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Dynamic Pythagorean fuzzy probabilistic linguistic TOPSIS method with psychological preference and its application for COVID-19 vaccination

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ABSTRACT

The probabilistic linguistic term set (PLTS) has been widely used in multiple criteria group decision making (MCGDM) problems where the linguistic information is uncertain and hesitant. To reflect the different preferences and uncertainties, we propose a new PLTS with probability in the form of Pythagorean fuzzy set (PFS), called Pythagorean fuzzy probabilistic linguistic term set (PFPLTS). In addition, considering the information integrity, uncertainty and DMs' preferences, the operation and aggregation operators for PFPLTS are introduced. Then, the weight method based on minimum deviation and dual ideal point-vector projection is proposed, which considers the time-varying characteristics of the weights and combines multi-dimensional influencing factors. Next, the psychological distance measure is proposed by dividing the psychological space into multiple vectors. Based on the proposed dynamic weight method, three psychological distance measures and TOPSIS method, we develop a dynamic Pythagorean fuzzy probabilistic linguistic TOPSIS method with psychological distance (Psy-TOPSIS), the psychological index ranges from 1 to 40. Finally, a practical case, site selecting of COVID-19 vaccination center, is given and compared with three approaches to illustrate the effectiveness and practicality of PFPLTS and the proposed decision-making method.

1. Introduction

Multiple criteria group decision making (MCGDM) is an essential branch in the field of decision making. It refers to the way decision-makers (DMs) apply decision-making methods to select the best alternative with multiple criteria. With the increasing complexity of decision-making problems and their backgrounds, in many cases, DMs cannot accurately quantify the evaluation objects and can only use natural language to evaluate. For example, in the site selection of the COVID-19 vaccination center, DMs may say that "the traffic conditions in this location are not bad, but the transportation and transformation cost is too high." In this context, the words "not bad" and "too high" are described in linguistic terms. Although linguistic terms are intuitive, flexible, and close to people's cognition, they are difficult to calculate. To deal with this problem, Zadeh [1] proposed the fuzzy linguistic approach (FLA), which can aptly describe the fuzziness and uncertainty of information. To further reflect the uncertainty and the preference degree of linguistic terms, Pang [2] proposed the concept of PLTS. Since it was submitted, PLTS has been widely used in medical level assessment [3], supplier selection [4], venture capital [5], and so on. On the other hand, some contributions have been made to the conversion, computation, and aggregation method of PLTS [6–15]. In the follow-up research, many scholars have carried on the related expansion to PLTS. For example, multiple linguistic terms are utilized in the probabilistic uncertain linguistic term set (PULTS) to express the hesitation of evaluation information [16]. The Interval-valued probabilistic linguistic term set (IVPLTS) extends the corresponding probability of linguistic terms to interval values [17]. By adding the unknown probability to linguistic term, the uncertain probabilistic linguistic term set (UPLTS) can be established [18], etc. It is evident that only membership degree is used to describe the importance of linguistic terms in the existing probabilistic linguistic terms, which ignores the uncertainty and preference of DMs about a particular linguistic evaluation.

Dynamic decision-making is a process that changes with time. Regarding the time pressure and affairs unpredictability at different development stages, DMs must be more cautious and effective in dealing with the fuzziness and uncertainty of information. Additionally, it also means that decision information and criteria weights are affected by time. Therefore, it is necessary to determine the time weight and criteria weights of each stage. At present, there are many methods for weight

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determination, mainly divided into three categories: subjective weighting method, objective weighting method, and combination weighting method. Subjective weighting methods mainly include BEM [19], AHP method [20], Delphi method [21], etc., which are obtained by the personal judgment of experts' experience and generally not affected by the attribute value. The advantage is that experts can determine the weights based on actual problems and their knowledge, and there will be no situation that contradicts the actual importance. However, the decision-making or evaluation results are subjective and vulnerable to the lack of decision-makers' knowledge. Because of the poor objectivity, it has great limitations in application. The objective weights mainly include the dispersion maximization method [22], entropy weight method [23], etc., calculated by attribute values. This is usually based on sound mathematical theories and techniques, so the weights are highly reasonable, and the method has a solid mathematical theoretical basis. However, this kind of empowerment method cannot reflect the preferences of DMs, and there may be situations that are contrary to the actual importance.

A combination weighting method was proposed to take into account the advantages of subjective and objective weighting methods. The most common is the linear weighted combination approach, and our personal perception often decides the combination coefficients. But in general, the mathematical theoretical basis of the subjective and objective integrated weighting method is relatively perfect, but the disadvantage lies in the high complexity of the algorithm. At the same time, many criteria weights methods are based on the needs of one or two sides of the DMs, which rarely contain the needs of multiple sides together. In addition, most of the existing weight methods are in a static environment. The criteria weights do not consider time-varying factors, so they cannot reflect the characteristics of dynamic decision-making.

The TOPSIS method is commonly used in MCGDM problems. It has been widely studied and applied, but it is rarely used in dynamic decision-making environments, and does not take into account the psychological changes of decision-makers. We propose a probabilistic linguistic term set that includes membership and non-membership of linguistic terms to overcome the above shortcomings. And on this basis, we have developed a new dynamic multi-criteria weight method that considers time-varying effects and a decision-making method that considers psychological changes. The main contributions of this paper can be summarized as follows:

1 Due to the advantages of PLTS in describing information, and its ignorance in linguistic preference and uncertainty, the corresponding probability is extended to a set that includes both the degree of membership and non-membership. Using the Pythagorean Fuzzy Set (PFS) to appropriately reduce the constraints on the set, we define the term set of Pythagorean probabilistic linguistic term set (PFPLTS). In addition, the basic operation and aggregation operators of PFPLTS are introduced.
2 In the existing criteria weights methods, DMs need to choose one or two angles for calculation, such as hesitation degree, identification degree, and so on. It is rare to consider the influence of multiple angles on the criteria weights, which leads to the inaccuracy of the fusion weight. Therefore, it is necessary to study the weight fusion method from multiple dimensions. In addition, the current weight methods are mostly single-stage and static [24,25], so they can not reflect the DMs' changing information and preferences. The subjective and objective time weight method of minimum deviation linear programming and a weight fusion method of dual ideal point-vector projection in a dynamic environment is proposed to solve these two problems.
3 The DMs' preferences for different alternatives and criteria will change accordingly with the decision-making background and environment. Therefore, we consider the multi-dimensional psychological space of DMs [26], and propose a dynamic psychological distance measurement that includes the psychological changes of DMs.
4 As a widely used multi-criteria decision-making method, the TOPSIS method has attracted extensive research from scholars since it was proposed. Based on the TOPSIS method and the aforementioned methods, a dynamic Pythagorean fuzzy probabilistic linguistic TOPSIS method with psychological preference is constructed and applied to the site selecting of the COVID-19 vaccination center.

The rest of this paper is organized as follows: In Section 2, some basic concepts about PLTS and PFS are briefly reviewed. Section 3 defines the concept of PFPLTS and proposes the related calculation method, operation and aggregation operators of PFPLTS. Section 4 constructs the time and criteria weights method with minimum deviation linear programming and dual ideal-point vector projection, respectively. Section 5 establishes the novel TOPSIS method with psychological distance measure and constructs an approach for MCGDM based on the Pythagorean fuzzy probabilistic linguistic psy-TOPSIS method. Section 6 gives a case study, site selection for the COVID-19 vaccination center, to illustrate the applicability and practicability of the proposed method. Finally, some conclusions are given in Section 7.

2. Preliminaries

In this section, we will briefly review some basic concepts related to PLTS and PFS.

2.1. Probabilistic linguistic term set

Definition 1. [2] Let $S = \{s_\alpha | \alpha = 0, 1, 2, \dots, \tau\}$ be a linguistic term set, then the probabilistic linguistic term set (PLTS) can be defined as $L(p) = \{L^{(k)}(p^{(k)}) | L^{(k)} \in S, p^{(k)} \geq 0, k = 1, 2, \dots, \#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1\}$. Where $L^{(k)}(p^{(k)})$ is the linguistic term $L^{(k)}$ associated with its corresponding probability $p^{(k)}$, and $\#L(p)$ is the cardinality of $L(p)$.

Definition 2. [27] Let $L(p) = \{L^{(k)}(p^{(k)}) | L^{(k)} \in S, p^{(k)} \geq 0, k = 1,2,\dots,\# L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1\}$, $S = \{s_\alpha | \alpha = 0,1,2,\dots,\tau\}$, and $\alpha^{(k)}$ is the subscript of linguistic term $L^{(k)}$. The score function and its inverse function are defined as:

$$
\begin{aligned}
&g : [0, \tau] \to [0, 1], g(L(p)) = \left\{ \left[\frac{\alpha^{(k)}}{\tau} \right] \left(p^{(k)} \right) \right\} = L_\varpi(p), \varpi \in [0, 1], \\
&g^{-1} : [0, 1] \to [0, \tau], g\left(L_\gamma(p)\right) = \left\{ s_{\tau\varpi}\left(p^{(k)}\right) \right\} = L(p)), \varpi \in [0, 1].
\end{aligned}
\tag{1}
$$

Definition 3. [27] Let $L_1(p) = \{{L_1}^{(k)}({p_1}^{(k)}) | k = 1, 2, \dots, \#L_1(p)$, $L_2(p) = \{{L_2}^{(k)}({p_2}^{(k)}) | k = 1, 2, \dots, \#L_2(p)$ and $L_3(p) = \{{L_3}^{(k)}({p_3}^{(k)}) | k = 1, 2, \dots, \#L_3(p)$ be three finite and ordered PLTSs. λ is a positive real number, $\gamma_1^{(k)} \in g(L_1)$, $\gamma_2^{(l)} \in g(L_2)$, $\gamma_3^{(r)} \in g(L_3)$ and $k = 1,2,\dots,\#L_1(p)$, $l = 1,2,\dots,\#L_2(p)$, $r = 1,2,\dots,\#L_3(p)$.

