3309.493	3429.643	3397.901	4100.74	3343.461	3609.224	9168.769	4156.371	12,042.06	3545.571	6527.371	5947.63	4077.874
	Best	3278.01	3418.349	3366.464	3852.6	3317.685	3589.05	7725.039	3990.782	9032.411	3501.478	6227.312	5265.963	3914.645
	Worst	3344.5	3442.534	3429.572	4385.755	3367.087	3622.274	10,617.89	4435.174	15,163.31	3582.021	6895.571	6703.2	4203.083
	Std	30.85754	11.48963	34.43176	309.159	22.55758	15.61134	1709.117	209.7897	3594.154	36.2999	313.7552	847.4585	155.4061
	Median	3307.732	3428.845	3397.783	4082.303	3344.536	3612.786	9166.075	4099.764	11,986.27	3549.393	6493.302	5910.677	4096.884
	Rank	1	4	3	8	2	6	12	9	13	5	11	10	7
C17-F28	Mean	3322.242	3561.799	3470.999	5507.887	3389.967	4422.82	20,433.68	4702.095	27,555.49	3792.938	15,381.29	10,207.74	9135.32
	Best	3318.742	3493.756	3451.04	5241.83	3368.22	4136.396	19,038.15	4408.373	24,684.07	3662.621	12,096.79	8725.156	7757.843
	Worst	3327.816	3609.887	3485.301	5833.541	3404.585	4651.482	23,030.74	4910.59	31,139.47	3886.048	17,877.05	11,173.94	11,111.71
	Std	4.767125	53.36449	16.40456	268.4895	17.34123	265.5143	1979.318	233.1433	2943.764	102.1826	3011.706	1135.099	1540.364
	Median	3321.205	3571.776	3473.827	5478.088	3393.532	4451.7	19,832.91	4744.708	27,199.21	3811.542	15,775.67	10,465.92	8835.861
	Rank	1	4	3	8	2	6	12	7	13	5	11	10	9
C17-F29	Mean	4450.696	4348.425	5052.217	34,399.2	4815.53	5814.836	178,184.5	9641.904	338,908	6986.325	18,198.3	16,354.87	8364.218
	Best	4169.151	4168.852	4827.658	20,305	4453.284	5423.067	101,634.1	8404.264	181,988.8	6143.902	13,999.2	13,644.05	8153.022
	Worst	4829.521	4653.939	5393.981	46,442.15	5147.692	6133.701	243,009.9	10,344.29	470,354.8	7797.362	22,990.22	18,712.87	8672.762
	Std	307.1569	231.9926	298.1015	12,045.47	328.4298	327.4102	65,483.34	929.6113	133,683.9	735.423	4085.493	2686.956	256.3964
	Median	4402.056	4285.454	4993.615	35,424.82	4830.571	5851.288	184,047	9909.531	351,644.2	7002.018	17,901.9	16,531.28	8315.544
	Rank	2	1	4	11	3	5	12	8	13	6	10	9	7
C17-F30	Mean	5407.166	5370.764	2,694,393	3.25E+09	709,024.5	1.15E+09	2.24E+10	27,170,981	3.65E+10	4,895,617	1.3E+10	1.45E+09	1.78E+09
	Best	5337.48	5334.985	1,561,539	3.04E+09	318,617.1	7.03E+08	1.97E+10	15,511,799	3.41E+10	2,181,355	7.89E+09	1.19E+09	7.3E+08
	Worst	5557.155	5446.07	4,341,297	3.52E+09	1,154,819	1.43E+09	2.44E+10	47,314,864	3.94E+10	7,994,222	1.6E+10	1.97E+09	2.32E+09
	Std	110.0477	55.33871	1,318,189	2.24E+08	420,441	3.47E+08	2.15E+09	15,437,007	2.51E+09	2,922,492	3.89E+09	3.81E+08	7.79E+08
	Median	5367.014	5351	2,437,369	3.23E+09	681,330.7	1.24E+09	2.28E+10	22,928,631	3.62E+10	4,703,446	1.39E+10	1.33E+09	2.03E+09
	Rank	2	1	4	10	3	7	12	6	13	5	11	8	9
Sum rank		32	73	113	214	79	178	346	226	364	171	315	287	241
Mean rank		1.103448	2.517241	3.896552	7.37931	2.724138	6.137931	11.93103	7.793103	12.55172	5.896552	10.86207	9.896552	8.310345
Total rank		1	2	4	7	3	6	12	8	13	5	11	10	9

Table 5. Optimization results of CEC 2017 test suite (dimension = 100).

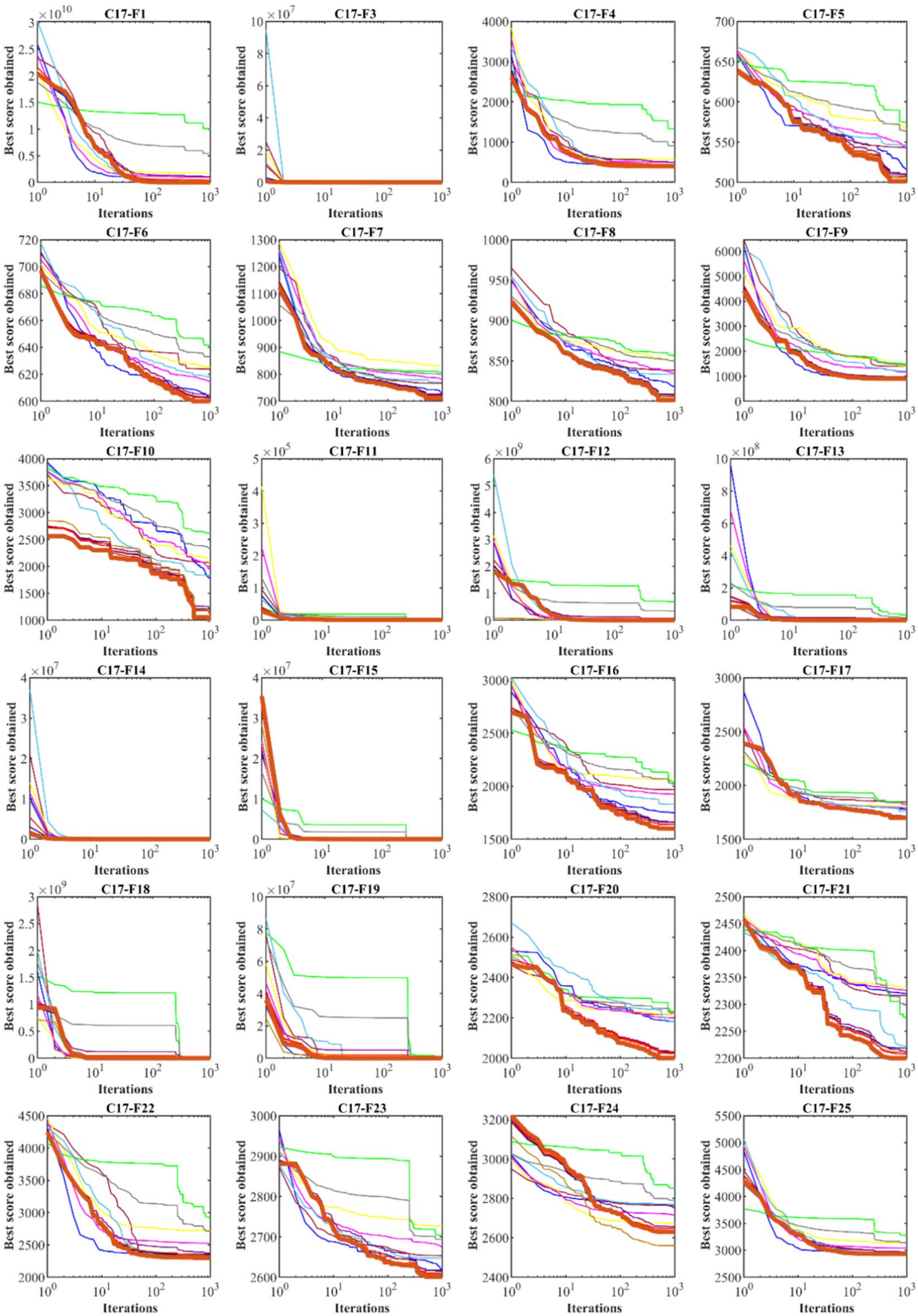

Figure 5. Convergence curves of algorithms performances on CEC 2017 test suite (dimension = 10).

The results of applying the Wilcoxon rank sum test on the performance of BOA in competition with each of the corresponding compared algorithms are reported in Table 7. Based on the obtained results, in cases where p-value is less than 0.05, BOA has a statistical superiority in competition with the corresponding algorithm. The findings obtained from the statistical analysis are that BOA has a significant statistical superiority compared to all twelve competitor algorithms in order to handle the CEC 2017 test suite for problem dimensions equal to 10, 30, 50, and 100, as well as CEC 2020 test suite.

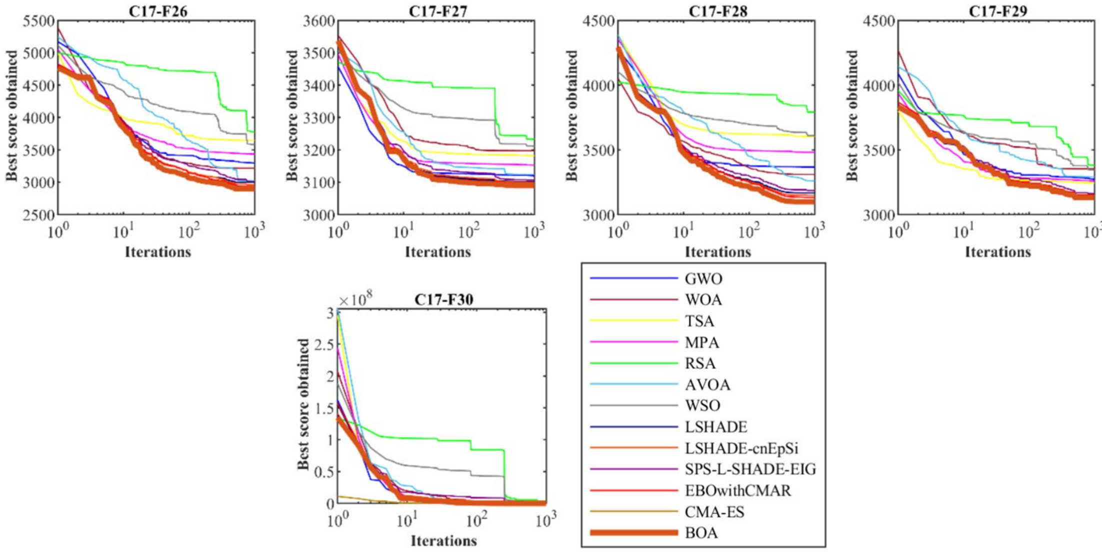

Figure 5. (continued)

In order to further analyze the results and performance of the optimization algorithms, another test, called the Friedman rank test[100], is used. The results of this test are presented in Table 8. Based on the results of the Friedman test, the proposed BOA ranks first in optimizing CEC 2017 test suite for all dimensions of 10, 30, 50, and 100, as well as CEC 2020 test suite compared to all twelve competitor algorithms.

At this stage, using the *t*-test[101], the superiority of BOA compared to competitor algorithms is checked. In a *t*-test, the two algorithms are taken into account simultaneously in order to compute the *t*-values for each function. The results of *t*-test implementation on the performance of metaheuristic algorithms to handle CEC 2017 test suite for different dimensions are reported in Table 9. Based on the obtained results, the proposed BOA approach has a significant superiority in the competition with all twelve corresponding algorithms to handle the CEC 2017 test suite in the dimensions of 10, 30, 50, and 100, as well as CEC 2020 test suite.

BOA for real-world applications

In this section, the performance of BOA and competitor algorithms for handsling optimization tasks in real-world applications is analyzed. In this regard, twenty-two constrained optimization problems from CEC 2011 test suite and four engineering design problems are selected from real-world applications.

CEC 2011 test suite

In this subsection, the performance of BOA and competitor algorithms in handling the CEC 2011 test suite is tested. CEC 2011 test suite consists of twenty-two constrained optimization prsoblems from real-world applications, the full description and details of which are available in[102].

Table 10 reports the implementation results of BOA and competitor algorithms on CEC 2011 test suite. Figure 10 shows the convergence curves obtained from the performance of metaheuristic algorithms in this implementation. The findings from the optsimization results are that BOA with high quality in balancing exploration and exploitation has provided good results for all twenty-two optimization problems C11-F1 to C11-F22. The findings obtained from the simulation results are that BOA has provided superior performance in comparison with competitor algorithms in order to handle CEC 2011 test suite by providing better results and obtaining the rank of the first best optimizer in C11-F1, C11-F2, C11-F4 to C11-F12, C11-F14 to C11-F22 (20 problems from 22 problems) problems. In addition, the results of the statistical analysis obtained from the Wilcoxon rank sum test reported in Table 8 confirm that BOA has significant statistical superiority in comparison with all twelve competitor algorithms for solving the CEC 2011 test suite.

Pressure vessel design problem

Pressure vessel design is presented as a challenge in real world applications with the schematic shown in Fig. 11. Minimizing construction cost is the main goal in this design. The mathematical model of this design is as follows[103]:

Consider: $X = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L]$.

Minimize: $f(x) = 0.6224x_1x_3x_4 + 1.778x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$.

Subject to:

$$g_1(x) = -x_1 + 0.0193x_3 \leq 0, \quad g_2(x) = -x_2 + 0.00954x_3 \leq 0,$$

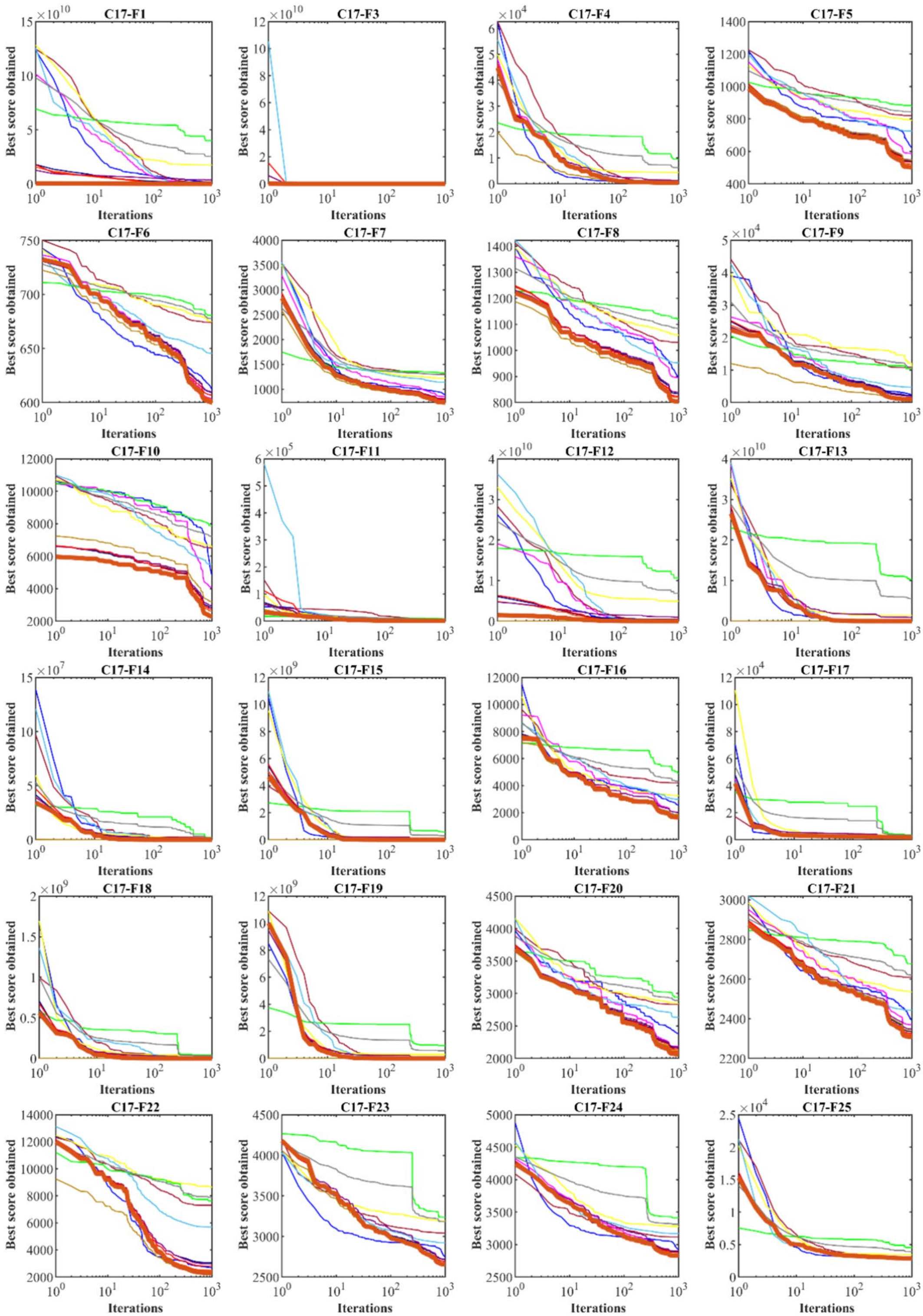

Figure 6. Convergence curves of algorithms performances on CEC 2017 test suite (dimension = 30).

$$g_3(x) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0, \quad g_4(x) = x_4 - 240 \leq 0.$$

With

$$0 \leq x_1, x_2 \leq 100 \text{ and } 10 \leq x_3, x_4 \leq 200.$$

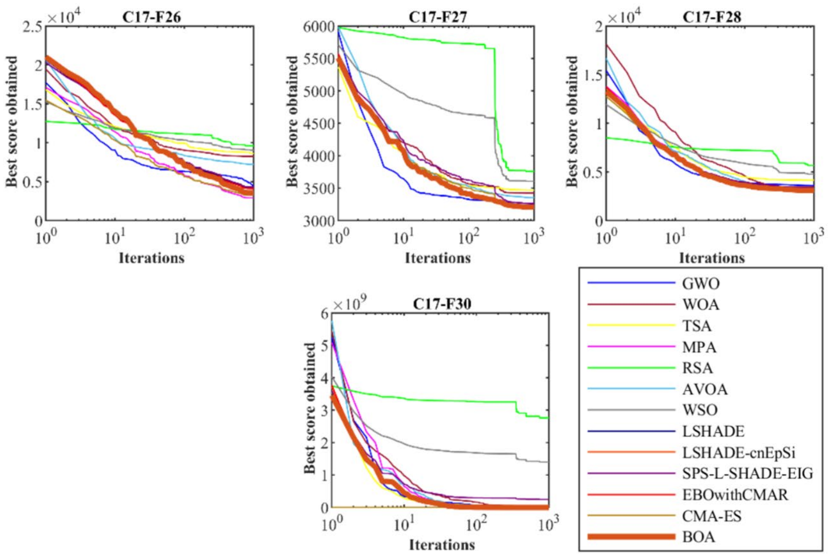

Figure 6. (continued)

Tables 11, 12 report the implementation results of BOA and metaheuristic algorithms on the pressure vessel design problem. Figure 12 shows the convergence curve of BOA while achieving the solution for pressure vessel design. Based on the obtained results, BOA has provided the optimal design for the pressure vessel with the values of the design variables equal to (0.7780271, 0.3845792, 40.312284, 200) and the value of the objective function equal to (5882.8955). The finding of the simulation results is that BOA has provided superior performance in comparison with competitor algorithms, by providing better results compared to competitor algorithms for handling pressure vessel design.

Speed reducer design problem

Speed reducer design is presented as a challenge in real world applications with the schematic shown in Fig. 13, where minimizing the weight of the speed reducer is the main goal in this design. The mathematical model of this design is as follows[104,105]:

Consider: $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7] = [b, m, p, l_1, l_2, d_1, d_2]$.

Minimize: $f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$.

Subject to:

$$g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \leq 0, \quad g_2(x) = \frac{397.5}{x_1x_2^2x_3} - 1 \leq 0,$$

$$g_3(x) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leq 0, \quad g_4(x) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leq 0,$$

$$g_5(x) = \frac{1}{110x_6^3}\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9 \times 10^6} - 1 \leq 0,$$

$$g_6(x) = \frac{1}{85x_7^3}\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5 \times 10^6} - 1 \leq 0,$$

$$g_7(x) = \frac{x_2x_3}{40} - 1 \leq 0, \quad g_8(x) = \frac{5x_2}{x_1} - 1 \leq 0,$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \leq 0, \quad g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0,$$

$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0.$$

With

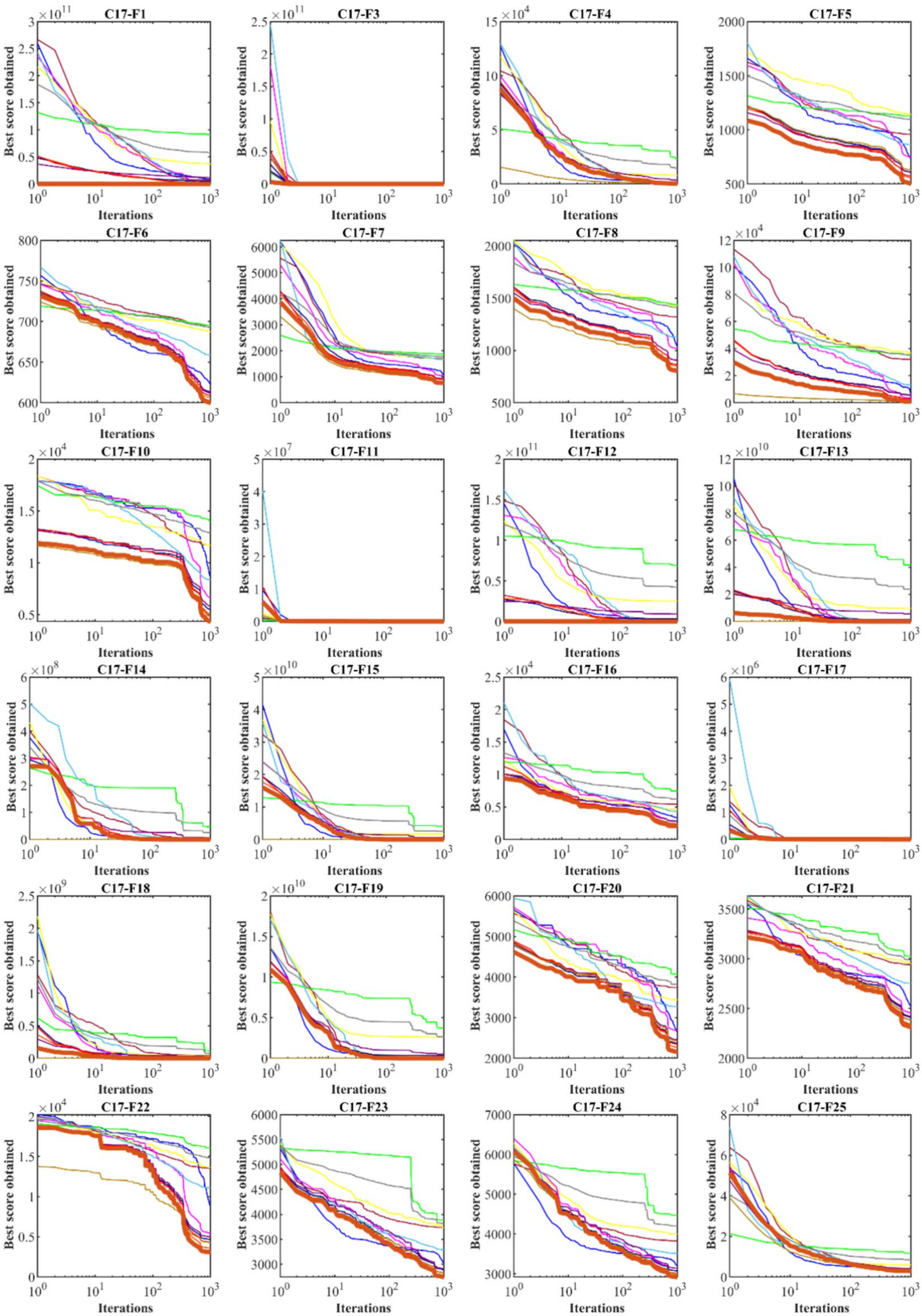

Figure 7. Convergence curves of algorithms performances on CEC 2017 test suite (dimension = 50).

$$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 7.3 \leq x_4 \leq 8.3, 7.8 \leq x_5 \leq 8.3, 2.9 \leq x_6 \leq 3.9, \text{ and } 5 \leq x_7 \leq 5.5.$$

Tables 13, 14 present the results of employing BOA and metaheuristic algorithms to deal with the speed reducer design problem. Figure 14 shows the convergence curve of BOA while achieving the solution for speed reducer design. Based on the obtained results, BOA has provided the optimal design for speed reducer with the values of the design variables equal to (3.5, 0.7, 17, 7.3, 7.8, 3.3502147, 5.2866832) and the value of the objective

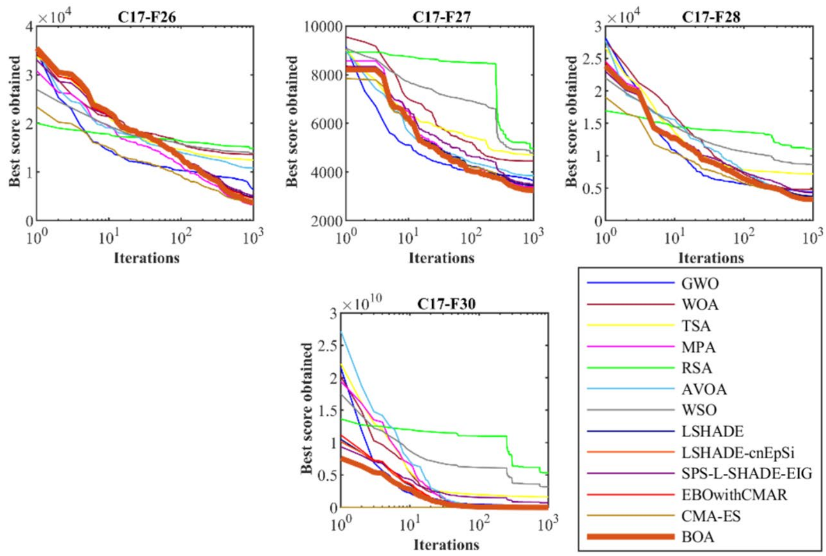

Figure 7. (continued)

function equal to (2996.3482). The finding of the simulation results is that BOA has provided superior performance by achieving better results compared to competitor algorithms for addressing speed reducer design.

Welded beam design problem

Welded beam design is presented as a challenge in real world applications with the schematic shown in Fig. 15, where minimizing the fabrication cost of the welded beam is the main goal in this design. The mathematical model of this design is as follows[95]:

Consider: $X = [x_1, x_2, x_3, x_4] = [h, l, t, b]$.

Minimize: $f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$.

Subject to:

$$g_1(x) = \tau(x) - 13600 \le 0, \quad g_2(x) = \sigma(x) - 30000 \le 0,$$

$$g_3(x) = x_1 - x_4 \le 0, g_4(x) = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5.0 \le 0,$$

$$g_5(x) = 0.125 - x_1 \le 0, \quad g_6(x) = \delta(x) - 0.25 \le 0,$$

$$g_7(x) = 6000 - p_c(x) \le 0.$$

where

$$\tau(x) = \sqrt{(\tau\prime)^2 + (2\tau\tau\prime)\frac{x_2}{2R} + (\tau")^2}, \tau' = \frac{6000}{\sqrt{2}x_1x_2}, \tau" = \frac{MR}{J},$$

$$M = 6000\left(14 + \frac{x_2}{2}\right), R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},$$

$$J = 2\sqrt{2}x_1x_2\left(\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right), \quad \sigma(x) = \frac{504000}{x_4x_3^2},$$

$$\delta(x) = \frac{65856000}{(30 \cdot 10^6)x_4x_3^3}, p_c(x) = \frac{4.013(30 \cdot 10^6)x_3x_4^3}{1176}\left(1 - \frac{x_3}{28}\sqrt{\frac{30 \cdot 10^6}{4(12 \cdot 10^6)}}\right).$$

With

$$0.1 \le x_1, x_4 \le 2 \text{ and } 0.1 \le x_2, x_3 \le 10.$$

Tables 15, 16 have published the results of using BOA and metaheuristic algorithms to address the welded beam design problem. Figure 16 shows the convergence curve of BOA while achieving the solution for welded beam design. Based on the obtained results, BOA has provided the optimal design for welded beam with the