1 $L_1(p) \oplus L_2(p) = g^{-1}(\cup_{\gamma_1^{(k)} \in g(L_1), \gamma_2^{(l)} \in g(L_2)} \{(\gamma_1^{(k)} + \gamma_2^{(l)})(p_1^{(k)} p_2^{(l)})\})$.
2 $L_1(p) \ominus L_2(p) = g^{-1}(\cup_{\gamma_1^{(k)} \in g(L_1), \gamma_2^{(l)} \in g(L_2)} \{\xi(p_1^{(k)} p_2^{(l)})\})$.

Where $\xi = \begin{cases} \dfrac{\gamma_1^{(k)} - \gamma_2^{(l)}}{1 - \gamma_2^{(l)}}, if \gamma_1^{(k)} \geq \gamma_2^{(l)} and \gamma_2^{(l)} \neq 1 \\ 0, otherwise \end{cases}$.

1 $L_1(p) \otimes L_2(p) = g^{-1}(\cup_{\gamma_1^{(k)} \in g(L_1), \gamma_2^{(l)} \in g(L_2)} \{(\gamma_1^{(k)} \gamma_2^{(l)})(p_1^{(k)} p_2^{(l)})\})$.
2 $L_1(p) \oslash L_2(p) = g^{-1}(\cup_{\gamma_1^{(k)} \in g(L_1), \gamma_2^{(l)} \in g(L_2)} \{\zeta(p_1^{(k)} p_2^{(l)})\})$.

Where $\zeta = \begin{cases} \frac{\gamma_1^{(k)}}{\gamma_2^{(l)}}, if \gamma_1^{(k)} \le \gamma_2^{(l)} and \gamma_2^{(l)} \ne 0 \\ 0, otherwise \end{cases}$.

1 $\lambda L_3(p) = g^{-1}(\cup_{\gamma_3^{(r)} \in g(L_3)} \{(1-(1-\gamma_3^{(r)})^{\lambda})(p_1^{(r)})\})$.

2 $L_3^{\lambda}(p) = g^{-1}(\cup_{\gamma_3^{(r)} \in g(L_3)} \{(\gamma_3^{(r)})^{\lambda}(p_1^{(r)})\})$.

3 $L_3^{-1}(p) = g^{-1}(\cup_{\gamma_3^{(r)} \in g(L_3)} \{(1-\gamma_3^{(r)})(p_1^{(r)})\})$.

And the distance measure is defined as:

$$d(L_1(p), L_2(p)) = \frac{1}{2}\left(\sum_{k=1}^{\#L_1(p)} g\left(L_1^{(k)}\right)\left(p_1^{(k)}\right) - \sum_{k=1}^{\#PL_1(p)} g\left(L_2^{(k)}\right)\left(p_2^{(k)}\right)\right)$$

2.2. Pythagorean fuzzy set

Definition 4. [28] Let X be the universe of discourse, the Pythagorean fuzzy set is defined as

$$P = \{ < x, \mu_p(x), \nu_p(x) > | x \in X, 0 \le \mu_p{}^2(x) + \nu_p{}^2(x) \le 1\}$$

Where $\mu_p(x) : X \to [0,1]$ and $\nu_p(x) : X \to [0,1]$ are the degree of membership and non-membership of x belonging to P respectively. For any $x \in X$, the hesitation of x belonging to P is $\pi_p(x) = \sqrt{1-\mu_p{}^2(x) - \nu_p{}^2(x)}$. For convenience, $< \mu_p, \nu_p >$ is called as Pythagorean fuzzy number (PFN), where $\mu_p, \nu_p \in [0,1]$, $\mu_p{}^2 + \nu_p{}^2 \le 1$, it is simply recorded as $P = < \mu_p, \nu_p >$.

3. Pythagoras-probabilistic linguistic term set

In order to describe the fuzziness and uncertainty of DMs, we introduce a new concept called PFPLTS. Then, the related comparison method, basic operation and aggregation operators are proposed.

3.1. The concept and comparation method of PFPLTS

By adding corresponding probability values to linguistic terms, PLTS has significantly progressed in describing uncertain information of hesitant fuzzy linguistic evaluation and comparative preference. But the fuzziness and uncertainty of linguistic terms have not been specifically expressed. With the increase of complexity and uncertainty of decision-making problems, the fuzziness of DMs' thinking, and the limitation of knowledge reserve, the probability value of hesitant linguistic evaluation is not completely certain. PFS introduced by Yager [28,29] on the basis of IFS can make the description of decision information more scientific and effective, because it contains membership and non-membership degrees whose square sum is not more than 1. For the fuzziness and uncertainty of PLTS and Pythagorean fuzzy sets in describing information, the Pythagorean fuzzy probabilistic linguistic term set (PFPLTS) is proposed.

Definition 5. Let X be a non-empty universe of discourse, $S = \{s_\alpha | \alpha = 0, 1, 2, \dots, \tau\}$ is the linguistic term set, a PFPLTS can be defined as follows:

$$\begin{aligned} PL(p) &= \{[x, PL^{(k)}(\tilde{p}^{(k)})] | x \in X, PL^{(k)} \in S, k = 1, 2, \dots, \#PL(p)\} \\ &= \{[x, PL^{(k)}\langle \mu^{(k)}, \nu^{(k)} \rangle] | x \in X, PL^{(k)} \in S, k = 1, 2, \dots, \#PL(p)\} \end{aligned}$$

where $PL^{(k)}(\tilde{p}^{(k)})$ represents the linguistic term $PL^{(k)}$ associated with its uncertain PFS probability $\tilde{p}^{(k)}$, and $\tilde{p}^{(k)} = \langle \mu^{(k)}, \nu^{(k)} \rangle$, in which $\mu^{(k)}$ and $\nu^{(k)}$ represent the membership and non-membership degrees of linguistic term $PL^{(k)}$, $\alpha^{(k)}$ is the subscript of linguistic term $PL^{(k)}$, and $\#PL(p)$ is the cardinal number of $PL(p)$. The hesitation degree can be calculated by $\pi^{(k)} = \sqrt{1-(\mu^{(k)})^2 - (\nu^{(k)})^2}$. We have noticed that the PFPLTS is an extension of the PLTS, when $PL^{(k)} = \langle \mu^{(k)} \rangle$, the PFPLTS degenerates into PLTS.

To compare two different PFPLTSs, the score and accuracy function are given in the following, and then the comparison method is developed.

Definition 6. Let X be a non-empty universe of discourse, and the score function of PFPLTSs on X can be defined as:

$$S(PL(p)) = \sum_{k=1}^{\#PL(p)} g(PL^{(k)}) \times \left[(\mu^{(k)})^2 - (\nu^{(k)})^2\right] \tag{2}$$

The accuracy function of PFPLTSs on X can be defined as:

$$H(PL(p)) = \sum_{k=1}^{\#PL(p)} g(PL^{(k)}) \times \left[(\mu^{(k)})^2 + (\nu^{(k)})^2\right] \tag{3}$$

Definition 7. Let X be a non-empty universe of discourse, for any two PFPLTSs $PL_1(p)$ and $PL_2(p)$.

(1) If $S(PL_1(p)) > S(PL_2(p))$, then $PL_1(p) \succ PL_2(p)$.
(2) If $S(PL_1(p)) < S(PL_2(p))$, then $PL_1(p) \prec PL_2(p)$.
(3) If $S(PL_1(p)) = S(PL_2(p))$, then:
  a If $H(PL_1(p)) > H(PL_2(p))$, then $PL_1(p) \succ PL_2(p)$.
  b If $H(PL_1(p)) < H(PL_2(p))$, then $PL_1(p) \prec PL_2(p)$.
  c If $H(PL_1(p)) = H(PL_2(p))$, then $PL_1(p) \sim PL_2(p)$.

Definition 8. Let X be a non-empty universe of discourse, give a PFPLTS $PL(p) = \{[x, PL^{(k)}\langle \mu^{(k)}, \nu^{(k)} \rangle] | x \in X, PL^{(k)} \in S, k = 1, 2, \dots, \#PL(p)\}$, and $\alpha^{(k)}$ is the subscript of linguistic term $PL^{(k)}$, $PL(p)$ is called an ordered PFPLTS, if the elements (PFPLEs) $PL^{(k)}(\tilde{p}^{(k)})$ in PFPLTS are sorted by the values of $S(PL^{(k)}(\tilde{p}^{(k)}))(k = 1, 2, \dots, \#PL(P))$ in ascending order.

3.2. Some basic operation for PFPLTS

With the introduction of PFPLTS, it is crucial to find the basic operation. Assume that all PFPLTSs are ordered and finite, then some basic operations are proposed as bellow:

Definition 9. Let $PL_1(p) = \{[x, PL_1{}^{(k)}\langle \mu_1{}^{(k)}, \nu_1{}^{(k)} \rangle] | x \in X, k = 1, 2, \dots, \#PL_1(p)\}$ and $PL_2(p) = \{[x, PL_2{}^{(l)}\langle \mu_2{}^{(l)}, \nu_2{}^{(l)} \rangle] | x \in X, l = 1, 2, \dots, \#PL_2(p)\}$ be any two PFPLTSs, then the distance measure is as follows:

$$\begin{aligned} d(PL_1(p), PL_2(p)) = \frac{1}{2}(& \left| \sum_{k=1}^{\#PL_1(p)} g\left(PL_1^{(k)}\right)\left(\mu_1^{(k)}\right)^2 - \sum_{k=1}^{\#PL_1(p)} g\left(PL_2^{(k)}\right)\left(\mu_2^{(k)}\right)^2 \right| \\ & + \left| \sum_{k=1}^{\#PL_1(p)} g\left(PL_1^{(k)}\right)\left(\nu_1^{(k)}\right)^2 - \sum_{k=1}^{\#PL_1(p)} g\left(PL_2^{(k)}\right)\left(\nu_2^{(k)}\right)^2 \right|) \end{aligned} \tag{4}$$

Definition 10. Let $PL_1(p)=\{[x,PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle]|x\in X,k=1,2,\ldots,\#PL_1(p)\}$ and $PL_2(p)=\{[x,PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle]|x\in X,l=1,2,\ldots,\#PL_2(p)\}$ be any two PFPLTSs, then some basic operations are defined as follows:

(1) $PL_1(p)\oplus PL_2(p) = g^{-1}\begin{pmatrix}\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p)}\\ \{(\gamma_1^{(k)}+\gamma_2^{(l)}-\gamma_1^{(k)}\gamma_2^{(l)})\langle\mu_1^{(k)}\mu_2^{(l)},\nu_1^{(k)}\nu_2^{(l)}\rangle\}\end{pmatrix}.$

(2) $PL_1(p)\ominus PL_2(p) = g^{-1}\begin{pmatrix}\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p)}\\ \{\xi\langle\mu_1^{(k)}\mu_2^{(l)},\nu_1^{(k)}\nu_2^{(l)}\rangle\}\end{pmatrix},$

where $\xi = \begin{cases}\dfrac{\gamma_1^{(k)}-\gamma_2^{(l)}}{1-\gamma_2^{(l)}}, if\gamma_1^{(k)}>\gamma_2^{(l)},\gamma_2^{(l)}\neq 0\\ 0, otherwise\end{cases}.$

(3) $PL_1(p)\otimes PL_2(p) = g^{-1}\begin{pmatrix}\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p)}\\ \{\gamma_1^{(k)}\gamma_2^{(l)}\langle\mu_1^{(k)}\mu_2^{(l)},\nu_1^{(k)}\nu_2^{(l)}\rangle\}\end{pmatrix}.$

(4) $PL_1(p)\oslash PL_2(p) = g^{-1}\begin{pmatrix}\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(k)},\nu_2^{(l)}\rangle\in PL_2(p)}\\ \{\zeta\langle\mu_1^{(k)}\mu_2^{(l)},\nu_1^{(k)}\nu_2^{(l)}\rangle\}\end{pmatrix},$

where $\zeta = \begin{cases}\dfrac{\gamma_1^{(k)}}{\gamma_2^{(l)}}, if\gamma_1^{(k)}\leq\gamma_2^{(l)},\gamma_2^{(l)}\neq 0\\ 0, otherwise\end{cases}.$

(5) $\lambda PL_1(p) = g^{-1}(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\{(1-(1-\gamma_1^{(k)})^{\lambda})\langle\mu^{(k)},\nu^{(k)}\rangle\}).$

(6) $(PL_1(p))^{\lambda} = g^{-1}(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\{(\gamma_1^{(k)})^{\lambda}\langle\mu^{(k)},\nu^{(k)}\rangle\}),\lambda\geq 0.$

(7) $(PL_1(p))^{-1} = g^{-1}(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\{(1-\gamma_1^{(k)})\langle\mu^{(k)},\nu^{(k)}\rangle\}).$

Where $\gamma_1^{(k)}\in g(PL_1(p))$, $\gamma_2^{(l)}\in g(PL_2(p))$, $g(\cdot)$ is the score function. $\langle\mu_1^{(k)},\nu_1^{(k)}\rangle$ and $\langle\mu_2^{(k)},\nu_2^{(k)}\rangle$ are the uncertain probability values of hesitant linguistic evaluations, which are in the form of PFS.

Theorem 1. *Let* $PL_1(p)=\{[x,PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle]|x\in X,k=1,2,\ldots,\#PL_1(p)\}$, $PL_2(p)=\{[x,PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle]|x\in X,l=1,2,\ldots,\#PL_2(p)\}$ *and* $PL_3(p)=\{[x,PL_3^{(r)}\langle\mu_3^{(r)},\nu_3^{(r)}\rangle]|x\in X,r=1,2,\ldots,\#PL_3(p)\}$ *be any three PFPLTSs,* $\lambda,\lambda_1,\lambda_2\geq 0$. *Then:*

(1) $PL_1(p)\oplus PL_2(p) = PL_2(p)\oplus PL_1(p)$.
(2) $(PL_1(p)\oplus PL_2(p))\oplus PL_3(p) = PL_1(p)\oplus(PL_2(p)\oplus PL_3(p))$.
(3) $\lambda(PL_1(p)\oplus PL_2(p)) = \lambda PL_2(p)\oplus\lambda PL_1(p)$.
(4) $(\lambda_1+\lambda_2)PL_1(p) = \lambda_1 PL_1(p)\oplus\lambda_2 PL_1(p)$.
(5) $PL_1(p)\otimes PL_2(p) = PL_2(p)\otimes PL_1(p)$.
(6) $(PL_1(p)\otimes PL_2(p))\otimes PL_3(p) = PL_1(p)\otimes(PL_2(p)\otimes PL_3(p))$.
(7) $(PL_1(p)\otimes PL_2(p))^{\lambda} = (PL_2(p))^{\lambda}\otimes(PL_1(p))^{\lambda}$.
(8) $(PL_1(p))^{(\lambda_1+\lambda_2)} = (PL_1(p))^{\lambda_1}\otimes(PL_1(p))^{\lambda_2}$.
(9) $\lambda(PL_1(p)\ominus PL_2(p)) = \lambda PL_1(p)\ominus\lambda PL_1(p)$.
(10) $\lambda_1 PL_1(p)\ominus\lambda_2 PL_1(p)) = (\lambda_1-\lambda_2)PL_1(p)$.
(11) $(PL_1(p)\oslash PL_2(p))^{\lambda} = (PL_1(p))^{\lambda}\oslash(PL_2(p))^{\lambda}$.
(12) $(PL_1(p))^{\lambda_1}\oslash(PL_1(p))^{\lambda_2} = (PL_1(p))^{\lambda_1}\oslash(PL_1(p))^{\lambda_2}$

Proof

$$
\begin{aligned}
&(1)PL_1(p)\oplus PL_2(p)\\
&=g^{-1}\left(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p)}\left\{\left(\gamma_1^{(k)}+\gamma_2^{(l)}-\gamma_1^{(k)}\gamma_2^{(l)}\right)\left\langle\mu_1^{(k)}\mu_2^{(l)},\nu_1^{(k)}\nu_2^{(l)}\right\rangle\right\}\right)\\
&=g^{-1}\left(\cup_{PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p),PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\left\{\left(\gamma_2^{(l)}+\gamma_1^{(k)}-\gamma_2^{(l)}\gamma_1^{(k)}\right)\left\langle\mu_2^{(l)}\mu_1^{(k)},\nu_2^{(l)}\nu_1^{(k)}\right\rangle\right\}\right)\\
&=PL_2(p)\oplus PL_1(p)
\end{aligned}
$$

$$
\begin{aligned}
&(2)(PL_1(p)\oplus PL_2(p))\oplus PL_3(p)\\
&=g^{-1}\begin{pmatrix}\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p)}\\ \left\{\left(\gamma_1^{(k)}+\gamma_2^{(l)}-\gamma_1^{(k)}\gamma_2^{(l)}\right)\left\langle\mu_1^{(k)}\mu_2^{(l)},\nu_1^{(k)}\nu_2^{(l)}\right\rangle\right\}+\cup_{PL_3^{(r)}\langle\mu_3^{(r)},\nu_3^{(r)}\rangle\in PL_3(p)}\{\gamma_3^{(r)}\langle\mu_3^{(r)},\nu_3^{(r)}\rangle\}\end{pmatrix}\\
&=g^{-1}\begin{pmatrix}\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p),PL_3^{(r)}\langle\mu_3^{(r)},\nu_3^{(r)}\rangle\in PL_3(p)}\\ \left\{\left(\gamma_1^{(k)}+\gamma_2^{(l)}+\gamma_3^{(r)}-\gamma_1^{(k)}\gamma_2^{(l)}-\gamma_1^{(k)}\gamma_3^{(r)}-\gamma_2^{(l)}\gamma_3^{(r)}+\gamma_1^{(k)}\gamma_2^{(l)}\gamma_3^{(r)}\right)\left\langle\mu_1^{(k)}\mu_2^{(l)}\mu_3^{(r)},\nu_1^{(k)}\nu_2^{(l)}\nu_3^{(r)}\right\rangle\right\}\end{pmatrix}\\
&=g^{-1}\begin{pmatrix}\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\left\{\gamma_1^{(k)}\left\langle\mu_1^{(k)},\nu_1^{(k)}\right\rangle\right\}\\ +\cup_{PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p),PL_3^{(r)}\langle\mu_3^{(r)},\nu_3^{(r)}\rangle\in PL_3(p)}\left\{\left(\gamma_2^{(l)}+\gamma_3^{(r)}-\gamma_2^{(l)}\gamma_3^{(r)}\right)\left\langle\mu_2^{(l)}\mu_3^{(r)},\nu_2^{(l)}\nu_3^{(r)}\right\rangle\right\}\end{pmatrix}\\
&=PL_1(p)\oplus(PL_2(p)\oplus PL_3(p)).
\end{aligned}
$$