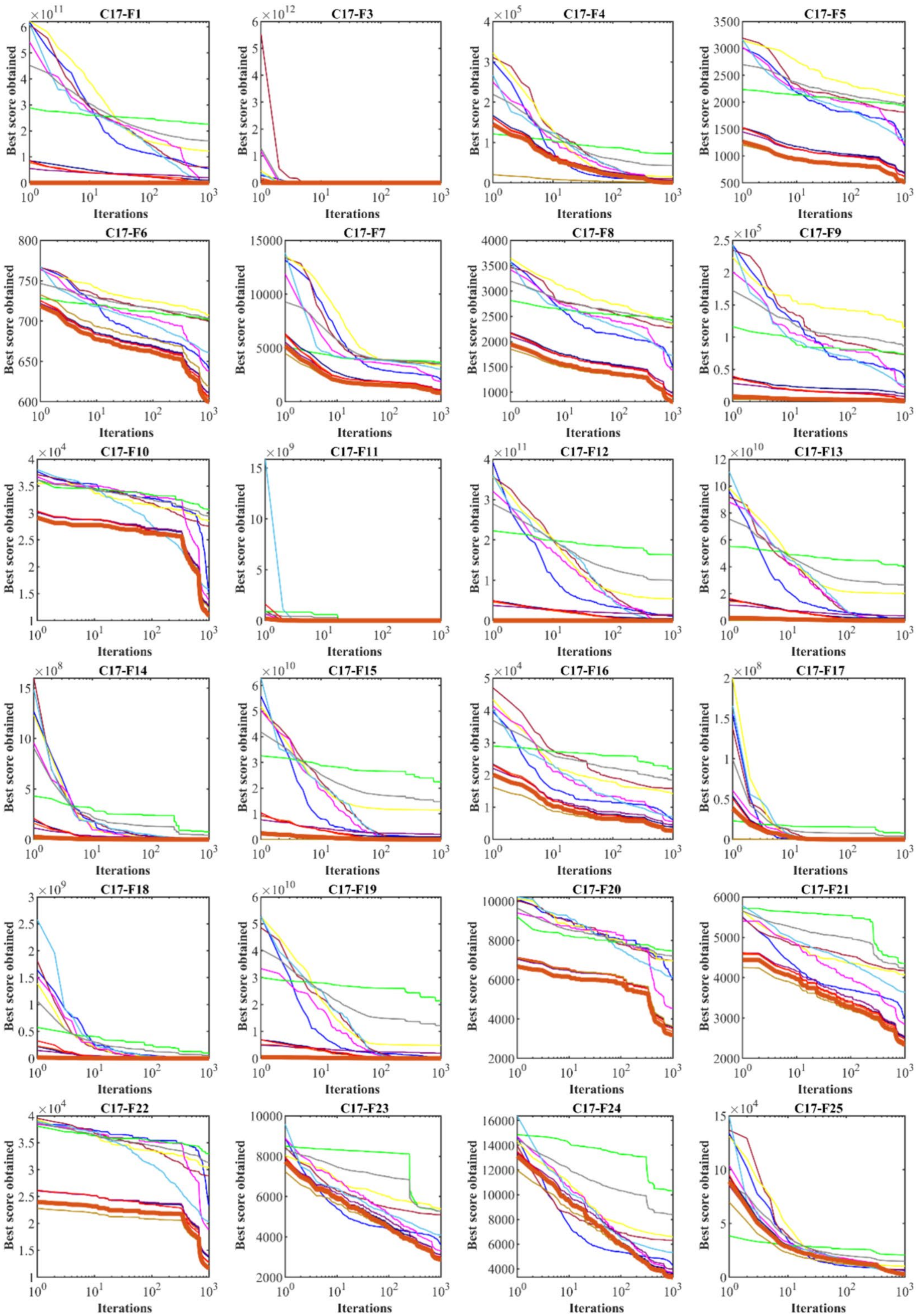

Figure 8. Convergence curves of algorithms performances on CEC 2017 test suite (dimension = 100).

values of design variables equal to (0.2057296, 3.4704887, 9.0366239, 0.2057296) and the value of the objective function equal to (1.7246798). The finding of the simulation results is that BOA has provided superior performance by achieving better results compared to competitor algorithms in order to deal with welded beam design.

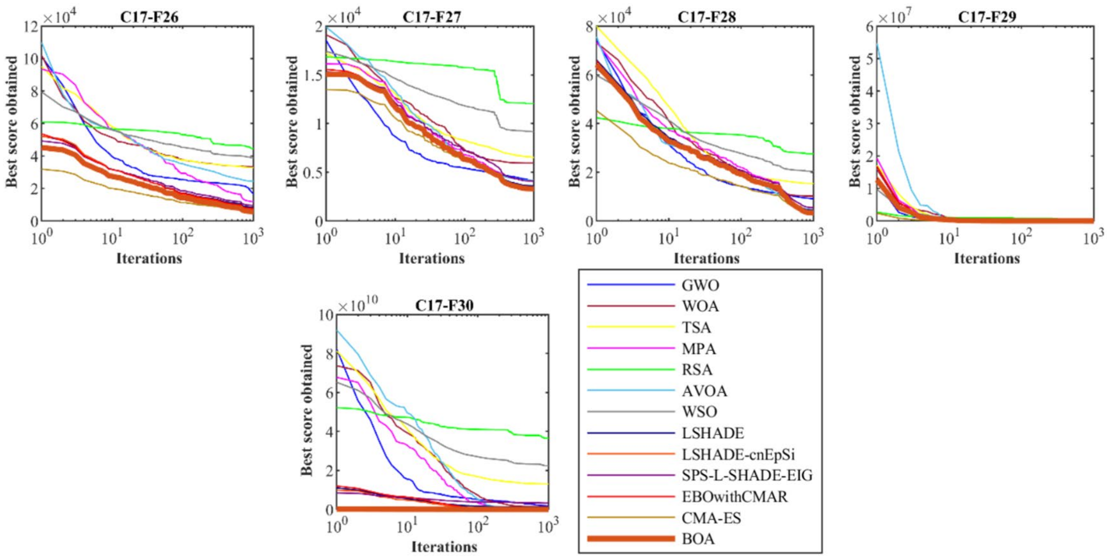

Figure 8. (continued)

Tension/compression spring design problem

Tension/compression spring design is presented as a challenge in real-world applications with the schematic shown in Fig. 17, where minimizing the weight of tension/compression spring is the main goal in this design. The mathematical model of this design is as follows[95]:

Consider: $X = [x_1, x_2, x_3] = [d, D, P]$.

Minimize: $f(x) = (x_3 + 2)x_2x_1^2$.

Subject to:

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785x_1^4} \le 0, g_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3)} + \frac{1}{5108x_1^2} - 1 \le 0,$$

$g_3(x) = 1 - \frac{140.45x_1}{x_2^2 x_3} \le 0$, $g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \le 0$.

With

$$0.05 \le x_1 \le 2, 0.25 \le x_2 \le 1.3 \text{and} 2 \le x_3 \le 15$$

Tables 17, 18 report the results of addressing the tension/compression spring design problem using BOA and metaheuristic algorithms. Figure 18 shows the convergence curve of BOA while achieving the solution for tension/compression spring design. Based on the obtained results, BOA has provided the optimal design for tension/compression spring with the values of the design variables equal to (0.0516891, 0.3567177, 11.288966) and the value of the objective function equal to (0.0126019). The finding of the simulation results is that BOA has provided superior performance in order to deal with tension/compression spring design, by providing better results in competition with the compared algorithms.

BOA for supply chain management (SCM)

To formulate the sustainable lot size optimization problem, we adopted mathematical models that considered economic costs and environmental impacts. The objective was to determine the optimal lot size at each stage of the supply chain that reduced both CO2 emissions and costs. The model incorporates various constraints such as production, inventory constraints, and demand satisfaction requirements. In addition, specific sustainability limits were added to limit the maximum CO2 allowed Disposals associated with manufacturing, transportation, and storage activities. The accounting system looks like a standard solution. Intelligence strikes a balance between economic prosperity and environmental sustainability. The company must minimize scarcity, maximize surplus, and choose the optimal quantity for the lot size. When customer needs arise, they must review the materials needed to start production or decide how much to order. Outstanding inventory represents a backlog, which should be checked to avoid excess inventory and, if attempted, actions should be planned to reduce it. The flowchart in Algorithm 1 represents an example of an optimized listing system.

TC = Cc * D + Cp * P * (Q + SS) + p * A * D + Ce * D

F [TC] = Total Cost [Cc, Q, D, Cp, SS, A, Ce, P, p]

where: Cc: Order cost/unit; Cp: Holding cost/unit; P: Price; p: Shortage cost/unit; A: Expected shortage/cycle; D: Annual demand; Ce: Footprint emission cost; Q: Quantity; SS: Shortage; TC: Total cost; F [TC]: Objective function for lot size.

Lot size optimization is influenced by demand data, which determines the number of items required to satisfy customer demand. The variability and uncertainty in demand patterns influence the lot sizing decision process. The optimal lot size can be identified to balance customer satisfaction and inventory costs by considering changes

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C20-F1	Mean	100	4654.416	269.3601	269.3601	269.3601	529,106.2	540.86	414.33	1379.043	5126.764	330.9591	947.9631	541.3533
	Best	100	218.7791	100	100	100	100	517.6785	227.5587	100	100	171.9595	333.0219	171.9595
	Worst	100	12,862.76	586.7017	586.7017	586.7017	2,111,279	586.7017	586.7017	3775.674	19,599.7	704.3544	2487.268	846.2411
	Std	1.11E−06	6388.102	241.4972	241.4972	241.4972	1,173,165	35.27051	220.4615	1823.838	10,733.99	278.5073	1147.849	311.3273
	Median	100	2768.062	195.3693	195.3693	195.3693	2522.622	529.5298	421.5297	820.2491	403.6797	223.7613	485.7811	573.6063
	Rank	1	10	2	2	3	12	6	5	9	11	4	8	7
C20-F2	Mean	1109.997	2385.246	1861.024	1789.1	1933.271	2005.351	2670.564	2556.306	6156.211	96,100.19	3227.281	2268.84	2523.301
	Best	1100	1533.602	1248.242	1119.942	1247.209	1508.382	1876.703	1780.75	1119.942	1897.054	1119.942	1447.958	2063.44
	Worst	1120.638	3247.528	2300.844	2300.844	2300.844	2615.147	3575.328	3423.353	10,169.03	170,027.9	4436.652	3009.309	2933.155
	Std	9.411746	920.5457	495.7543	546.2998	521.8125	517.5414	1012.515	965.4905	4169.781	85,099.8	1662.701	710.8755	455.5074
	Median	1109.674	2379.928	1947.504	1867.808	2092.516	1948.937	2615.113	2510.56	6667.939	106,237.9	3676.265	2309.047	2548.304
	Rank	1	7	3	2	4	5	10	9	12	13	11	6	8
C20-F3	Mean	710.7399	946.5239	1030.93	921.6721	928.7036	2140.083	1280.415	855.566	1098.417	842.7941	899.4639	1622.219	911.7972
	Best	700.0001	701.0209	703.0943	833.6536	700	833.6536	700	700.0174	700	700	759.5655	736.4907	825.1043
	Worst	714.3198	1179.969	1526.272	1030.329	1040.175	3012.079	1909.583	1021.259	1497.344	1034.631	1196.107	2948.858	1063.042
	Std	7.963452	293.0101	404.4373	92.58541	176.4788	1047.268	565.5208	178.9907	472.6718	156.6231	222.3403	1164.253	119.5282
	Median	714.3198	952.5529	947.1777	911.3529	987.3195	2357.299	1256.038	850.4937	1098.163	818.2726	821.0917	1401.764	879.5213
	Rank	1	8	9	6	7	13	11	3	10	2	4	12	5
C20-F4	Mean	1941.536	2488.139	6176.896	2587.731	2744.026	2529.797	3257.763	2304.616	3135.115	2259.914	2570.478	3140.962	2394.711
	Best	1900	2126.172	2126.172	2126.172	1900	2070.681	2126.172	2078.726	1900	2005.014	1900	2049.193	1900
	Worst	1983.073	2831.743	9355.236	2992.157	3531.494	3070.15	4541.494	2526.219	5411.428	2504.562	3376.309	5136.934	3195.114
	Std	46.35488	320.7347	3341.428	417.9987	895.1909	563.3419	1300.7	271.2238	1742.225	258.0201	675.9606	1544.024	622.431
	Median	1941.536	2497.32	6613.087	2616.297	2772.304	2489.178	3181.692	2306.759	2614.516	2265.041	2502.802	2688.861	2241.866
	Rank	1	5	13	8	9	6	12	3	10	2	7	11	4
C20-F5	Mean	1702.285	2239.525	1858.166	2164.36	1788.468	3207.768	2005.77	1907.013	2085.434	1892.865	1781.148	1790.773	2070.973
	Best	1700	1835.073	1776.535	1781.423	1700	1700	1705.567	1705.567	1700	1700	1730.202	1708.17	1705.567
	Worst	1704.57	2849.541	1976.775	2627.538	1900.977	6222.57	2181.269	2292.72	2652.44	2036.165	1862.216	1842.893	2340.974
	Std	2.680999	504.0245	94.06495	387.6702	109.7219	2368.197	243.0359	292.2743	505.8121	179.68	64.50852	69.43882	299.7735
	Median	1702.285	2136.742	1839.677	2124.24	1776.448	2454.251	2068.123	1814.883	1994.648	1917.647	1766.086	1806.014	2118.676
	Rank	1	12	5	11	3	13	8	7	10	6	2	4	9
C20-F6	Mean	1603.727	2483.368	1941.492	1838.004	1832.404	2686.847	1939.455	3468.956	2894.062	2096.874	5826.813	3369.879	2283.484
	Best	1600	1755.053	1755.053	1674.294	1600	1755.053	1775.014	1909.611	1600	1755.053	1909.611	1755.053	1755.053
	Worst	1609.322	2891.929	2177.142	1914.703	1989.928	3212.981	2074.295	4800.85	3855.79	2295.9	8875.029	5397.799	2723.037
	Std	5.070415	564.6686	201.7032	125.9336	210.9064	732.4838	166.7318	1369.214	1078.798	264.6256	3305.629	1738.474	458.3604
	Median	1602.793	2643.244	1916.887	1881.51	1869.845	2889.678	1954.255	3582.682	3060.23	2168.271	6261.306	3163.332	2327.923
	Rank	1	8	5	3	2	9	4	12	10	6	13	11	7
C20-F7	Mean	2206.269	3902.404	2958.347	2580.351	3965.246	3002.99	3379.417	3131.289	3038.459	3429.377	2835.707	3567.499	3383.971
	Best	2100	2380.15	2172.035	2172.035	2172.035	2834.606	2820.134	2491.375	2380.15	2343.686	2632.634	2343.686	2181.634
	Worst	2327.209	5400.796	4233.579	3063.596	6111.454	3155.574	4696.493	4705.774	4283.549	4348.672	3092.797	4354.73	4373.827
	Std	106.0625	1404.331	1038.734	407.8927	1835.668	149.6089	992.6682	1181.171	947.2925	1068.908	213.1073	1016.463	1167.87
	Median	2198.933	3914.336	2713.887	2542.885	3788.747	3010.89	3000.519	2664.004	2745.068	3512.575	2808.699	3785.79	3490.211
	Rank	1	12	4	2	13	5	8	7	6	10	3	11	9
C20-F8	Mean	2218.874	2568.297	2497.947	2377.194	2551.459	2541.787	2362.74	2500.844	2519.123	2375.379	2351.969	2365.86	2405.083
	Best	2200	2284.306	2267.622	2284.306	2233.587	2267.622	2284.306	2233.587	2284.306	2284.306	2267.622	2267.622	2267.622
	Worst	2237.305	2691.31	2675.148	2471.186	2826.468	2882.077	2397.15	2742.492	2685.164	2552.073	2436.719	2505.304	2563.966
	Std	21.86637	212.4807	195.617	85.71142	352.9057	295.8129	58.56492	252.1761	208.096	134.8033	104.6396	113.6305	136.6297
	Median	2219.095	2648.786	2524.509	2376.642	2572.891	2508.724	2384.751	2513.649	2553.512	2332.569	2351.768	2345.257	2394.372
	Rank	1	13	8	6	12	11	3	9	10	5	2	4	7
C20-F9	Mean	2404.914	2505.427	2928.06	2930.358	2493.553	2643.167	2696.627	2462.19	2498.552	2519.963	2443.155	2588.241	2495.838
	Best	2400	2413.197	2426.192	2434.644	2416.702	2416.702	2416.702	2416.702	2413.197	2426.192	2418.582	2416.702	2426.192
	Worst	2411.095	2602.213	3278.573	3337.795	2586.208	2936.937	3079.996	2493.618	2621.356	2612.492	2480.385	2701.826	2545.829
	Std	5.296123	95.25177	401.2837	429.2936	78.36549	257.5471	348.6891	39.39015	106.2098	110.0469	29.28298	139.1959	65.39078
	Median	2404.28	2503.149	3003.739	2974.495	2485.651	2609.514	2644.905	2469.221	2479.828	2520.583	2436.827	2617.218	2505.665
	Rank	1	7	12	13	4	10	11	3	6	8	2	9	5

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C20-F10	Mean	2500.005	5501.834	2506.285	2508.548	2500.793	2505.697	2514.842	2502.45	2505.446	2503.433	29,528.4	2503.527	2501.114
	Best	2500	2500.002	2501.462	2500.629	2500.018	2500.002	2500.826	2500.629	2500.002	2500.629	2501.462	2500.024	2500.629
	Worst	2500.017	6539.912	2518.054	2514.524	2501.462	2516.926	2546.822	2503.583	2511.038	2509.539	38,553.66	2511.966	2501.879
	Std	0.008848	2226.021	8.761668	6.46181	0.659229	8.498574	23.89477	1.412787	5.072157	4.572588	20,040.18	6.293057	0.594907
	Median	2500.001	6483.71	2502.813	2509.519	2500.847	2502.93	2505.86	2502.794	2505.372	2501.782	38,529.23	2501.059	2500.975
	Rank	1	12	9	10	2	8	11	4	7	5	13	6	3
Sum rank		10	94	70	63	59	92	84	62	90	68	61	82	64
Mean rank		1	9.4	7	6.3	5.9	9.2	8.4	6.2	9	6.8	6.1	8.2	6.4
Total rank		1	13	8	5	2	12	10	4	11	7	3	9	6

Table 6. Optimization results of CEC 2020 test suite.

in demand and forecast accuracy. The optimal lot size is considered an important decision in terms of travel expenses. Lot sizes can increase economies and reduce transportation costs per unit, resulting in cost savings, but lot sizes can increase storage and warehousing costs and drive transportation cost savings equal lot The goal of size optimization is to minimize inventory costs associated with storage and maintenance at the inventory level. Higher lot sizes can increase inventory costs due to longer storage times. Small lot sizes ordered, there are often setup costs to consider. Regularly increasing lot sizes incorporates environmental considerations such as CO_2 emissions. Transport demand increased.