$$
\begin{aligned}
&(4)(\lambda_1+\lambda_2)PL_1(p)\\
&=g^{-1}\left(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\left\{\left(1-\left(1-\gamma_1^{(k)}\right)^{\lambda_1+\lambda_2}\right)\left\langle\mu^{(k)},\nu^{(k)}\right\rangle\right\}\right)\\
&=g^{-1}\left(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\left\{\left(1-\left(1-\gamma_1^{(k)}\right)^{\lambda_1}\right)+\left(1-\left(1-\gamma_1^{(k)}\right)^{\lambda_2}\right)-\left(1-\left(1-\gamma_1^{(k)}\right)^{\lambda_1}\right)\left(1-\left(1-\gamma_1^{(k)}\right)^{\lambda_2}\right)\left\langle\mu^{(k)},\nu^{(k)}\right\rangle\right\}\right)\\
&=g^{-1}\left(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\left\{\left(1-\left(1-\gamma_1^{(k)}\right)^{\lambda_1}\right)\left\langle\mu^{(k)},\nu^{(k)}\right\rangle\right\}\right)\oplus g^{-1}\left(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p)}\left\{\left(1-\left(1-\gamma_1^{(k)}\right)^{\lambda_2}\right)\left\langle\mu^{(k)},\nu^{(k)}\right\rangle\right\}\right)\\
&=\lambda_1 PL_1(p)\oplus\lambda_2 PL_1(p).
\end{aligned}
$$

$$
\begin{aligned}
&(5)PL_1(p)\otimes PL_2(p)\\
&=g^{-1}\left(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p)}\left\{\gamma_1^{(k)}\gamma_2^{(l)}\left\langle\mu_1^{(k)}\mu_2^{(l)},\nu_1^{(k)}\nu_2^{(l)}\right\rangle\right\}\right)\\
&=PL_2(p)\otimes PL_1(p).
\end{aligned}
$$

$$
\begin{aligned}
&(6)(PL_1(p)\otimes PL_2(p))\otimes PL_3(p)\\
&=g^{-1}\left(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p),PL_3^{(r)}\langle\mu_3^{(r)},\nu_3^{(r)}\rangle\in PL_3(p)}\right.\\
&\left.\left\{\gamma_1^{(k)}\gamma_2^{(l)}\gamma_3^{(r)}\left\langle\mu_1^{(k)}\mu_2^{(l)}\mu_3^{(r)},\nu_1^{(k)}\nu_2^{(l)}\nu_3^{(r)}\right\rangle\right\}\right)=PL_1(p)\otimes(PL_2(p)\otimes PL_3(p)).
\end{aligned}
$$

$$
\begin{aligned}
&(6)(PL_1(p)\otimes PL_2(p))\otimes PL_3(p)\\
&=g^{-1}\left(\cup_{PL_1^{(k)}\langle\mu_1^{(k)},\nu_1^{(k)}\rangle\in PL_1(p),PL_2^{(l)}\langle\mu_2^{(l)},\nu_2^{(l)}\rangle\in PL_2(p),PL_3^{(r)}\langle\mu_3^{(r)},\nu_3^{(r)}\rangle\in PL_3(p)}\right.\\
&\left.\left\{\gamma_1^{(k)}\gamma_2^{(l)}\gamma_3^{(r)}\left\langle\mu_1^{(k)}\mu_2^{(l)}\mu_3^{(r)},\nu_1^{(k)}\nu_2^{(l)}\nu_3^{(r)}\right\rangle\right\}\right)=PL_1(p)\otimes(PL_2(p)\otimes PL_3(p)).
\end{aligned}
$$

$$
\begin{aligned}
&(8)(PL_1(p))^{(\lambda_1+\lambda_2)} \\
&= g^{-1}\left(\cup PL_1^{(k)}\left\langle \mu_1^{(k)}, \nu_1^{(k)} \right\rangle \in PL_1(p)\left\{\left(\gamma_1^{(k)}\right)^{(\lambda_1+\lambda_2)}\left\langle \mu_1^{(k)}, \nu_1^{(k)} \right\rangle\right\}\right) = (PL_1(p))^{\lambda_1} \otimes (PL_1(p))^{\lambda_2}
\end{aligned}
$$

$$
\begin{aligned}
&(8)(PL_1(p))^{(\lambda_1+\lambda_2)} \\
&= g^{-1}\left(\cup PL_1^{(k)}\left\langle \mu_1^{(k)}, \nu_1^{(k)} \right\rangle \in PL_1(p)\left\{\left(\gamma_1^{(k)}\right)^{(\lambda_1+\lambda_2)}\left\langle \mu_1^{(k)}, \nu_1^{(k)} \right\rangle\right\}\right) = (PL_1(p))^{\lambda_1} \otimes (PL_1(p))^{\lambda_2}
\end{aligned}
$$

$$
\begin{aligned}
&(11)(PL_1(p) \oslash PL_2(p))^{\lambda} \\
&= g^{-1}\left(\cup_{PL_1{}^{(k)}\langle \mu_1{}^{(k)}, \nu_1{}^{(k)}\rangle \in PL_1(p), PL_2{}^{(l)}\langle \mu_2{}^{(k)}, \nu_2{}^{(l)}\rangle \in PL_2(p)}\left\{\left(\frac{\gamma_1{}^{(k)}}{\gamma_2{}^{(l)}}\right)^{\lambda}\left\langle \mu_1{}^{(k)}\mu_2{}^{(l)}, \nu_1{}^{(k)}\nu_2{}^{(l)}\right\rangle\right\}\right) \\
&= g^{-1}\left(\cup_{PL_1{}^{(k)}\langle \mu_1{}^{(k)}, \nu_1{}^{(k)}\rangle \in PL_1(p), PL_2{}^{(l)}\langle \mu_2{}^{(k)}, \nu_2{}^{(l)}\rangle \in PL_2(p)}\left\{\frac{\left(\gamma_1{}^{(k)}\right)^{\lambda}}{\left(\gamma_2{}^{(l)}\right)^{\lambda}}\left\langle \mu_1{}^{(k)}\mu_2{}^{(l)}, \nu_1{}^{(k)}\nu_2{}^{(l)}\right\rangle\right\}\right) \\
&= (PL_1(p))^{\lambda} \oslash (PL_2(p))^{\lambda}, when \gamma_1{}^{(k)} \le \gamma_2{}^{(l)}, \gamma_2{}^{(l)} \ne 0.
\end{aligned}
$$

$$
\begin{aligned}
&(12)(PL_1(p))^{\lambda_1} \oslash (PL_1(p))^{\lambda_2} \\
&= g^{-1}\left(\cup_{PL_1{}^{(k)}\langle \mu_1{}^{(k)}, \nu_1{}^{(k)}\rangle \in PL_1(p)}\left\{\left(\gamma_1{}^{(k)}\right)^{\lambda_1}\left\langle \mu_1{}^{(k)}, \nu_1{}^{(k)}\right\rangle\right\}\right) \oslash g^{-1}\left(\cup_{PL_1{}^{(k)}\langle \mu_1{}^{(k)}, \nu_1{}^{(k)}\rangle \in PL_1(p)}\left\{\left(\gamma_1{}^{(k)}\right)^{\lambda_2}\left\langle \mu_1{}^{(k)}, \nu_1{}^{(k)}\right\rangle\right\}\right) \\
&= g^{-1}\left(\cup_{PL_1{}^{(k)}\langle \mu_1{}^{(k)}, \nu_1{}^{(k)}\rangle \in PL_1(p)}\left\{\left(\gamma_1{}^{(k)}\right)^{\lambda_1-\lambda_2}\left\langle \mu_1{}^{(k)}, \nu_1{}^{(k)}\right\rangle\right\}\right) \\
&= (PL_1(p))^{\lambda_1} \oslash (PL_1(p))^{\lambda_2}, when \gamma_1{}^{(k)} \le \gamma_2{}^{(l)}, \gamma_2{}^{(l)} \ne 0.
\end{aligned}
$$

3.3. The aggregation operators for PFPLTS

In order to make better use of PFPLTS in decision-making problems, some aggregation operators are proposed in this subsection.

Definition 11. Let $PL_i(p) = \{[x, PL_i{}^{(k)}\langle \mu_i{}^{(k)}, \nu_i{}^{(k)}\rangle] | x \in X, PL_i{}^{(k)} \in S, k = 1, 2, \ldots, \#PL_i(p)\}$, $(i = 1, 2, \ldots, n)$, where $PL_i{}^{(k)}$ is the kth linguistic term, and $\langle \mu_i{}^{(k)}, \nu_i{}^{(k)}\rangle$ is the corresponding uncertain probability. Then the Pythagorean fuzzy probabilistic linguistic average (PFPLA) operator is defined as:

$$
PFPLA(PL_1(p), PL_2(p), \ldots, PL_n(p)) = \frac{1}{n}(PL_1(p) \oplus PL_2(p) \oplus \cdots \oplus PL_n(p)) \tag{5}
$$

Definition 12. Let $PL_i(p) = \{[x, PL_i{}^{(k)}\langle \mu_i{}^{(k)}, \nu_i{}^{(k)}\rangle] | x \in X, PL_i{}^{(k)} \in S, k = 1, 2, \ldots, \#PL_i(p)\}$, $(i = 1, 2, \ldots, n)$, where $PL_i{}^{(k)}$ is the kth linguistic term, and $\langle \mu_i{}^{(k)}, \nu_i{}^{(k)}\rangle$ is the corresponding uncertain probability. Then the Pythagorean fuzzy probabilistic linguistic weighted average (PFPWLA) operator is defined as:

$$
\begin{aligned}
PFPLWA(PL_1(p), PL_2(p), \ldots, PL_n(p)) = \omega_1 PL_1(p) \oplus \omega_2 PL_2(p) \oplus \cdots \\
\oplus \omega_n PL_n(p)
\end{aligned} \tag{6}
$$

Where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $PL_i(p)(i = 1, 2, \ldots, n)$, $\omega_i \ge 0$, $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} \omega_i = 1$. Especially, if $\omega = (1/n, 1/n, \ldots, 1/n)^T$, then the PFPLWA operator degenerates to the PFPLA operator.

Definition 13. Let $PL_i(p) = \{[x, PL_i{}^{(k)}\langle \mu_i{}^{(k)}, \nu_i{}^{(k)}\rangle] | x \in X, PL_i{}^{(k)} \in S, k = 1, 2, \ldots, \#PL_i(p)\}$, $(i = 1, 2, \ldots, n)$, where $PL_i{}^{(k)}$ is the kth linguistic term, and $\langle \mu_i{}^{(k)}, \nu_i{}^{(k)}\rangle$ is the corresponding uncertain probability. Then the Pythagorean fuzzy probabilistic linguistic geometric (PFPLG) operator is defined as:

$$
PFPLG(PL_1(p), PL_2(p), \ldots, PL_n(p)) = (PL_1(p) \otimes PL_2(p) \otimes \cdots \otimes PL_n(p))^{\frac{1}{n}} \tag{7}
$$

Definition 14. Let $PL_i(p) = \{[x, PL_i{}^{(k)}\langle \mu_i{}^{(k)}, \nu_i{}^{(k)}\rangle] | x \in X, PL_i{}^{(k)} \in S, k = 1, 2, \ldots, \#PL_i(p)\}$, $(i = 1, 2, \ldots, n)$, where $PL_i{}^{(k)}$ is the kth linguistic term, and $\langle \mu_i{}^{(k)}, \nu_i{}^{(k)}\rangle$ is the corresponding uncertain probability. Then the Pythagorean fuzzy probabilistic linguistic geometric (PFPLWG) operator is defined as:

$$
\begin{aligned}
PFPLWG(PL_1(p), PL_2(p), \ldots, PL_n(p)) = (PL_1(p))^{\omega_1} \otimes (PL_2(p))^{\omega_2} \otimes \cdots \\
\otimes (PL_n(p))^{\omega_n}
\end{aligned} \tag{8}
$$

Where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $PL_i(p)(i = 1, 2, \ldots, n)$, $\omega_i \ge 0$, $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} \omega_i = 1$. Especially, if $\omega = (1/n, 1/n, \ldots, 1/n)^T$, then the PFPLWG operator degenerates to the PFPLG operator.

4. The weight calculation method

The objectivity and accuracy of criteria weights are quite important in decision-making problems, which directly affect the validity of decision-making results. With the existing research methods and time-varying factors, we use the minimum deviation and the vector projection method to conduct in-depth research on the time weight method and the criteria's fusion weight method.

4.1. Time weight method based on minimum deviation of AHP and entropy method

In dynamic decision-making problems, the importance of different stages is quite different. To make the time weights at different stages more convincing and improve the accuracy of the decision-making results, we combine the subjective and objective weights to calculate the time weights. The entropy method calculates the objective time weight, reflecting the advantages in illustrating data information. Then, the DMs compare the importance of different stages in pairs according to their experience and use the AHP method to determine their objective weights. Finally, a model is set to solve the distribution coefficient by minimizing the deviation of the subjective and objective weights to the combined weight.

Definition 15. Let X be a non-empty universe of discourse, $A = \{A_1, A_2, \cdots, A_m\}$ is the alternatives set, $C = \{c_1, c_2, \cdots, c_n\}$ is the criteria sets, $S = \{s_\alpha | \alpha = 0, 1, 2, \ldots, \tau\}$ is the linguistic term set. In the eth stage, the PFPLTS evaluation of the ith alternative under the jth criterion is recorded as $PL_{ij}(p) = \{[x, PL^{(ek)}\langle\mu^{(ek)}, \nu^{(ek)}\rangle] | x \in X, PL^{(ek)} \in S, k = 1, 2, \ldots, \#PL_{ij}(p)\}$. The entropy of eth stage is defined as:

$$H^{(e)} = -\frac{1}{2\ln mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(P^{(e)}{}_{ij}\ln P^{(e)}{}_{ij} + Q^{(e)}{}_{ij}\ln Q^{(e)}{}_{ij}\right) \tag{9}$$

Where, $P^{(e)}{}_{ij}, Q^{(e)}{}_{ij}$ represent the degree of membership and non-membership contribution at the eth stage, respectively. In order to obtain entropy $H^{(e)}$, the membership and non-membership contribution $P^{(e)}{}_{ij}$ and $Q^{(e)}{}_{ij}$need to be evaluated m*n times. And

$$P^{(e)}{}_{ij} = \frac{\sum_{k=1}^{\#PL_{ij}(P)} g\left(PL_{ij}^{(ek)}\right)\left(\mu_{ij}^{(ek)}\right)^2}{\sum_{i=1}^{m}\sum_{k=1}^{\#PL_{ij}(P)} g\left(PL_{ij}^{(ek)}\right)\left(\mu_{ij}^{(ek)}\right)^2} \tag{10}$$

$$Q^{(e)}{}_{ij} = \frac{\sum_{k=1}^{\#PL_{ij}(P)} g\left(PL_{ij}^{(ek)}\right)\left(\nu_{ij}^{(ek)}\right)^2}{\sum_{i=1}^{m}\sum_{k=1}^{\#PL_{ij}(P)} g\left(PL_{ij}^{(ek)}\right)\left(\nu_{ij}^{(ek)}\right)^2} \tag{11}$$

Where, the function $g(\cdot)$ is given as Definition 2. The entropy weight of the eth stage is defined as:

$$\theta^{(1)}(t_e) = \frac{1 - H^{(e)}}{\sum_{e=1}^{p}(1 - H^{(e)})} \tag{12}$$

Definition 16. The 1∼9 scale method is used to construct the pairwise judgment matrix $A(a_{ij})_{q\times q}$ about q stages, it is necessary to have q(q-1)/2 pairwise comparisons. The normalized eth row vector is

$$\overline{\theta}^{(2)}(t_e) = \sqrt[p]{\prod_{i=1}^{p} a_{ei}} \tag{13}$$

And the subjective weight of the eth stage can be calculated as:

$$\theta^{(2)}(t_e) = \frac{\overline{\theta}^{2}(t_e)}{\sum_{e=1}^{p}\overline{\theta}^{2}(t_e)} \tag{14}$$

The consistency test is performed on the judgment matrix. When $CR < 0.1$, the judgment matrix passes the consistency test, where $CR = CI/RI$, CI is the maximum eigenvalue of the judgment matrix, and RI is directly obtained by looking up the table. Otherwise, the judgment matrix should be reconstructed.

Considering subjective and objective weights, we propose a combination weight method that minimizes the deviation between AHP and entropy weight method, which is defined as follows:

Definition 17. The objective weight is $\theta^{(1)}(t_e)$, the subjective weight is $\theta^{(2)}(t_e)$, and the combination weight is $\theta(t_e) = \alpha\theta^{(1)}(t_e) + \beta\theta^{(2)}(t_e)$, where $0 \le \alpha, \beta \le 1, \alpha + \beta = 1$. In order to obtain the coefficients, the nonlinear multi-objective programming model is constructed as follows:

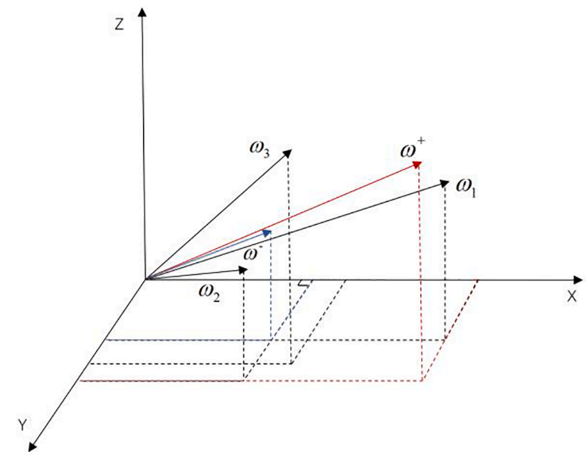

Fig. 1.. The cosine projected vector on the positive and negative ideal weights.

$$\begin{cases} \min \frac{1}{p-1}\sum_{e=1}^{p}\left(\left|\theta(t_e) - \theta^{(1)}(t_e)\right| + \left|\theta(t_e) - \theta^{(2)}(t_e)\right|\right) \\ \alpha + \beta = 1, 0 \le \alpha, \beta \le 1 \end{cases} \tag{15}$$

4.2. Fusion criteria weights based on time-varying

Since the criteria selection has been subjectively analyzed and selected by experts, this paper tends to use objective methods to determine the criteria weights. The objective weight method based on the idea of data analysis can avoid the error of subjective cognitive uncertainty and help reduce the pressure of DMs. This subsection selects three common objective weight analysis methods and proposes the dual ideal point-vector projection method to get the fusion criteria weights with time-varying factors.