Lot size can increase CO_2 emissions. By optimizing lot sizes, a balance can be achieved between inventory costs, transportation costs and CO_2 emissions.

The implementation results of BOA and competitor algorithms for sustainable lot size optimization are reported in Table 19. The results and analysis showed the indisputable superiority of the Bobcat algorithm in optimizing lot sizes in supply chain management. Comparative analysis with conventional methods such as EOQ and other algorithms revealed a consistent reduction in total relative costs obtained by the Bobcat system in supply chain planning and demand models. Convergence analysis showed its fast convergence behavior, and it outperformed genetic algorithms and simulated annealing. Furthermore, scalability testing in large scale demonstrated its robust performance, demonstrating linear scalability and suitability for complex supply chain networks including disruption of supply. The comparative analysis with benchmark solutions confirmed the effectiveness of the Bobcat algorithm in finding a nearly always optimal lot-size policy. Finally, sensitivity analysis confirmed its versatility and ease of implementation in parameter settings. Overall, these findings firmly establish the Bobcat algorithm as the first choice for lot size optimization, providing better solutions, convergence speed, scalability, and robustness to uncertainties in real-world supply in chain scenarios.

Discussion

This section delves into a comprehensive analysis of the experiments conducted in this paper, evaluating the advantages, disadvantages, and specific shortcomings of the Bobcat Optimization Algorithm (BOA) compared to existing methods. The experiments conducted in this study are designed to evaluate the performance of the Bobcat Optimization Algorithm (BOA) across a range of optimization problems. These has been included the CEC 2017 test suite for various problem dimensions, the CEC 2011 constrained optimization problems, and several engineering design problems. Additionally, BOA is tested on sustainable lot size optimization in supply chain management (SCM) applications.

CEC 2017 test suite

Benchmark functions in CEC 2017 test suite are of four types: unimodal, multimodal, hybrid, and composition. Unimodal functions are suitable criteria for measuring the ability to exploit metaheuristic algorithms to manage local search in the problem space. The findings show that BOA, with its ability in exploitation, has been able to provide suitable solutions for dealing with unimodal functions. Multimodal functions challenge the discovery ability of metaheuristic algorithms due to having multiple local optima. The findings show that BOA with a high ability in discovery in order to manage global search has provided an effective performance for handling multi-model functions. Hybrid functions and composition are complex optimization problems that challenge the ability of metaheuristic algorithms to balance exploration and exploitation. The findings obtained from the optimization results of these functions show that BOA, with its high ability to balance exploration and exploitation during the search process, has been able to provide an effective performance for handling hybrid and composition functions. The findings obtained from the CEC 2017 test suite optimization confirm that the proposed BOA approach has a high capability in exploring, exploiting, and balancing them during the search process.

The results obtained from BOA have been compared with the performance of twelve well-known algorithms. Based on the analysis of simulation results, BOA has shown high efficiency in solving optimization problems in different dimensions, with a success rate of 89.65% for dimensions 10 and 100, 79.31% for dimension 30, and 93.10% for dimension 50. This shows BOA's strong ability to maintain a balance between exploration and exploitation.

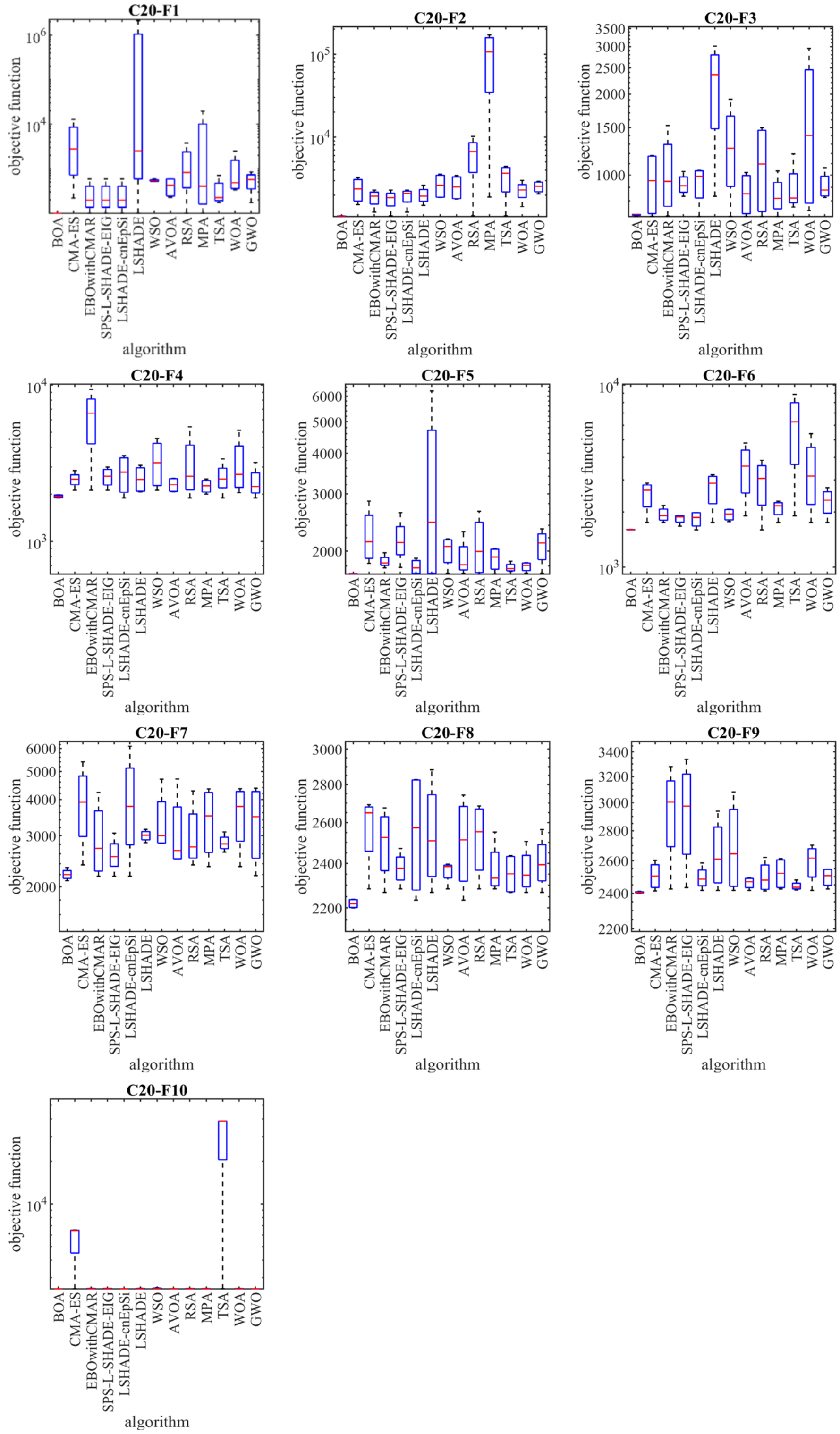

Figure 9. Boxplot diagrams of BOA and competitor algorithms performances on CEC 2020 test suite.

CEC 2020 test suite

The evaluation of the Bobcat Optimization Algorithm (BOA) on the CEC 2020 test suite revealed exceptional performance. This test suite consists of ten numerical optimization functions with specific boundary constraints, ranging from unimodal (C20-F1) to more complex hybrid and composition functions (C20-F2 to C20-F10). BOA successfully handled 100% of these functions and outperformed twelve other algorithms, securing the top rank across all benchmark problems. The experimental results demonstrate BOA's robustness, efficiency, and

Compared algorithm	Objective function type				
	CEC 2017				CEC 2020
	D = 10	D = 30	D = 50	D = 100	
BOA vs. CMA–ES	1.34E–08	1.18E–11	1.26E–12	4.72E–16	3.57E–08
BOA vs. EBOwithCMAR	3.78E–21	3.02E–21	1.97E–21	1.97E–21	3.85E–08
BOA vs. SPS_L_SHADE_EIG	1.97E–21	1.97E–21	1.97E–21	1.97E–21	3.85E–08
BOA vs. LSHADE_cnEpSi	4.51E–21	2.95E–21	6.62E–18	1.97E–21	1.10E–07
BOA vs. LSHADE	7.79E–21	1.97E–21	1.97E–21	1.97E–21	4.48E–08
BOA vs. WSO	1.77E–34	1.77E–34	1.77E–34	1.77E–34	3.85E–08
BOA vs. AVOA	2.08E–25	2.91E–21	1.77E–34	1.77E–34	3.57E–08
BOA vs. RSA	1.77E–34	1.77E–34	1.77E–34	1.77E–34	1.10E–07
BOA vs. MPA	1.11E–29	3.61E–16	5.96E–31	1.77E–34	5.63E–08
BOA vs. TSA	5.24E–34	1.77E–34	1.77E–34	1.77E–34	3.85E–08
BOA vs. WOA	5.24E–34	1.77E–34	1.77E–34	1.77E–34	3.57E–08
BOA vs. GWO	5.63E–32	1.77E–34	1.77E–34	1.77E–34	3.85E–08

Table 7. Wilcoxon signed-rank test results.

Compared algorithm	Objective function type									
	CEC 2017								CEC 2020	
	D = 10		D = 30		D = 50		D = 100			
	Avg. rank	Overall rank	Avg. rank	Overall	Avg. rank	Overall	Avg. rank	Overall rank	Avg. rank	Overall rank
BOA	1.137931	1	1.275862	1	1.137931	1	1.103448	1	1.148965	1
CMA-ES	2	2	2.482759	2	2.724138	3	2.517241	2	2.286667	2
EBOwithCMAR	3.37931	3	4.206897	4	4.034483	4	3.896552	4	3.751035	4
SPS_L_SHADE_EIG	7.068966	8	7.62069	7	7.758621	7	7.37931	7	7.209196	7
LSHADE_cnEpSi	5.103448	4	3.068966	3	2.413793	2	2.724138	3	3.55954	3
LSHADE	5.344828	5	6.275862	6	6.241379	6	6.137931	6	5.80115	5
WSO	11.44828	12	11.72414	12	11.7931	11	11.93103	12	11.46713	12
AVOA	6.931034	6	7.827586	9	7.758621	7	7.793103	8	7.366896	8
RSA	12.48276	13	12.62069	13	12.75862	12	12.55172	13	12.30069	13
MPA	8.965517	10	4.482759	5	5.275862	5	5.896552	5	6.31931	6
TSA	10.51724	11	10.7931	11	10.58621	10	10.86207	11	10.50965	11
WOA	8.37931	9	10.03448	10	9.896552	9	9.896552	10	9.248045	10
GWO	7	7	7.758621	8	7.793103	8	8.310345	9	7.535862	9

Table 8. Friedman test results.

Compared algorithm	Objective function type				
	CEC 2017				CEC 2020
	D = 10	D = 30	D = 50	D = 100	
BOA vs. CMA–ES	0.025659	0.01308	0.012415	1.45E–08	1.23E–08
BOA vs. EBOwithCMAR	0.023591	0.06815	0.003216	0.002527	6.32E–05
BOA vs. SPS_L_SHADE_EIG	0.003543	0.000362	4.38E–05	3.4E–05	2.12E–05
BOA vs. LSHADE_cnEpSi	0.003885	0.066835	0.002224	0.000894	1.13E–04
BOA vs. LSHADE	0.024488	0.002107	0.000132	0.000113	4.13E–05
BOA vs. WSO	0.003696	0.00046	4.73E–05	5.08E–05	4.55E–05
BOA vs. AVOA	0.022884	0.068085	0.004259	0.002902	3.57E–05
BOA vs. RSA	0.003425	0.000363	4.38E–05	3.38E–05	8.69E–08
BOA vs. MPA	0.023707	0.001606	0.002224	0.000894	4.38E–05
BOA vs. TSA	0.024687	0.002113	0.000132	0.000113	2.09E–05
BOA vs. WOA	0.019632	0.005948	0.002747	0.001704	1.64E–03
BOA vs. GWO	0.029303	0.050449	0.002851	0.001458	3.45E–04

Table 9. T-test results.