4.2.1. Weight analysis method based on criterion recognition

Generally, a criterion has a higher recognition degree for the distinction and selection of alternatives, the more critical it is in the decision-making process. Then, it should be given greater weight. Conversely, if the criterion has a low degree of recognition in evaluating the alternatives, the criterion is not conducive to decision-making and should be given a smaller weight. The weight method based on criterion recognition of PFPLTS is proposed in this subsection.

Definition 18. Let $PL_{ij}{}^{(e)} = \{\langle c_j, PL_{ij}{}^{(ek)}(\langle\mu_{ij}{}^{(ek)}, \nu_{ij}{}^{(ek)}\rangle)\rangle | c_j \in C, PL_{ij}{}^{(ek)} \in S, k = 1,2,\ldots,\#PL_{ij}(p)\}$, which means the PFPLTS evaluation information of alternative A_i about criterion c_j at the eth stage. The degree of recognition is defined as:

$$O_j{}^{(e)} = \sum_{i=1}^{m}\sum_{l=1, l\neq i}^{m} d\left(PL_{ij}{}^{(e)}, PL_{lj}{}^{(e)}\right) \tag{16}$$

Where, $d(PL_{ij}{}^{(e)}, PL_{lj}{}^{(e)})$ means the deviation between alternative A_i and A_l at the eth stage.

Definition 19. Let $\omega^{(1)} = (\omega_1^{(1)}, \omega_2^{(1)}, \ldots, \omega_n^{(1)})$ denote the fusion weight based on the recognition degree of the criteria, then the recognition weight $\omega_j^{(1)}$ with time varying of criterion c_j is

$$\omega_j^{(1)} = \sum_{e=1}^{q}\theta(t_e)\frac{O_j{}^{(e)}}{\sum_{p=1}^{n} O_p{}^{(e)}} = \sum_{e=1}^{q}\theta(t_e)\frac{\sum_{i=1}^{m}\sum_{l=1,l\neq i}^{m} d\left(PL_{ij}{}^{(e)}, PL_{lj}{}^{(e)}\right)}{\sum_{p=1}^{n}\sum_{i=1}^{m}\sum_{l=1,l\neq i}^{m} d\left(PL_{ip}{}^{(e)}, PL_{lp}{}^{(e)}\right)} \tag{17}$$

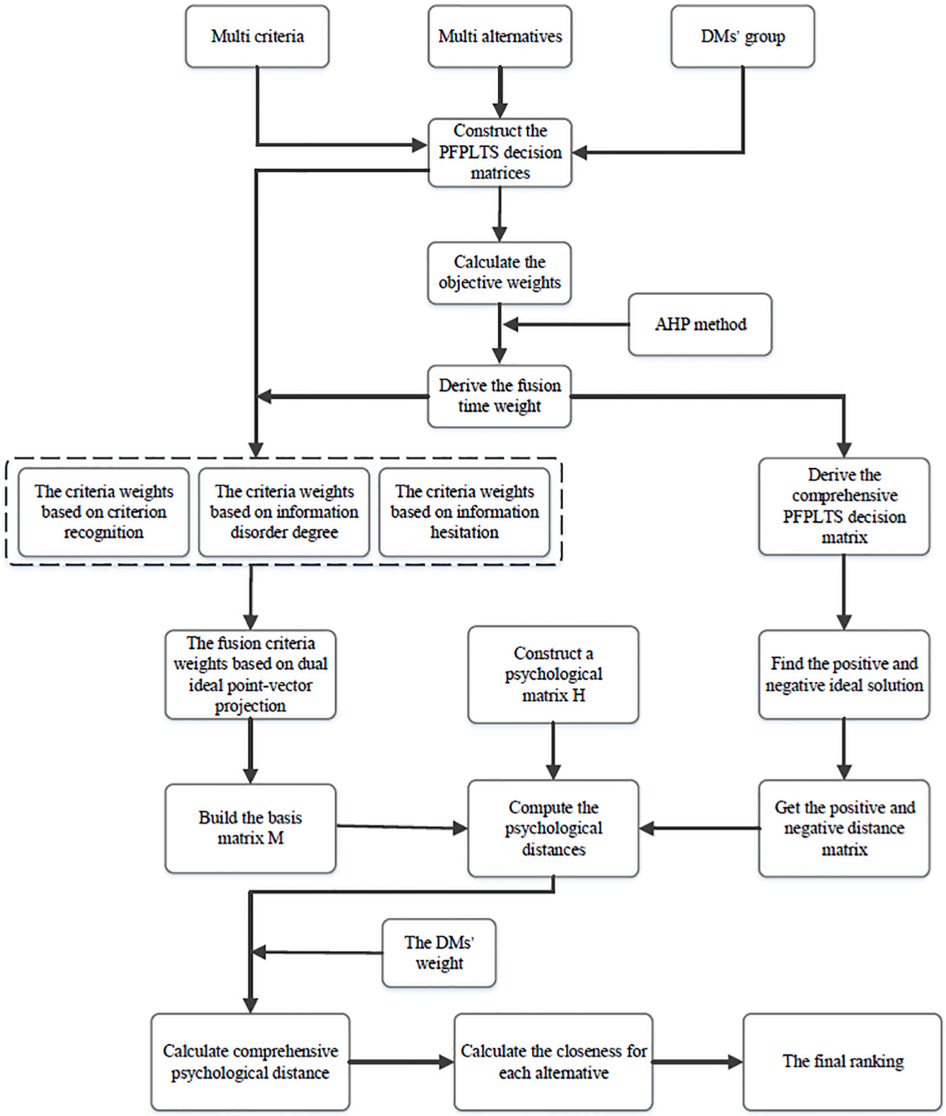

Fig. 2.. Process of PFPL-PT method for dynamic MCGDM.

In particular, if the identification degree of the criterion to the alternatives is 0, it is completely impossible to distinguish the pros and cons of the alternatives, which means that the criterion has no meaning in decision-making, and the weight should be set to 0.

4.2.2. Weight analysis method based on information disorder degree

The orderliness of decision-making information is also crucial to the impact of decision making, which can be described as the lower the disorder degree, the greater the utility value of the information. Information entropy is usually used to measure the disorder of information. The smaller the information entropy, the lower the information disorder degree, and the greater the effect value, then the greater the weight of the criterion. The weight method based on the information disorder degree of PFPLTS is proposed in this subsection.

Definition 20. Let $PL_{ij}{}^{(e)} = \{\langle c_j, PL_{ij}{}^{(ek)}(\langle \mu_{ij}{}^{(ek)}, \nu_{ij}{}^{(ek)}\rangle)\rangle | c_j \in C, PL_{ij}{}^{(ek)} \in S, k = 1,2,\ldots,\#PL_{ij}(p)\}$, which means the PFPLTS evaluation information of alternative A_i about criterion c_j at the eth stage. The information entropy of criterion c_j is defined as:

$$E_j{}^{(e)} = \frac{1}{2\ln m}\sum_{i=1}^{m}\left(P_{ij}{}^{(e)}\ln P_{ij}{}^{(e)} + Q_{ij}{}^{(e)}\ln Q_{ij}{}^{(e)}\right) \tag{18}$$

Where, $P_{ij}{}^{(e)}, Q_{ij}{}^{(e)}$ represent the degree of membership and non-membership contribution, respectively. And

$$P_{ij}{}^{(e)} = \frac{\sum_{k=1}^{\#PL_{ij}(P)} g\left(PL_{ij}^{(ek)}\right)\left(\mu_{ij}^{(ek)}\right)^2}{\sum_{i=1}^{m}\sum_{k=1}^{\#PL_{ij}(P)} g\left(PL_{ij}^{(ek)}\right)\left(\mu_{ij}^{(ek)}\right)^2} \tag{19}$$

$$Q_{ij}{}^{(e)} = \frac{\sum_{k=1}^{\#PL_{ij}(P)} g\left(PL_{ij}^{(ek)}\right)\left(\nu_{ij}^{(ek)}\right)^2}{\sum_{i=1}^{m}\sum_{k=1}^{\#PL_{ij}(P)} g\left(PL_{ij}^{(ek)}\right)\left(\nu_{ij}^{(ek)}\right)^2} \tag{20)}$$

Where the function $g(\cdot)$ is given as definition 2. Especially, $E_i{}^{(e)} = 1$, when $P_{ij}{}^{(e)} = Q_{ij}{}^{(e)} = \frac{1}{m}$.

Definition 21. Let $\omega^{(2)} = (\omega_1^{(2)}, \omega_2^{(2)}, \ldots, \omega_n^{(2)})$ denote the fusion weight based on information disorder degree, then the information disorder weight $\omega_j^{(2)}$ with time-varying of criterion c_j is

$$\omega_j^{(2)} = \sum_{e=1}^{q} \theta(t_e) \frac{1 - E_j^{(e)}}{\sum_{j=1}^{n} \left(1 - E_j^{(e)}\right)} \quad (21)$$

When the degree of membership and non-membership contributions tend to be the same, that is, $E_j^{(e)} = 1$, we can ignore this criterion in decision-making, and the corresponding weight is 0.

4.2.3. Weight analysis method based on information hesitation degree

The advantage of PFPLTS is that it can help DMs express the uncertainty of linguistic evaluation, reflect the probability and ambiguity of each linguistic evaluation. Obviously, the lower the hesitation of the DM's evaluation information, the more certain the decision-making is. It further shows that the evaluation information has higher accuracy, and the corresponding alternative should be given greater weight. The weight method based on the information hesitation degree of PFPLTS is proposed in this subsection.

Definition 22. Let $PL_{ij}{}^{(e)} = \{\langle c_j, PL_{ij}{}^{(ek)}(\langle \mu_{ij}{}^{(ek)}, \nu_{ij}{}^{(ek)}\rangle)\rangle | c_j \in C, PL_{ij}{}^{(ek)} \in S, k = 1,2,\ldots,\#PL_{ij}(p)\}$, which means the PFPLTS evaluation information of alternative A_i about criterion c_j at the eth stage. The information hesitation degree $H_j{}^{(e)}$ is defined as:

$$H_j{}^{(e)} = \sum_{i=1}^{m} \sum_{k=1}^{\#LP_{ij}(p)} \left[1 - \left(\mu_{ij}^{(ek)}\right)^2 - \left(\nu_{ij}^{(ek)}\right)^2\right] \quad (22)$$

Definition 23. Let $\omega^{(3)} = (\omega_1^{(3)}, \omega_2^{(3)}, \ldots, \omega_n^{(3)})$ denote the fusion weight based on information hesitation degree, then the information hesitation weight $\omega_j^{(3)}$ with time-varying of criterion c_j is

$$\omega_j^{(3)} = \sum_{e=1}^{q} \frac{1 - H_j{}^{(e)} \Big/ \sum_{j=1}^{n} H_j{}^{(e)}}{\sum_{j=1}^{n} \left(1 - H_j{}^{(e)} \Big/ \sum_{j=1}^{n} H_j{}^{(e)}\right)} \quad (23)$$

4.2.4. Fusion weight method based on dual ideal point-vector projection

The ideal point method has a wide range of applications in MCGDM problems, and many scholars have carried out in-depth research on it. The vector projection method has become a commonly used tool in studying multiple indexes due to its simple operation and easy understanding characteristics. The determination of the fusion weight is essentially a multi-index problem. In view of the advantages of the ideal point and vector projection method in multi-criteria problems, they will be integrated to get the fusion weight in this section.

In MCGDM problems, the effects of criteria are often divided into positive and negative effects. Generally, we expect that the positive criterion weight to be larger and the negative criterion weight to be smaller. Each weight vector is cosine projected on the positive and negative ideal weights, and the projection diagram is shown in Fig. 1.

The definitions of positive and negative ideal weight ω^+ and ω^- are presented below.

Definition 24. Let $\omega^{(i)} = (\omega_1^{(i)}, \omega_2^{(i)}, \ldots, \omega_n^{(i)}), i = 1,2,3$, which represent the criteria weight based on criterion recognition, information disorder degree and information hesitation degree, respectively. When the criterion is positive,

$$\omega^+ = \left(\max_j \omega_j^{(1)}, \max_j \omega_j^{(2)}, \max_j \omega_j^{(3)}\right) \quad (24)$$

$$\omega^- = \left(\min_j \omega_j^{(1)}, \min_j \omega_j^{(2)}, \min \omega_j^{(3)}\right) \quad (25)$$

When the criterion is negative,

$$\omega^+ = \left(\min_j \omega_j^{(1)}, \min_j \omega_j^{(2)}, \min \omega_j^{(3)}\right) \quad (26)$$

$$\omega^- = \left(\max_j \omega_j^{(1)}, \max_j \omega_j^{(2)}, \max_j \omega_j^{(3)}\right) \quad (27)$$

Definition 25. We have the criterion weight vector $\omega^{(i)} = (\omega_1^{(i)}, \omega_2^{(i)}, \ldots, \omega_n^{(i)}), i = 1, 2, 3$, positive and negative ideal weight ω^+ and ω^-. Let vector $\omega_j^{ide} = (\omega_j^{(1)}, \omega_j^{(2)}, \omega_j^{(3)})^T$. The positive projection intensity of the weight vector in the positive ideal weight is denoted as B_j, and the ratio of the positive projection intensity to the total sum is the positive fusion weight of the vector, denoted as ω_j^+.

$$B_j = \frac{< \omega^+, \omega_j^{ide} >}{\| \omega^+ \| \| \omega_j^{ide} \|} \| \omega_j^{ide} \| = \frac{< \omega^+, \omega_j^{ide} >}{\| \omega^+ \|} \quad (28)$$

$$\omega_j^+ = \frac{B_j}{\sum_{i=1}^{n} B_j} \quad (29)$$

The negative projection intensity of the weight vector in the negative ideal weight is denoted as D_j, and the ratio of the negative projection intensity to the total sum is the negative fusion weight of the vector, denoted as ω_j^-.

$$D_j = \frac{< \omega^-, \omega_j^{ide} >}{\| \omega^- \| \| \omega_j^{ide} \|} \| \omega_j^{ide} \| = \frac{< \omega^-, \omega_j^{ide} >}{\| \omega^+ \|} \quad (30)$$

$$\omega_j^- = \frac{D_j}{\sum_{j=1}^{n} D_j} \quad (31)$$

Then, the fusion weight is defined as:

$$\omega_j = \frac{\omega_j^- + \omega_j^+}{2} \quad (32)$$

5. Dynamic Pythagorean fuzzy probabilistic linguistic MCGDM with Psy-TOPSIS method

In this section, we mainly introduce the Pythagorean fuzzy probabilistic linguistic TOPSIS method with psychological distance measure (Psy-TOPSIS) method. Then an approach for MCGDM based on Pythagorean fuzzy probabilistic linguistic Psy-TOPSIS is proposed.

5.1. Novel TOPSIS method with psychological distance measure

Individuals do not consistently distribute their attention equally to each dimension when describing objects, and it differs their psychological space. A psychological distance measure, which can clarify the influence of different psychological factors and background information of the DMs and alternatives, is proposed. In psychological space, the preferential relationship between the alternatives is reflected by indifferent vectors ($v_{fj}(j = 1, 2, \ldots, n-1)$) and dominant vector ($v_d$). The indifferent vectors v_{fj} quantitatively describe the relative gain due to criterion substitution and the dominant vector v_d can manifest the direction of the optimal alternative.

In MCGDM problems, the alternatives set is$A = (A_1, A_2, \ldots, A_m)$, $C = \{c_1, c_2, \cdots, c_n\}$ is the criteria sets, and the criteria weight is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$. To calculate the indifferent vectors, we compare each criterion weight with the position weighted average weight ($\widetilde{\omega}$) based on the normal distribution. Then, the indifferent vectors can be calculated as:

$$v_{fj} = \left(-\frac{\omega_{j+1}}{\widetilde{\omega}}, 0, \ldots, 0, \frac{\omega_1}{\widetilde{\omega}}\right)^T \quad (34)$$

Table 1
Individual PFPLTS decision matrix D^{11}.

	c_1	c_2	c_3	c_4	c_5
A_1	$\{s_5\langle 0.8,0.1\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_2\langle 0.7,0.3\rangle, s_3\langle 0.2,0.4\rangle\}$	$\{s_3\langle 0.2,0.5\rangle, s_4\langle 0.7,0.2\rangle\}$	$\{s_5\langle 0.8,0.1\rangle\}$
A_2	$\{s_2\langle 0.5,0.2\rangle, s_3\langle 0.3,0.1\rangle\}$	$\{s_3\langle 0.6,0.2\rangle, s_4\langle 0.3,0.2\rangle\}$	$\{s_5\langle 0.9,0.2\rangle\}$	$\{s_3\langle 0.8,0.1\rangle\}$	$\{s_1\langle 0.9,0.1\rangle\}$
A_3	$\{s_4\langle 0.6,0.1\rangle, s_5\langle 0.3,0.2\rangle\}$	$\{s_4\langle 0.6,0.3\rangle, s_5\langle 0.3,0.1\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_4\langle 0.9,0.1\rangle\}$	$\{s_1\langle 0.9,0.1\rangle\}$
A_4	$\{s_1\langle 0.9,0.1\rangle\}$	$\{s_1\langle 0.6,0.2\rangle, s_2\langle 0.4,0.3\rangle\}$	$\{s_2\langle 0.5,0.2\rangle, s_3\langle 0.4,0.3\rangle\}$	$\{s_3\langle 0.7,0.2\rangle, s_4\langle 0.3,0.2\rangle\}$	$\{s_1\langle 0.7,0.3\rangle, s_2\langle 0.2,0.2\rangle\}$

where $\frac{\omega_1}{\varpi}$ is at the $(j+1)$th position. According to Berkowitsch [26], the dominance vector v_d is orthogonal to all indifference vectors $(v_{fi})_{n-1}$, which means $v_d \cdot v_{fj} = 0, j = 1,2,\ldots,n-1$. Then, the dominant vector is

$$v_d = \left(\frac{\omega_1}{\varpi}, \frac{\omega_2}{\varpi}, \ldots, \frac{\omega_n}{\varpi}\right)^T \tag{35}$$

To calculate the projection in each direction, the basis matrix(M) can be built as

$$M = \left(v_{f1}, v_{f2}, \ldots, v_{fn-1}, v_d\right)^T \tag{36}$$

And the basis matrix (M^*) after length normalization is

$$M^* = \left(\frac{v_{f1}}{\| v_{f1} \|}, \frac{v_{f2}}{\| v_{f2} \|}, \ldots, \frac{v_{fn-1}}{\| v_{fn-1} \|}, \frac{v_d}{\| v_d \|}\right)^T \tag{37}$$

It is significant to weigh the dominance vector more strongly in psychological distance measure by adjusting the parameter w_{dom}. Hence, we construct a psychological matrix H:

$$H = diag(1, 1, \ldots, 1, w_{dom}) \tag{38}$$

Then, the psychological distance between alternatives A_k and A_l can be computed as follows:

$$\| D \|_i = \| HM^{*-1} d' \|_i, i = 1, 2, \infty. \tag{39}$$

Where, d is the standard distance matrix between two alternatives A_k and A_l, $d(A_k, A_l) = (PL_{k1} - PL_{l1}, PL_{k2} - PL_{l2}, \ldots, PL_{kn} - PL_{ln})$, PL_{ij} is the PFPLTS evaluation information of alternative A_i about criterion c_j. Especially, $\| D \|$is 1-norm, when $i = 1$. $\| D \|$is 2-norm, when$i = 2$. $\| D \|$is infinity-norm, when $i = \infty$.