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C11-F1	Mean	5.920103	6.183228	6.738518	7.663933	6.039163	7.299507	13.57783	15.02735	11.12614	20.03647	23.68149	19.48093	25.589
	Best	2E-10	0.059473	0.981494	2.086606	0.026815	1.917445	8.453028	11.70977	1.171431	17.66003	22.05158	11.75673	24.27333
	Worst	12.30606	12.36437	12.7852	13.42714	12.33324	12.89891	17.51285	18.02634	18.2779	22.43279	24.77817	25.79976	27.26328
	Std	7.476538	7.198024	7.106069	6.833248	7.348851	6.659064	4.489697	2.968292	7.876573	2.230811	1.454978	7.575792	1.390322
	Median	5.687176	6.154532	6.593691	7.570993	5.898301	7.190838	14.17272	15.18664	12.52762	20.02652	23.9481	20.18362	25.40969
	Rank	1	3	4	6	2	5	8	9	7	11	12	10	13
C11-F2	Mean	−26.3179	−26.118	−25.712	−24.7491	−26.2297	−24.7205	−18.4083	−6.80337	−22.4843	−9.13516	−14.3114	−22.2721	−11.4007
	Best	−27.0676	−26.8407	−26.4316	−25.4532	−26.9672	−25.1705	−21.957	−9.02685	−24.6968	−10.3883	−19.8679	−23.7385	−14.0856
	Worst	−25.4328	−25.3903	−25.0044	−23.972	−25.4155	−24.3606	−14.3388	−5.11857	−18.7559	−8.05643	−9.79705	−19.5489	−9.44663
	Std	0.767703	0.724938	0.671114	0.691313	0.744805	0.383093	4.234766	1.834314	2.856253	1.056973	5.031236	2.029882	2.352976
	Median	−26.3856	−26.1205	−25.706	−24.7856	−26.268	−24.6755	−18.6688	−6.53403	−23.2422	−9.04796	−13.7903	−22.9005	−11.0353
	Rank	1	3	4	5	2	6	9	13	7	12	10	8	11
C11-F3	Mean	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05
	Best	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05
	Worst	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05
	Std	2.08E−19	2.02E−16	2.65E−10	5.21E−12	9.1E−17	2.43E−15	1.02E−16	1.14E−12	3.98E−15	8.96E−14	2.3E−19	6.59E−20	3.16E−18
	Median	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05
	Rank	3	1	13	11	2	8	7	12	9	10	5	4	6
C11-F4	Mean	0	0	0	0	0	0	0	0	0	0	0	0	0
	Best	0	0	0	0	0	0	0	0	0	0	0	0	0
	Worst	0	0	0	0	0	0	0	0	0	0	0	0	0
	Std	0	0	0	0	0	0	0	0	0	0	0	0	0
	Median	0	0	0	0	0	0	0	0	0	0	0	0	0
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1
C11-F5	Mean	−34.1274	−33s.9869	−33.4735	−32.6509	−34.0665	−33.375	−27.5271	−26.2509	−31.4941	−8.28938	−26.6454	−5.892	−6.84436
	Best	−34.7494	−34.6029	−34.1393	−33.4062	−34.6859	−33.5294	−27.7046	−31.484	−34.1989	−10.7244	−31.2638	−9.82996	−8.43368
	Worst	−33.3862	−33.4208	−32.8248	−31.8708	−33.4042	−33.2165	−27.2122	−23.6131	−27.3312	−6.42043	−23.2282	−3.93246	−4.93201
	Std	0.612958	0.546605	0.591415	0.735292	0.577418	0.14038	0.23884	4.037264	3.202875	1.998215	3.849145	3.023656	1.676274
	Median	−34.1871	−33.962	−33.465	−32.6633	−34.088	−33.3771	−27.5958	−24.9532	−32.2231	−8.00632	−26.0448	−4.90278	−7.00588
	Rank	1	3	4	6	2	5	8	10	7	11	9	13	12
C11-F6	Mean	−24.1119	−23.8724	−23.516	−22.9111	−24.0056	−22.3573	−19.8144	−7.9834	−19.4891	0	−21.6613	−0.95891	−1.96296
	Best	−27.4298	−27.1611	−26.6314	−25.8646	−27.3106	−25.0833	−22.9152	−16.8457	−22.2461	0	−26.9874	−3.83566	−7.85186
	Worst	−23.0059	−22.7382	−22.3718	−21.8038	−22.8869	−21.0226	−12.7528	0	−17.8223	0	−16.7906	0	0
	Std	2.415463	2.394967	2.27064	2.152354	2.406217	2.08112	5.248052	10.09718	2.315651	0	4.76581	2.094307	4.287194
	Median	−23.0059	−22.7951	−22.5303	−21.9881	−22.9125	−21.6616	−21.7947	−7.54395	−18.944	0	−21.4335	0	0
	Rank	1	3	4	5	2	6	8	10	9	13	7	12	11

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C11-F7	Mean	0.860699	0.871324	0.907054	0.970842	0.865551	0.908844	1.750956	0.883102	1.073368	1.804349	1.101417	1.149726	1.82829
	Best	0.582266	0.609562	0.650113	0.730509	0.594615	0.646949	1.64062	0.800399	0.808639	1.580978	0.914546	0.816436	1.426014
	Worst	1.025027	1.013836	1.059499	1.129486	1.020055	1.059054	1.913631	0.966353	1.306208	1.94451	1.323653	1.428968	2.040515
	Std	0.219737	0.203006	0.20541	0.190162	0.212148	0.208132	0.127446	0.098176	0.224655	0.174751	0.206477	0.349056	0.302915
	Median	0.91775	0.930948	0.959302	1.011686	0.923767	0.964686	1.724787	0.882827	1.089312	1.845955	1.083734	1.176751	1.923316
	Rank	1	3	5	7	2	6	11	4	8	12	9	10	13
C11-F8	Mean	220	220.3405	222.1997	230.7489	220.1693	223.8941	266.5276	224.5	227.5731	224.5	249.0859	495.4306	222.75
	Best	220	219.956	220.6414	226.762	219.996	220	245.4229	220	220	220	220	251	220
	Worst	220	220.725	224.0346	235.1052	220.3427	233.822	313.0551	238	235.1462	238	301	606.3905	231
	Std	0	0.484819	1.604077	3.75	0.218589	7.283881	34.14759	9.828182	9.549379	9.828182	41.94558	183.6816	6.006111
	Median	220	220.3405	222.0615	230.5642	220.1693	220.8771	253.8161	220	227.5731	220	237.6718	562.166	220
	Rank	1	3	4	9	2	6	11	7	8	7	10	12	5
C11-F9	Mean	8789.286	10,582.77	46,680.33	115,573.8	9598.543	15,223.47	378,067.3	146,438.6	44,149.83	450,581.2	908,719.2	1,195,008	2,145,417
	Best	5457.674	7894.025	39,942.32	83,096.81	7106.216	11,013.91	209,009	82,594.58	18,796.43	372,659.4	776,984.3	959,191.4	2,055,477
	Worst	14,042.29	15,642.6	51,101.42	131,782.7	14,764.83	20,468.25	639,683.2	221,696.4	77,480.91	577,846.5	978,482.7	1,463,389	2,271,153
	Std	4040.59	3772.539	5184.928	24,089.06	3856.725	4542.441	214,422.1	62,641.89	27,082.33	98,531.16	97,986.7	294,523.9	115,752.2
	Median	7828.591	9397.229	47,838.78	123,707.9	8261.564	14,705.85	331,788.6	140,731.8	40,160.98	425,909.4	939,704.9	1,178,726	2,127,520
	Rank	1	3	6	7	2	4	9	8	5	10	11	12	13
C11-F10	Mean	−21.4889	−21.0979	−20.8902	−20.4228	−21.3142	−20.6354	−12.6331	−14.0006	−13.871	−10.232	−12.2919	−10.3421	−10.016
	Best	−21.8299	−21.4464	−21.2161	−20.7713	−21.6586	−21.1909	−13.3341	−21.1288	−14.3623	−10.3156	−12.8412	−10.3763	−10.0336
	Worst	−20.7878	−20.4449	−20.2387	−19.7949	−20.6347	−19.9709	−12.1541	−10.4071	−12.6687	−10.1842	−11.4186	−10.2991	−10.0025
	Std	0.518028	0.490697	0.484134	0.470434	0.505223	0.548418	0.544438	5.27389	0.883739	0.064444	0.74327	0.038528	0.017366
	Median	−21.669	−21.2501	−21.053	−20.5626	−21.4817	−20.6899	−12.5221	−12.2332	−14.2264	−10.2142	−12.454	−10.3466	−10.0139
	Rank	1	3	4	6	2	5	9	7	8	12	10	11	13
C11-F11	Mean	571,712.3	742,421.9	675,338.1	1,467,342	648,720.9	1,173,966	1,304,182	1,384,293	3,936,396	5,689,705	1,497,804	5,701,929	6,697,096
	Best	260,837.9	466,634.7	386,344.9	1,170,537	353,643.8	894,529.2	1,190,534	603,550.3	3,739,692	5,689,705	1,344,136	5,689,705	6,654,808
	Worst	828,560.9	980,059.1	917,618.6	1,710,560	896,926.1	1,411,503	1,456,040	2,963,819	4,315,401	5,689,705	1,679,064	5,714,152	6,784,052
	Std	271,080	246,881.2	254,206.4	256,302	260,140	287,705.3	120,975.1	1,167,134	282,438.7	0	152,222.7	15,413.55	64,409.66
	Median	598,725.2	761,497	698,694.5	1,494,135	672,156.8	1,194,916	1,285,077	984,901.8	3,845,245	5,689,705	1,484,009	5,701,929	6,674,762
	Rank	1	4	3	8	2	5	6	7	10	11	9	12	13
C11-F12	Mean	1,199,805	1,211,309	1,422,984	2,415,324	1,205,078	1,589,444	5,845,383	1,340,623	1,431,117	15,685,253	6,263,817	2,428,911	15,862,796
	Best	1,155,937	1,169,359	1,373,887	2,289,705	1,162,072	1,563,793	5,424,799	1,170,807	1,262,817	14,751,661	5,942,068	2,236,087	15,726,437
	Worst	1,249,353	1,265,507	1,476,057	2,544,926	1,256,726	1,612,360	6,051,865	1,504,078	1,572,455	16,408,417	6,496,367	2,663,553	16,005,029
	Std	48,993.46	52,223.07	52,541.69	117,800	50,406.9	26,873.37	316,690.3	148,633.4	140,192.2	759,437.6	261,428	192,429.7	124,270.4
	Median	1,196,965	1,205,184	1,420,995	2,413,333	1,200,757	1,590,812	5,952,435	1,343,803	1,444,597	15,790,467	6,308,418	2,408,001	15,859,858
	Rank	1	3	5	8	2	7	10	4	6	12	11	9	13

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C11-F13	Mean	15,444.2	15,444.09	15,445.65	15,532.49	15,445.26	15,449.9	15,537.51	15,514.52	15,502.94	15,984.05	140,082	15,495.68	31,517.17
	Best	15,444.19	15,443.73	15,445.55	15,490.5	15,445.1	15,448.78	15,493.87	15,492.18	15,495.79	15,646.58	100,822.9	15,476.66	15,462.15
	Worst	15,444.21	15,444.72	15,445.85	15,636.83	15,445.54	15,451.17	15,596.65	15,557.03	15,515.31	16,605.73	193,331	15,536	79,293.98
	Std	0.009445	0.478391	0.145075	76.58996	0.215158	1.194419	52.32485	32.82946	9.444644	474.0736	45,481.82	29.66999	34,782.32
	Median	15,444.2	15,443.96	15,445.61	15,501.32	15,445.2	15,449.83	15,529.76	15,504.44	15,500.33	15,841.94	133,087	15,485.03	15,656.28
	Rank	2	1	4	9	3	5	10	8	7	11	13	6	12
C11-F14	Mean	18,295.35	18,340.82	18,335.58	39,546.26	18,317.17	18,437.51	19,256.06	19,536.54	19,263.05	341,308.2	19,175.82	19,211.78	19,197.69
	Best	18,241.58	18,282.19	18,280.3	33,471.35	18,261.2	18,373.24	19,098.84	19,430.12	19,113.29	31,488.89	18,855.78	19,030.02	18,882.71
	Worst	18,388.08	18,430.95	18,427.97	49,676.34	18,408.73	18,504.76	19,382.88	19,630.9	19,455.49	660,566.8	19,413.1	19,363.98	19,532.85
	Std	74.38679	71.57923	75.54823	7936.489	73.00297	60.7239	141.7149	91.07887	164.8416	329,799.5	261.0218	150.1261	290.1312
	Median	18,275.87	18,325.07	18,317.03	37,518.68	18,299.37	18,436.02	19,271.26	19,542.58	19,241.71	336,588.6	19,217.2	19,226.55	19,187.6
	Rank	1	4	3	12	2	5	9	11	10	13	6	8	7
C11-F15	Mean	32,883.58	32,887.2	40,442.45	222,574.1	32,887.58	35,071.35	219,895.5	33,122.71	33,084.29	17,036,307	327,796.7	33,330.48	8,766,184
	Best	32,782.17	32,789.41	33,989.56	110,195.9	32,787.79	32,840	33,017.83	33,039.96	33,047.44	3,566,969	289,724.9	33,319.18	3,988,196
	Worst	32,956.46	32,959.66	47,742.85	533,443.4	32,960.27	41,562.65	314,838.5	33,187.07	33,149.24	25,406,717	353,867.8	33,338.43	15,026,014
	Std	79.94256	77.67837	8120.385	226,670.8	78.92186	4726.113	138,936.3	71.03783	50.84676	10,844,349	32,583.82	9.361516	5,526,697
	Median	32,897.86	32,899.86	40,018.7	123,328.5	32,901.13	32,941.36	265,862.9	33,131.91	33,070.24	19,585,771	333,797	33,332.16	8,025,262
	Rank	1	2	8	10	3	7	9	5	4	13	11	6	12
C11-F16	Mean	133,550	134,169.5	133,946.4	316,794.7	133,838.9	134,949.1	142,544.9	142,673.9	146,279	98,220,886	20,674,185	87,910,864	84,408,504
	Best	131,374.2	132,008.7	131,841.4	168,845.8	131,669.7	132,724.9	136,575.8	133,079.6	143,646	95,713,439	10,493,845	72,716,866	68,217,489
	Worst	136,310.8	137,075.2	136,516.6	611,139.3	136,665.3	137,375.5	147,961.5	152,615.5	151,904	1.01E+08	37,415,902	1.05E+08	1.08E+08
	Std	2485.329	2515.941	2309.548	217,959.4	2498.282	2282.485	5188.973	8996.661	4151.659	2,442,036	12,712,449	15,220,715	18,439,517
	Median	133,257.5	133,797.1	133,713.8	243,596.9	133,510.4	134,848	142,821.2	142,500.2	144,783	98,060,477	17,393,496	86,936,590	80,723,913
	Rank	1	4	3	9	2	5	6	7	8	13	10	12	11
C11-F17	Mean	1,926,615	1,983,558	2.35E+08	1.57E+09	1,952,428	1.31E+08	9.76E+09	3,236,561	3,055,798	2.47E+10	1.24E+10	2.3E+10	2.42E+10
	Best	1,916,953	1,922,926	2.14E+08	1.13E+09	1,919,784	1.08E+08	6.96E+09	2,333,961	2,042,163	2.37E+10	1.09E+10	2.03E+10	2.26E+10
	Worst	1,942,685	2,075,218	2.57E+08	1.91E+09	1,990,296	1.49E+08	1.3E+10	3,933,469	4,978,873	2.57E+10	1.31E+10	2.66E+10	2.73E+10
	Std	12,470.83	72,807.99	20,877,833	3.69E+08	33,768.09	23,145,715	2.76E+09	806,246.3	1,444,811	9.08E+08	1.1E+09	3.1E+09	2.33E+09
	Median	1,923,412	1,968,045	2.35E+08	1.61E+09	1,949,815	1.33E+08	9.55E+09	3,339,406	2,601,077	2.46E+10	1.28E+10	2.26E+10	2.34E+10
	Rank	1	3	7	8	2	6	9	5	4	13	10	11	12
C11-F18	Mean	942,057.5	946,540.9	1,509,543	12,785,127	944,146.7	1,055,430	9,641,866	992,950.7	1,034,076	34,159,342	12,198,065	1.49E+08	1.26E+08
	Best	938,416.2	940,015.9	1,245,098	9,086,579	939,204.9	1,028,519	4,144,222	965,927.8	967,816.8	27,054,473	9,077,664	1.25E+08	1.22E+08
	Worst	944,706.9	957,937.7	1,987,616	14,482,383	950,740.2	1,085,666	16,934,301	1,005,714	1,209,033	36,959,114	15,413,362	1.65E+08	1.31E+08
	Std	2882.138	8582.375	378,440.5	2,754,779	5262.437	28,895.02	5,897,910	19,955.69	127,634.6	5,193,739	3,091,078	19,727,839	4,154,662
	Median	942,553.5	944,104.9	1,402,730	13,785,774	943,320.8	1,053,767	8,744,471	1,000,080	979,727.2	36,311,890	12,150,617	1.53E+08	1.26E+08
	Rank	1	3	7	10	2	6	8	4	5	11	9	13	12

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
C11-F19	Mean	1,025,341	1,042,836	1,599,321	12,616,413	1,033,303	1,176,661	10,291,018	1,524,075	1,373,985	39,223,360	6,818,060	1.91E+08	1.27E+08
	Best	967,927.7	986,740.3	1,496,947	10,986,585	976,479.3	1,115,159	2,076,737	1,145,008	1,240,381	27,439,684	2,515,303	1.73E+08	1.24E+08
	Worst	1,167,142	1,186,697	1,701,410	15,579,735	1,176,043	1,291,762	18,641,683	2,062,414	1,556,914	48,957,518	8,991,630	2.2E+08	1.31E+08
	Std	103,555.5	104,922.3	110,315.2	2,297,524	104,166.9	87,598.01	8,515,809	421,168.3	144,637.7	10,180,470	3,211,288	22,502,803	3,102,132
	Median	983,146.6	998,953.7	1,599,462	11,949,667	990,344.2	1,149,861	10,222,827	1,444,439	1,349,322	40,248,119	7,882,654	1.85E+08	1.27E+08
	Rank	1	3	7	10	2	4	9	6	5	11	8	13	12
C11-F20	Mean	941,250.4	944,067.1	1,441,411	13,471,341	942,588.1	1,031,567	7,326,695	976,129.4	1,000,440	38,105,472	15,671,493	1.76E+08	1.27E+08
	Best	936,143.2	940,087.2	1,366,949	11,896,204	937,988.7	1,011,525	6,903,007	964,906.8	978,541.6	37,267,078	10,376,829	1.61E+08	1.21E+08
	Worst	946,866.6	948,285.3	1,514,452	15,865,552	947,574.4	1,066,940	7,892,426	988,890.8	1,017,515	39,011,796	24,312,978	1.91E+08	1.32E+08
	Std	5208.733	4026.358	66,988.52	1,843,844	4675.262	27,750.41	462,083.5	11,664.05	18,346.54	792,412.9	6,650,888	18,415,907	4,991,383
	Median	940,995.9	943,948	1,442,122	13,061,803	942,394.6	1,023,902	7,255,674	975,360	1,002,851	38,071,507	13,998,083	1.76E+08	1.28E+08
	Rank	1	3	7	9	2	6	8	4	5	11	10	13	12
C11-F21	Mean	12.71443	13.21891	13.79709	19.32464	12.9428	14.62207	39.23486	29.10064	22.70134	109.9653	43.65333	115.531	112.0717
	Best	9.974206	10.56784	11.22903	18.86382	10.24258	12.34717	35.89634	25.53524	20.87773	51.93913	38.02115	99.51578	63.46985
	Worst	14.97499	15.4836	16.01574	19.82398	15.20538	16.70867	43.43102	32.44838	25.05194	162.7197	46.73021	128.9644	137.425
	Std	2.506594	2.464445	2.376962	0.43465	2.487614	2.288612	3.637903	4.087745	1.999736	49.54801	4.310747	15.66338	37.53049
	Median	12.95425	13.41211	13.9718	19.30539	13.16162	14.71622	38.80603	29.20946	22.43784	112.6011	44.93097	116.8218	123.6959
	Rank	1	3	4	6	2	5	9	8	7	11	10	13	12
C11-F22	Mean	16.12513	16.58732	17.43416	21.06867	16.33467	17.90737	46.79181	34.00153	25.2922	111.7436	49.91934	116.2026	100.6545
	Best	11.50133	12.23683	12.84301	17.27831	11.83377	13.43906	40.60225	25.89615	24.07836	71.91816	41.07087	97.62745	99.72026
	Worst	19.55286	19.81785	20.8036	24.87509	19.67374	21.17591	51.45752	39.30212	26.20769	132.3202	60.00375	128.152	102.0605
	Std	4.36122	4.089225	4.36193	3.566982	4.238879	4.068777	5.337981	6.715222	1.102373	29.69358	8.479631	15.15482	1.127333
	Median	16.72317	17.14729	18.04502	21.06064	16.91559	18.50726	47.55373	35.40392	25.44138	121.368	49.30137	119.5155	100.4187
	Rank	1	3	4	6	2	5	9	8	7	12	10	13	11
Sum rank		25	62	111	168	45	118	183	158	147	241	201	222	237
Mean rank		1.136364	2.818182	5.045455	7.636364	2.045455	5.363636	8.318182	7.181818	6.681818	10.95455	9.136364	10.09091	10.77273
Total rank		1	3	4	8	2	5	9	7	6	13	10	11	12
Wilcoxon: p-value			1.68E−15	8.56E−15	1.50E−15	6.22E−15	3.21E−15	1.58E−15	9.00E−15	1.58E−15	6.54E−15	3.37E−15	1.58E−15	3.68E−12

Table 10. Optimization results of CEC 2011 test suite.

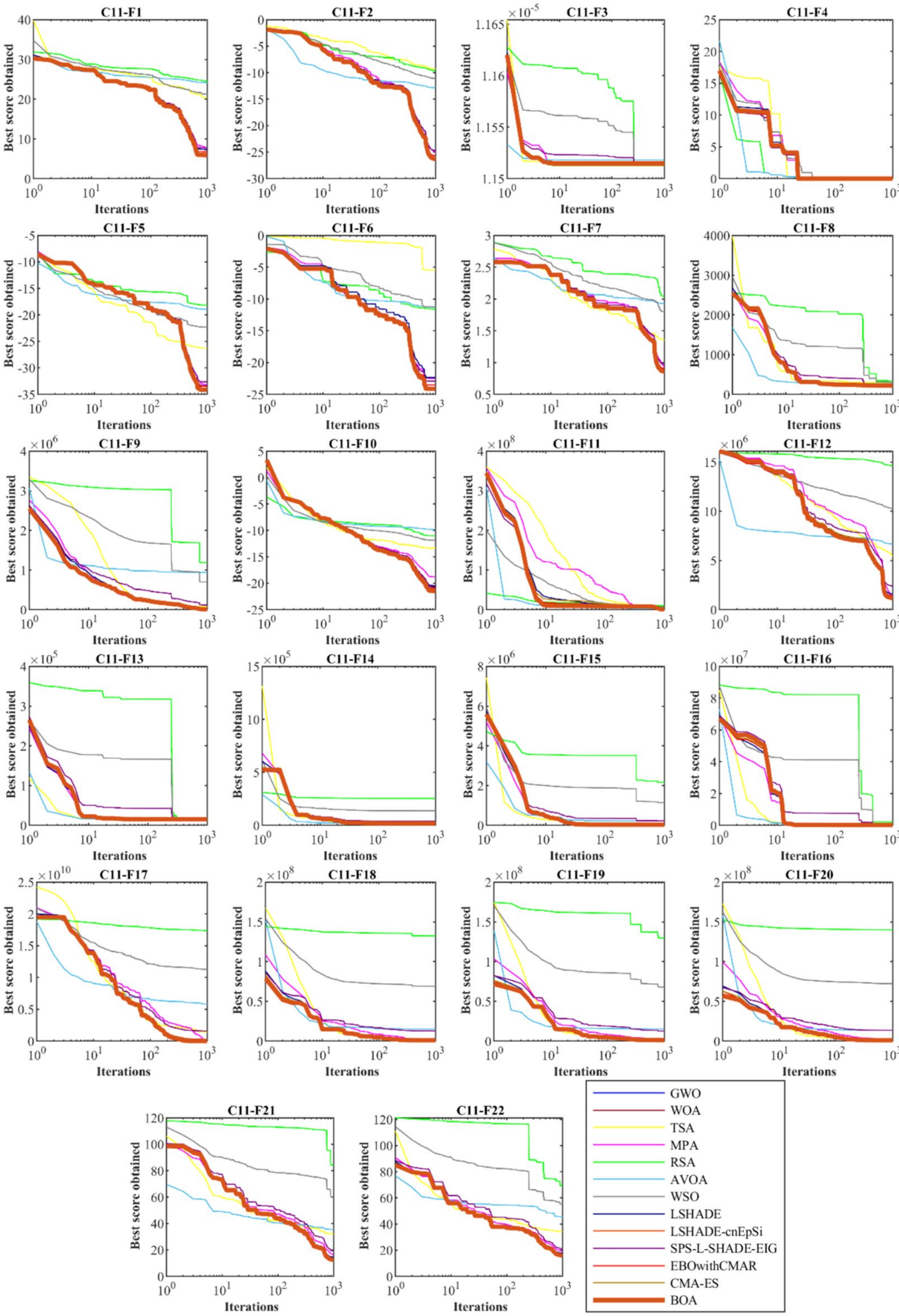

Figure 10. Convergence curves of BOA and competitor algorithms performances on CEC 2011 test suite.

superiority in solving diverse and intricate optimization scenarios. Consequently, BOA's outstanding performance validates its effectiveness as a powerful optimizer for complex numerical problems.

CEC 2011 constrained optimization problems

In order to evaluate the efficiency of BOA to deal with real-world applications, CEC 2011 test suite has been used, which has twenty-two constrained optimization problems. The findings show that BOA, by maintaining the balance between exploration and exploitation, has provided an effective framework for dealing with real-world

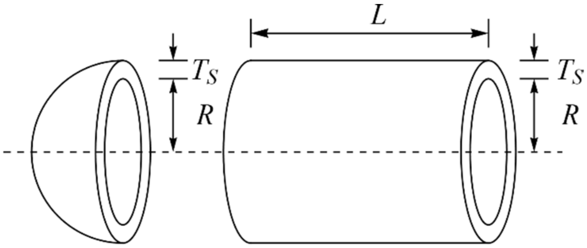

Figure 11. Schematics of the pressure vessel design[103].

Algorithm	Optimum variables				Optimum cost
	T_s	T_h	R	L	
BOA	0.7780271	0.3845792	40.312284	200	5882.8955
CMA-ES	0.7780271	0.3845792	40.312284	200	5882.9013
EBOwithCMAR	0.7780276	0.3845794	40.312309	199.99965	5882.9022
SPS_L_SHADE_EIG	0.8320098	0.4176948	42.932141	180.32644	6125.8849
LSHADE_cnEpSi	0.7780271	0.3845792	40.312284	200	5882.9013
LSHADE	0.7782177	0.3847397	40.321894	200	5886.3286
WSO	0.7780271	0.3845792	40.312284	200	5882.9013
AVOA	0.7780305	0.3845809	40.312462	199.99752	5882.9072
RSA	1.1637964	0.6212285	59.034202	59.409618	7619.3008
MPA	0.7780271	0.3845792	40.312284	200	5882.9013
TSA	0.7793892	0.3857262	40.380959	200	5907.3928
WOA	0.9015228	0.446094	45.787672	138.79276	6241.816
GWO	0.7784275	0.3857204	40.320003	199.96709	5889.6633

Table 11. Performance of optimization algorithms on pressure vessel design problem.

Algorithm	Mean	Best	Worst	Std	Median	Rank
BOA	5882.8955	5882.8955	5882.8955	9.83E-14	5882.8955	1
CMA-ES	5883.979	5882.9013	5893.5344	0.1514476	5882.9014	4
EBOwithCMAR	5926.4819	5882.9022	6033.5159	2.4017279	5904.2353	6
SPS_L_SHADE_EIG	6727.8363	6125.8849	7709.4146	21.314541	6597.5689	11
LSHADE_cnEpSi	5882.9013	5882.9013	5882.9013	3.71E-08	5882.9013	2
LSHADE	5933.1611	5886.3286	6020.8365	2.2708573	5916.653	7
WSO	5890.6028	5882.9013	5958.8872	1.0822684	5882.9017	5
AVOA	6194.3351	5882.9072	6959.217	17.163132	6035.3576	9
RSA	11,920.942	7619.3008	18,935.458	152.31712	10,990.03	13
MPA	5882.9013	5882.9013	5882.9013	2.274E-07	5882.9013	3
TSA	6242.0657	5907.3928	6868.6083	16.22791	6124.096	10
WOA	7840.235	6241.816	12,287.111	81.924031	7452.6013	12
GWO	6002.6746	5889.6633	6611.9911	11.660298	5897.3778	8

Table 12. Statistical results of optimization algorithms on pressure vessel design problem.

applications from the CEC 2011 test suite. Also, in the competition with the compared algorithms, BOA has successfully managed 90.90% of these problems, demonstrating its robustness in dealing with real-world constraints.

Engineering design problems

In addition to the CEC 2011 test suite, the efficiency of BOA to address four engineering design challenges has also been evaluated. These design challenges are: pressure vessel design, speed reducer design, welded beam design, and tension/compression spring design. BOA has been achieved a 100% success rate in the selected engineering design problems, underscoring its practical applicability and effectiveness.

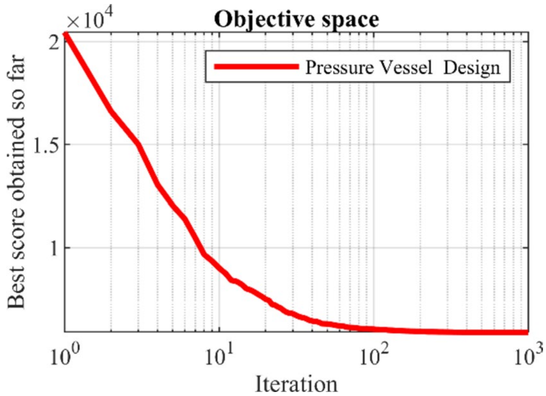

Figure 12. Convergence curves of BOA on the pressure vessel design.

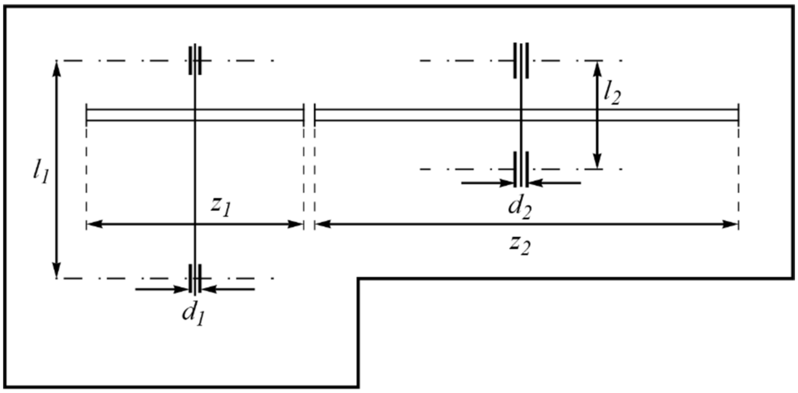

Figure 13. Schematics of the speed reducer design[104,105].

Algorithm	Optimum variables							Optimum cost
	b	*M*	*p*	l_1	l_2	d_1	d_2	
BOA	3.5	0.7	17	7.3	7.8	3.3502147	5.2866832	2996.3482
CMA-ES	3.5000001	0.7	17	7.3000011	7.8	3.3502147	5.2866832	2996.3482
EBOwithCMAR	3.5	0.7	17	7.3000001	7.8	3.3502147	5.2866832	2996.3482
SPS_L_SHADE_EIG	3.5105122	0.7	17	7.4051215	7.8525608	3.3508361	5.3091074	3017.617
LSHADE_cnEpSi	3.5	0.7	17	7.3	7.8	3.3502147	5.2866832	2996.3482
LSHADE	3.5014711	0.7	17	7.3	7.8525608	3.3502518	5.2870862	2998.3472
WSO	3.5000004	0.7	17	7.3000081	7.8000003	3.3502148	5.2866833	2996.3483
AVOA	3.5	0.7	17	7.3000006	7.8	3.3502147	5.2866832	2996.3482
RSA	3.5751215	0.7	17	8.0512152	8.1756076	3.3546557	5.44693	3148.3385
MPA	3.5	0.7	17	7.3	7.8	3.3502147	5.2866832	2996.3482
TSA	3.5105126	0.7	17	7.3	8.1756076	3.3504803	5.2895627	3010.6338
WOA	3.5712923	0.7	17	7.3	7.9706109	3.3595034	5.2867424	3030.4996
GWO	3.5005226	0.7	17	7.3041919	7.8	3.3614073	5.2884166	3000.5584

Table 13. Performance of optimization algorithms on speed reducer design problem.

Sustainable lot size optimization

The capability of the proposed BOA approach has been evaluated in SCM applications to deal with sustainable lot size optimization. Findings show that BOA is very efficient to solve this challenge of SCM applications. What is evident from the analysis of simulation results, BOA with superior performance in all ten study cases has been 100% successful in dealing with sustainable lot size optimization in competition with compared algorithms.

Algorithm	Mean	Best	Worst	Std	Median	Rank
BOA	2996.3482	2996.3482	2996.3482	4.91E-14	2996.3482	1
CMA-ES	2996.3805	2996.3482	2996.6277	0.003512	2996.35	4
EBOwithCMAR	2996.8623	2996.3482	2998.0278	0.023828	2996.8509	6
SPS_L_SHADE_EIG	3028.3327	3017.617	3034.983	0.3453842	3030.0295	11
LSHADE_cnEpSi	2996.3482	2996.3482	2996.3482	2.82E-08	2996.3482	2
LSHADE	3000.4295	2998.3472	3001.9956	0.0608894	3000.6334	8
WSO	2996.5792	2996.3483	2998.3459	0.0250976	2996.3614	5
AVOA	3000.0222	2996.3482	3008.3513	0.1702786	2999.9411	7
RSA	3224.9145	3148.3385	3272.4387	2.4681705	3237.0405	13
MPA	2996.3482	2996.3482	2996.3482	1.733E-07	2996.3482	3
TSA	3025.514	3010.6338	3036.7053	0.4351255	3026.9712	10
WOA	3121.6245	3030.4996	3362.1108	4.5615672	3094.4525	12
GWO	3003.0913	3000.5584	3007.9529	0.107595	3002.6692	9

Table 14. Statistical results of optimization algorithms on speed reducer design problem.

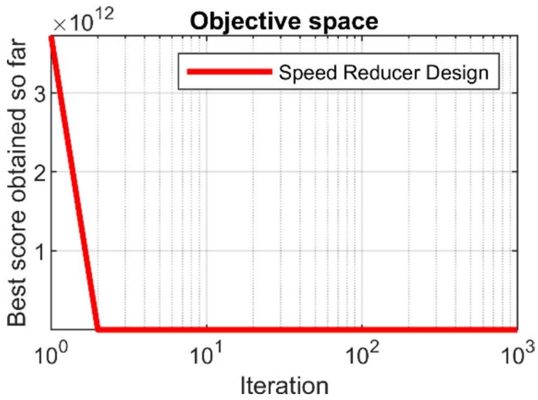

Figure 14. Convergence curves of BOA on the speed reducer design.

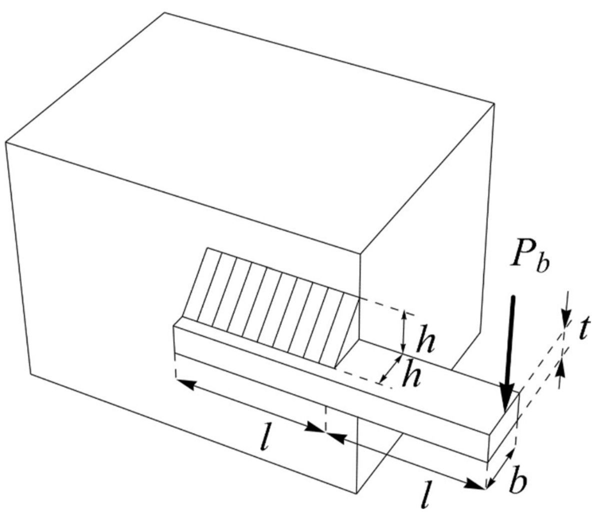

Figure 15. Schematics of the welded beam design[95].

Algorithm	Optimum variables				Optimum cost
	h	*l*	*t*	*b*	
BOA	0.2057296	3.4704887	9.0366239	0.2057296	1.7246798
CMA-ES	0.2057296	3.4704887	9.0366239	0.2057296	1.7248523
EBOwithCMAR	0.2056424	3.4723799	9.0366117	0.2057302	1.724974
SPS_L_SHADE_EIG	0.2046995	3.477807	9.1378955	0.2071056	1.753423
LSHADE_cnEpSi	0.2057296	3.4704887	9.0366239	0.2057296	1.7248523
LSHADE	0.2055548	3.4733263	9.0397667	0.2057783	1.7258775
WSO	0.2057296	3.4704887	9.0366239	0.2057296	1.7248523
AVOA	0.2051064	3.4840038	9.036537	0.2057336	1.7257219
RSA	0.1983682	3.5227869	9.7603267	0.2155622	1.9290231
MPA	0.2057296	3.4704887	9.0366239	0.2057296	1.7248523
TSA	0.2044799	3.490767	9.0590825	0.2060773	1.7321784
WOA	0.2122464	3.3558135	8.9854546	0.2181687	1.8034458
GWO	0.2056175	3.4730604	9.0363112	0.2057859	1.7253992

Table 15. Performance of optimization algorithms on welded beam design problem.

Algorithm	Mean	Best	Worst	Std	Median	Rank
BOA	1.7246798	1.7246798	1.7246798	1.20E-17	1.7246798	1
CMA-ES	1.7248523	1.7248523	1.7248529	7.509E-09	1.7248523	4
EBOwithCMAR	1.7289972	1.724974	1.7382836	0.0002189	1.7274137	8
SPS_L_SHADE_EIG	1.7769188	1.753423	1.8165447	0.0008651	1.7740648	11
LSHADE_cnEpSi	1.7248523	1.7248523	1.7248523	2.97E-11	1.7248523	2
LSHADE	1.7269379	1.7258775	1.7279858	3.364E-05	1.7269489	7
WSO	1.7248526	1.7248523	1.7248568	5.365E-08	1.7248523	5
AVOA	1.754472	1.7257219	1.8208342	0.0015641	1.7431561	10
RSA	2.0969278	1.9290231	2.380101	0.0061821	2.0765327	12
MPA	1.7248523	1.7248523	1.7248523	1.823E-10	1.7248523	3
TSA	1.7397565	1.7321784	1.7472446	0.0002404	1.739835	9
WOA	2.2020843	1.8034458	3.6157276	0.0275223	2.0188504	13
GWO	1.7268075	1.7253992	1.7301016	5.845E-05	1.7266077	6

Table 16. Statistical results of optimization algorithms on welded beam design problem.

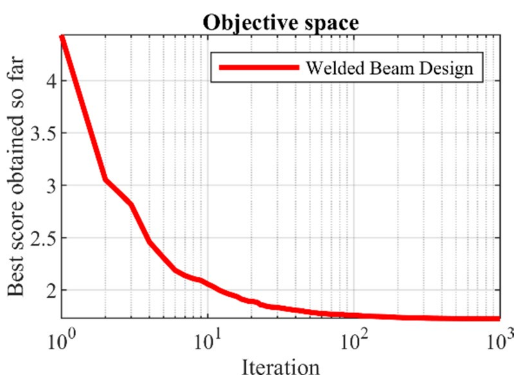

Figure 16. Convergence curves of BOA on the welded beam design.

The proposed BOA offers several advantages for addressing global optimization problems. One notable benefit of BOA is its parameter-free design, which means there are no control parameters to adjust or fine-tune, simplifying its implementation and reducing the potential for human error during setup. Another key advantage of BOA is its high efficiency in tackling a wide array of optimization problems across various scientific disciplines. It is particularly effective when dealing with complex, high-dimensional challenges, making it a versatile tool for many types of optimization scenarios. Additionally, BOA excels in balancing exploration and exploitation

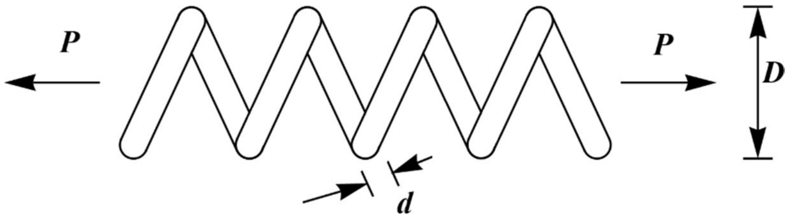

Figure 17. Schematics of the tension/compression spring design[95].

Algorithm	Optimum variables			Optimum cost
	d	*D*	*P*	
BOA	0.0516891	0.3567177	11.288966	0.0126019
CMA-ES	0.0516888	0.3567126	11.289268	0.0126652
EBOwithCMAR	0.0516323	0.3553671	11.37253	0.0126658
SPS_L_SHADE_EIG	0.0515113	0.3518632	11.679323	0.0127214
LSHADE_cnEpSi	0.0516893	0.3567246	11.288566	0.0126652
LSHADE	0.0516093	0.3548256	11.409529	0.0126671
WSO	0.0516875	0.3566801	11.291173	0.0126652
AVOA	0.0512837	0.3470699	11.885893	0.0126693
RSA	0.0504195	0.3220395	14.077742	0.0130666
MPA	0.0516908	0.3567596	11.286511	0.0126652
TSA	0.051119	0.3431896	12.150935	0.0126789
WOA	0.0512632	0.3465879	11.917489	0.0126697
GWO	0.0519059	0.3619489	10.993905	0.0126697

Table 17. Performance of optimization algorithms on tension/compression spring design problem.

Algorithm	Mean	Best	Worst	Std	Median	Rank
BOA	0.0126019	0.0126019	0.0126019	3.62E−19	0.0126019	1
CMA-ES	0.0126665	0.0126652	0.0126833	2.123E−07	0.0126653	4
EBOwithCMAR	0.0127415	0.0126658	0.0128326	3.301E−06	0.0127338	9
SPS_L_SHADE_EIG	0.0127306	0.0127214	0.0127468	4.109E−07	0.0127282	8
LSHADE_cnEpSi	0.0126652	0.0126652	0.0126652	2.49E−11	0.0126652	2
LSHADE	0.0126987	0.0126671	0.0127621	1.431E−06	0.0126904	6
WSO	0.0126742	0.0126652	0.0127945	1.517E−06	0.0126656	5
AVOA	0.01321	0.0126693	0.013861	2.359E−05	0.0131551	13
RSA	0.0131323	0.0130666	0.013248	2.936E−06	0.0131154	11
MPA	0.0126652	0.0126652	0.0126652	1.528E−10	0.0126652	3
TSA	0.0129041	0.0126789	0.0133573	1.022E−05	0.0128449	10
WOA	0.0131533	0.0126697	0.0141393	2.557E−05	0.0129939	12
GWO	0.0127116	0.0126697	0.0128911	2.34E−06	0.0127096	7

Table 18. Statistical results of optimization algorithms on tension/compression spring design problem.

within the search process. This balance enables the algorithm to converge rapidly, effectively identifying suitable values for decision variables. This capability is especially beneficial in complex optimization tasks, where swift and accurate convergence is crucial. Moreover, BOA demonstrates robust performance in real-world optimization applications. Whether applied to theoretical problems or practical, real-world scenarios, BOA consistently delivers reliable and powerful results, making it a valuable tool across various industries and research fields.

However, despite these advantages, BOA has several disadvantages and specific shortcomings. Firstly, like all metaheuristic algorithms, BOA's performance is based on random search principles, and thus, there is no guarantee of achieving the global optimum. This inherent uncertainty is a common drawback of metaheuristic approaches. Secondly, according to the No Free Lunch (NFL) theorem, it cannot be claimed that BOA is the best optimizer for all optimization applications. The performance of any optimization algorithm is problem-dependent, and no single algorithm excels in every scenario. Lastly, there is always the possibility that newer metaheuristic algorithms will be developed by researchers that outperform BOA. The field of optimization is

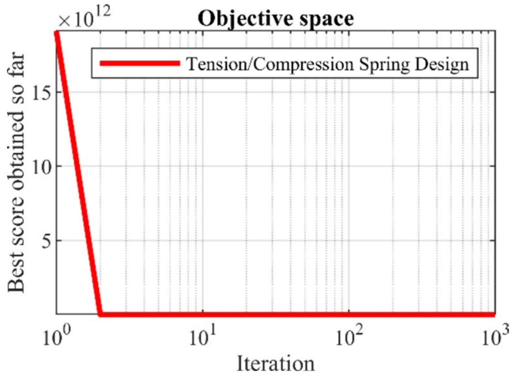

Figure 18. Convergence curves of BOA on the tension/compression spring design.

continually evolving, and advancements may lead to the creation of more efficient algorithms in the future, potentially surpassing the capabilities of BOA.

Open research questions (ORQ)

In this paper, a new metaheuristic algorithm called Bobcat Optimization Algorithm (BOA) is introduced and designed to deal with optimization tasks in various sciences. The efficiency of BOA has been evaluated to solve CEC 2017 tests suite, CEC 2011 test suite, four engineering design problems, and sustainable lot size optimization. While BOA has successfully achieved good results in these implementations and has provided superior performance in competition with the compared algorithms, several open research questions (ORQ) are raised as follows:

- ORQ 1: How can the Bobcat Optimization Algorithm (BOA) be further improved to handle even more complex and higher-dimensional optimization problems?
- ORQ 2: What are the limitations of BOA when applied to different types of real-world problems outside of the current test suites and engineering design problems?
- ORQ 3: How does BOA perform in dynamic and uncertain environments where problem parameters change over time?
- ORQ 4: What are the comparative advantages and disadvantages of BOA compared to other state-of-the-art bio-inspired algorithms in terms of computational efficiency and solution quality?
- ORQ 5: How can BOA be integrated into decision-support systems for real-time optimization in industrial applications?
- ORQ 6: Can the principles of BOA be extended or modified to create new variants of the algorithm that are tailored for specific problem domains?
- ORQ 7: How does BOA perform when applied to multi-objective optimization problems, and what modifications are necessary to handle multiple conflicting objectives effectively?
- ORQ 8: What is the impact of different initialization strategies on the performance and convergence rate of BOA?
- ORQ 9: How can BOA be adapted to improve its scalability for large-scale optimization problems involving thousands of variables and constraints?
- ORQ 10: How does BOA handle noisy and imprecise data in optimization problems, and what techniques can be integrated to improve its robustness in such conditions?
- ORQ 11: What are the impacts of different fitness landscape characteristics on the performance of BOA, and how can it be tuned to perform better on various types of landscapes?
- ORQ 12: How can BOA be combined with machine learning models to predict optimal solutions or guide the search process more intelligently?
- ORQ 13: How does BOA perform in comparison to other bio-inspired algorithms in terms of adaptability to different problem domains and flexibility in handling constraints?
- ORQ 14: How can BOA be extended or modified to incorporate learning mechanisms that adaptively improve its performance over time based on previous optimization experiences?
- ORQ 15: What are the implications of using BOA in real-time optimization scenarios where quick decision-making is critical, and how can its response time be optimized?

By addressing these open research questions, researchers can further expand the understanding and application of the Bobcat Optimization Algorithm (BOA), pushing the boundaries of its capabilities and effectiveness in solving a wide range of optimization problems.

		BOA	**CMA-ES**	**EBOwithCMAR**	**SPS_L_SHADE_EIG**	**LSHADE_cnEpSi**	**LSHADE**	**WSO**	**AVOA**	**RSA**	**MPA**	**TSA**	**WOA**	**GWO**
Part 1	Mean	129,905.4	129,919	129,914.2	129,914.2	129,914.2	129,914.2	129,952.8	129,952.8	129,952.8	129,952.8	129,952.8	129,952.8	129,952.8
	Best	129,905.4	129,906	129,905.8	129,905.8	129,905.8	129,905.8	129,907.6	129,907.6	129,907.6	129,907.6	129,907.6	129,907.6	129,907.6
	Worst	129,905.4	129,950	129,934.2	129,934.2	129,934.2	129,934.2	130,061.1	130,061.1	130,061.1	130,061.1	130,061.1	130,061.1	130,061.1
	Std	2.99E−11	13.41759	8.667081	8.667081	8.667081	8.667081	46.86961	46.86961	46.86961	46.86961	46.86961	46.86961	46.86961
	Median	129,905.4	129,913.7	129,910.8	129,910.8	129,910.8	129,910.8	129,934.4	129,934.4	129,934.4	129,934.4	129,934.4	129,934.4	129,934.4
	Rank	1	3	2	2	2	2	4	4	4	4	4	4	4
Part 2	Mean	14,450.67	14,452.04	14,451.56	14,451.56	14,451.56	14,451.56	14,455.46	14,455.46	14,455.46	14,455.46	14,455.46	14,455.46	14,455.46
	Best	14,450.67	14,450.68	14,450.68	14,450.68	14,450.68	14,450.68	14,450.71	14,450.71	14,450.71	14,450.71	14,450.71	14,450.71	14,450.71
	Worst	14,450.67	14,455.72	14,453.94	14,453.94	14,453.94	14,453.94	14,468.32	14,468.32	14,468.32	14,468.32	14,468.32	14,468.32	14,468.32
	Std	3.73E−12	1.485459	0.959531	0.959531	0.959531	0.959531	5.188926	5.188926	5.188926	5.188926	5.188926	5.188926	5.188926
	Median	14,450.67	14,451.54	14,451.23	14,451.23	14,451.23	14,451.23	14,453.7	14,453.7	14,453.7	14,453.7	14,453.7	14,453.7	14,453.7
	Rank	1	3	2	2	2	2	4	4	4	4	4	4	4
Part 3	Mean	111,778.3	111,778.3	111,796.2	111,814	111,778.3	111,778.3	111,778.3	111,797.4	111,816.4	111,778.3	111,778.3	111,778.3	111,778.3
	Best	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3	111,778.3
	Worst	111,778.3	111,778.3	111,906.6	112,034.9	111,778.3	111,778.3	111,778.3	111,915.4	112,052.4	111,778.3	111,778.3	111,778.3	111,778.3
	Std	1.33E−10	6.94E−05	35.16629	70.33257	4.48E−05	4.56E−05	0.000242	37.5708	75.14148	0.000242	0.000243	0.000242	0.000243
	Median	111,778.3	111,778.3	111,783.5	111,788.6	111,778.3	111,778.3	111,778.3	111,783.8	111,789.3	111,778.3	111,778.3	111,778.3	111,778.3
	Rank	1	4	9	11	2	3	5	10	12	5	8	6	7
Part 4	Mean	124,853.9	124,855.9	124,855.2	124,855.2	124,855.2	124,855.2	124,860.8	124,860.8	124,860.8	124,860.8	124,860.8	124,860.8	124,860.8
	Best	124,853.9	124,854.1	124,854	124,854	124,854	124,854	124,854.6	124,854.6	124,854.6	124,854.6	124,854.6	124,854.6	124,854.6
	Worst	124,853.9	124,860.3	124,858	124,858	124,858	124,858	124,876	124,876	124,876	124,876	124,876	124,876	124,876
	Std	0	2.054932	1.327381	1.327381	1.327381	1.327381	7.178177	7.178177	7.178177	7.178177	7.178177	7.178177	7.178177
	Median	124,853.9	124,855	124,854.6	124,854.6	124,854.6	124,854.6	124,857.5	124,857.5	124,857.5	124,857.5	124,857.5	124,857.5	124,857.5
	Rank	1	3	2	2	2	2	4	4	4	4	4	4	4
Part 5	Mean	120,571.8	120,585.3	120,580.5	120,580.5	120,580.5	120,580.5	120,618.8	120,618.8	120,618.8	120,618.8	120,618.8	120,618.8	120,618.8
	Best	120,571.8	120,572	120,572	120,572	120,572	120,572	120,572.6	120,572.6	120,572.6	120,572.6	120,572.6	120,572.6	120,572.6
	Worst	120,571.8	120,634.9	120,612.6	120,612.6	120,612.6	120,612.6	120,792.1	120,792.1	120,792.1	120,792.1	120,792.1	120,792.1	120,792.1
	Std	4.48E−11	16.58916	10.71575	10.71575	10.71575	10.71575	57.94835	57.94835	57.94835	57.94835	57.94835	57.94835	57.94835
	Median	120,571.8	120,578.6	120,576.2	120,576.2	120,576.2	120,576.2	120,595.7	120,595.7	120,595.7	120,595.7	120,595.7	120,595.7	120,595.7
	Rank	1	3	2	2	2	2	4	4	4	4	4	4	4
Part 6	Mean	287,556.1	287,583.9	287,574	287,574	287,574	287,574	287,653	287,653	287,653	287,653	287,653	287,653	287,653
	Best	287,556.1	287,557	287,556.7	287,556.7	287,556.7	287,556.7	287,559.2	287,559.2	287,559.2	287,559.2	287,559.2	287,559.2	287,559.2
	Worst	287,556.1	287,646.6	287,614.6	287,614.6	287,614.6	287,614.6	287,872.2	287,872.2	287,872.2	287,872.2	287,872.2	287,872.2	287,872.2
	Std	0	22.96211	14.83235	14.83235	14.83235	14.83235	80.21001	80.21001	80.21001	80.21001	80.21001	80.21001	80.21001
	Median	287,556.1	287,581.4	287,572.5	287,572.5	287,572.5	287,572.5	287,644.5	287,644.5	287,644.5	287,644.5	287,644.5	287,644.5	287,644.5
	Rank	1	3	2	2	2	2	4	4	4	4	4	4	4
Part 7	Mean	128,804.9	128,804.9	128,830.7	128,856.6	128,804.9	128,804.9	128,804.9	128,832.5	128,860.1	128,804.9	128,804.9	128,804.9	128,804.9
	Best	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9	128,804.9
	Worst	128,804.9	128,804.9	128,864.6	128,924.2	128,804.9	128,804.9	128,804.9	128,868.7	128,932.4	128,804.9	128,804.9	128,804.9	128,804.9
	Std	2.01E−10	0.001365	25.11465	50.22921	0.000882	0.000882	0.00477	26.83216	53.66387	0.00477	0.004769	0.00477	0.004772
	Median	128,804.9	128,804.9	128,823.5	128,842.1	128,804.9	128,804.9	128,804.9	128,824.7	128,844.6	128,804.9	128,804.9	128,804.9	128,804.9
	Rank	1	4	8	10	2	3	5	9	11	5	7	5	6
Part 8	Mean	20,368.81	20,370.41	20,369.84	20,369.84	20,369.84	20,369.84	20,374.4	20,374.4	20,374.4	20,374.4	20,374.4	20,374.4	20,374.4
	Best	20,368.81	20,368.92	20,368.88	20,368.88	20,368.88	20,368.88	20,369.21	20,369.21	20,369.21	20,369.21	20,369.21	20,369.21	20,369.21
	Worst	20,368.81	20,373.6	20,371.91	20,371.91	20,371.91	20,371.91	20,385.56	20,385.56	20,385.56	20,385.56	20,385.56	20,385.56	20,385.56
	Std	3.73E−12	1.117506	0.721852	0.721852	0.721852	0.721852	3.903613	3.903613	3.903613	3.903613	3.903613	3.903613	3.903613
	Median	20,368.81	20,370.32	20,369.79	20,369.79	20,369.79	20,369.79	20,374.1	20,374.1	20,374.1	20,374.1	20,374.1	20,374.1	20,374.1
	Rank	1	5	2	3	4	4	6	6	6	6	6	6	6
Part 9	Mean	4366.721	4366.721	4366.74	4366.759	4366.721	4366.721	4366.721	4366.741	4366.762	4366.721	4366.721	4366.721	4366.721
	Best	4366.721	4366.721	4366.721	4366.721	4366.721	4366.721	4366.721	4366.721	4366.721	4366.721	4366.721	4366.721	4366.721
	Worst	4366.721	4366.721	4366.912	4367.104	4366.721	4366.721	4366.721	4366.926	4367.13	4366.721	4366.721	4366.721	4366.721
	Std	2.3E−12	3.64E−12	0.044425	0.08885	9.95E−09	2.1E−08	1.21E−11	0.047462	0.094925	1.23E−11	2.25E−08	1.22E−11	5.38E−08
	Median	4366.721	4366.721	4366.722	4366.724	4366.721	4366.721	4366.721	4366.723	4366.725	4366.721	4366.721	4366.721	4366.721
	Rank	1	2	8	10	4	5	3	9	11	3	6	3	7

Continued

		BOA	CMA-ES	EBOwithCMAR	SPS_L_SHADE_EIG	LSHADE_cnEpSi	LSHADE	WSO	AVOA	RSA	MPA	TSA	WOA	GWO
Part 10	Mean	15,556.91	15,558.26	15,557.78	15,557.78	15,557.78	15,557.78	15,561.61	15,561.61	15,561.61	15,561.61	15,561.61	15,561.61	15,561.61
	Best	15,556.91	15,557.02	15,556.98	15,556.98	15,556.98	15,556.98	15,557.28	15,557.28	15,557.28	15,557.28	15,557.28	15,557.28	15,557.28
	Worst	15,556.91	15,560.59	15,559.29	15,559.29	15,559.29	15,559.29	15,569.75	15,569.75	15,569.75	15,569.75	15,569.75	15,569.75	15,569.75
	Std	7.46E−12	0.972823	0.628394	0.628394	0.628394	0.628394	3.398214	3.398214	3.398214	3.398214	3.398214	3.398214	3.398214
	Median	15,556.91	15,558.6	15,558	15,558	15,558	15,558	15,562.79	15,562.79	15,562.79	15,562.79	15,562.79	15,562.79	15,562.79
	Rank	1	3	2	2	2	2	4	4	4	4	4	4	4
Sum rank		10	33	39	46	24	27	43	58	64	43	51	44	50
Mean rank		1	3.3	3.9	4.6	2.4	2.7	4.3	5.8	6.4	4.3	5.1	4.4	5
Total rank		1	4	5	8	2	3	6	11	12	6	10	7	9

Table 19. Optimization results of the sustainable lot size optimization.

Conclusion and future works

In this paper, a new biomimetics metaheuristic algorithm named Bobcat Optimization Algorithm (BOA) was introduced, which imitates the behavior of bobcats in nature. The basic inspiration of BOA is derived from the hunting strategy of bobcats, which has two parts: tracking prey and chasing. The theory of BOA was expressed and then mathematically modeled in two phases (i) exploration based on the simulation of the bobcat's position change while moving towards the prey and (ii) exploitation based on simulating the bobcat's position change during the chase process to catch the prey. The performance of BOA in handling optimization applications was evaluated on the CEC 2017 test suite for problem dimensions of 10, 30, 50, and 100, as well as CEC 2020 test suite. The finding of the optimization results was that BOA has a high quality in exploration, exploitation and balance between them during the search process in the problem solving space. The performance of BOA was compared with the performance of twelve competitor metaheuristic algorithms. The finding from the simulation results was that BOA has provided superior performance compared to competitor algorithms, achieving better results for most of the benchmark functions and being ranked as the first best optimizer overall. The finding obtained from the statistical analysis was that BOA has a significant statistical superiority over the corresponding compared algorithms. The performance of BOA and competitor algorithms to address real-world applications was evaluated on twenty-two constrained optimization problems from the CEC 2011 test suite and four engineering design problems. The finding of this implementation was that BOA has an effective performance to tackle optimization tasks in real-world applications.

Following the introduction of BOA, several research suggestions for future work are presented which are listed below:

- *Multi-objective version development:* Developing a multi-objective version of BOA and using it to address multi-objective optimization applications is one of the key research potentials of this paper.
- *Binary version development:* The design of the binary version of BOA makes it suitable for handling optimization applications that require handling using the binary version.
- *Exploring hybrid approaches:* Combining BOA with other optimization techniques or machine learning methods could potentially enhance its performance and applicability to more complex problems.
- *Real-world applications:* Extending the application of BOA to a wider array of real-world problems in diverse industries will help validate its effectiveness and identify any potential limitations.
- *Dynamic optimization:* Adapting BOA for dynamic and uncertain environments where problem parameters evolve over time will be critical for applications in rapidly changing fields.
- *Benchmarking and comparison:* Conducting comprehensive benchmarking studies against other state-of-the-art algorithms to provide a clearer picture of BOA's relative strengths and weaknesses.
- *Integration into decision-support systems:* Developing practical frameworks for integrating BOA into decision-support systems to facilitate real-time optimization in industrial and commercial settings.
- *Algorithm variants:* Creating and testing new variants of BOA tailored for specific problem domains could unlock further potential and specialized applications.
- *Automated parameter tuning:* Investigating and developing automated parameter tuning mechanisms to enhance BOA's performance across different problem instances without extensive manual intervention.

By addressing mentioned open research questions and pursuing the suggested future research directions, the scientific community can continue to refine and enhance the capabilities of the Bobcat Optimization Algorithm and its applications in various fields.

Data availability

All data generated or analyzed during this study are included directly in the text of this submitted manuscript. There are no additional external files with datasets.

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Competing interests

The authors declare no competing interests.

Additional information

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