5.2. An approach for MCGDM problems based on PFPLF-PT method

We mainly introduce the dynamic Pythagorean fuzzy probabilistic linguistic MCGDM with the Psy-TOPSIS method. The procedure is visualized in Fig. 2 and summarized as follows:

Step1: Collect and preprocess the evaluation information of the PFPLTS decision matrices $D^{(ge)} = (PL_{ij}^{(ge)})_{m\times n}$ given by the gth DM under the same criteria at eth stage.

Step2: Calculate the objective weights vector $\theta_1^{(g)}(t_e)$ of the gth DM according to the method in Definition 15.

Step3: To subjectively evaluate the importance of the three stages involved in the problem, use the 1~9 scale method in Definition 16 to construct pairwise judgment matrix $A(a_{ij})_{q\times q}$, and calculate the subjective weight vector $\theta_2^{(g)}(t_k)$ through the AHP method.

Step4: Combine the subjective and objective time weights, use a nonlinear optimization model to calculate the coefficients of the time combination weights, and then get the time combination weights vector $\theta^{(g)} = (\theta^{(g)}(t_1, t_2, \ldots, t_q))$, which is assigned by the method in Section 4.1.

Step5: Derive the comprehensive PFPLTS decision matrix $D^{(g)} = (V_{ij}^{(com-g)})_{m\times n}$ of each expert. And

Table 2.
Individual PFPLTS decision matrix D^{12}.

	c_1	c_2	c_3	c_4	c_5
A_1	$\{s_2\langle 0.7,0.4\rangle, s_3\langle 0.3,0.4\rangle\}$	$\{s_4\langle 0.7,0.3\rangle, s_5\langle 0.3,0.1\rangle\}$	$\{s_2\langle 0.7,0.3\rangle, s_3\langle 0.2,0.4\rangle\}$	$\{s_4\langle 0.9,0.2\rangle, s_5\langle 0.1,0.3\rangle\}$	$\{s_4\langle 0.7,0.2\rangle, s_5\langle 0.2,0.5\rangle\}$
A_2	$\{s_2\langle 0.5,0.3\rangle, s_3\langle 0.4,0.1\rangle\}$	$\{s_4\langle 0.7,0.2\rangle\}$	$\{s_5\langle 0.8,0.1\rangle\}$	$\{s_3\langle 0.9,0.1\rangle\}$	$\{s_2\langle 0.7,0.4\rangle, s_3\langle 0.1,0.3\rangle\}$
A_3	$\{s_4\langle 0.7,0.2\rangle, s_5\langle 0.2,0.3\rangle\}$	$\{s_3\langle 0.6,0.3\rangle, s_4\langle 0.4,0.2\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_4\langle 0.8,0.2\rangle\}$	$\{s_1\langle 0.8,0.3\rangle, s_2\langle 0.2,0.2\rangle\}$
A_4	$\{s_1\langle 0.8,0.2\rangle, s_2\langle 0.2,0.4\rangle\}$	$\{s_1\langle 0.7,0.3\rangle, s_2\langle 0.3,0.3\rangle\}$	$\{s_3\langle 0.6,0.2\rangle\}$	$\{s_3\langle 0.9,0.1\rangle\}$	$\{s_1\langle 0.8,0.2\rangle\}$

Table 3.
Individual PFPLTS decision matrix D^{13}.

	c_1	c_2	c_3	c_4	c_5
A_1	$\{s_3\langle 0.7,0.3\rangle, s_4\langle 0.3,0.4\rangle\}$	$\{s_4\langle 0.9,0.1\rangle\}$	$\{s_2\langle 0.8, 0.2\rangle\}$	$\{s_4\langle 0.9,0.2\rangle, s_5\langle 0.1,0.3\rangle\}$	$\{s_4\langle 0.7,0.3\rangle, s_5\langle 0.2,0.5\rangle\}$
A_2	$\{s_2\langle 0.5,0.3\rangle, s_3\langle 0.4,0.1\rangle\}$	$\{s_4\langle 0.8,0.2\rangle\}$	$\{s_5\langle 0.8, 0.1\rangle\}$	$\{s_3\langle 0.9,0.1\rangle\}$	$\{s_1\langle 0.9,0.1\rangle\}$
A_3	$\{s_4\langle 0.7,0.3\rangle, s_5\langle 0.2,0.4\rangle\}$	$\{s_3\langle 0.6,0.3\rangle, s_4\langle 0.4,0.3\rangle\}$	$\{s_5\langle 0.9, 0.1\rangle\}$	$\{s_4\langle 0.8,0.1\rangle\}$	$\{s_4\langle 0.7,0.3\rangle, s_5\langle 0.3,0.2\rangle\}$
A_4	$\{s_1\langle 0.7,0.3\rangle, s_2\langle 0.3,0.1\rangle\}$	$\{s_1\langle 0.8,0.2\rangle, s_2\langle 0.2,0.1\rangle\}$	$\{s_3\langle 0.7, 0.1\rangle\}$	$\{s_3\langle 0.9,0.1\rangle\}$	$\{s_1\langle 0.9,0.1\rangle\}$

Table 4.
Individual PFPLTS decision matrix D^{21}.

	c_1	c_2	c_3	c_4	c_5
A_1	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_3\langle 0.6,0.4\rangle, s_4\langle 0.3,0.1\rangle\}$	$\{s_3\langle 0.8,0.2\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$
A_2	$\{s_2\langle 0.5,0.2\rangle, s_3\langle 0.3,0.1\rangle\}$	$\{s_3\langle 0.5,0.3\rangle, s_4\langle 0.5,0.4\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_2\langle 0.6,0.4\rangle, s_3\langle 0.4,0.1\rangle\}$	$\{s_1\langle 0.9,0.2\rangle\}$
A_3	$\{s_4\langle 0.8,0.1\rangle\}$	$\{s_5\langle 0.8,0.2\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_4\langle 0.8,0.3\rangle, s_5\langle 0.1,0.3\rangle\}$	$\{s_1\langle 0.9,0.2\rangle\}$
A_4	$\{s_3\langle 0.8,0.2\rangle\}$	$\{s_2\langle 0.7,0.3\rangle, s_3\langle 0.2,0.1\rangle\}$	$\{s_3\langle 0.6,0.3\rangle, s_4\langle 0.4,0.3\rangle\}$	$\{s_3\langle 0.8,0.1\rangle\}$	$\{s_1\langle 0.8,0.3\rangle, s_2\langle 0.2,0.1\rangle\}$

Table 5.
Individual PFPLTS decision matrix D^{22}.

	c_1	c_2	c_3	c_4	c_5
A_1	$\{s_2\langle 0.9,0.2\rangle, s_3\langle 0.1,0.1\rangle\}$	$\{s_4\langle 0.9,0.2\rangle\}$	$\{s_3\langle 0.7,0.3\rangle\}$	$\{s_4\langle 0.8,0.3\rangle, s_5\langle 0.2,0.1\rangle\}$	$\{s_4\langle 0.8, 0.2\rangle\}$
A_2	$\{s_3\langle 0.6,0.2\rangle\}$	$\{s_4\langle 0.8,0.2\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_2\langle 0.3,0.3\rangle, s_3\langle 0.7,0.2\rangle\}$	$\{s_3\langle 0.9, 0.2\rangle\}$
A_3	$\{s_4\langle 0.8,0.1\rangle\}$	$\{s_3\langle 0.7,0.3\rangle, s_4\langle 0.3,0.2\rangle\}$	$\{s_5\langle 0.9,0.1\rangle\}$	$\{s_4\langle 0.9,0.2\rangle\}$	$\{s_1\langle 0.8, 0.2\rangle\}$
A_4	$\{s_2\langle 0.9,0.1\rangle\}$	$\{s_2\langle 0.7,0.3\rangle\}$	$\{s_3\langle 0.7,0.2\rangle, s_4\langle 0.3,0.2\rangle\}$	$\{s_4\langle 0.7,0.3\rangle, s_5\langle 0.3,0.4\rangle\}$	$\{s_1\langle 0.8, 0.2\rangle\}$

Table 6.
Individual PFPLTS decision matrix D^{23}.

	c_1	c_2	c_3	c_4	c_5
A_1	$\{s_3\langle 0.8, 0.2\rangle\}$	$\{s_4\langle 0.9,0.2\rangle\}$	$\{s_3\langle 0.8, 0.2\rangle\}$	$\{s_4\langle 0.9,0.2\rangle, s_5\langle 0.1, 0.3\rangle\}$	$\{s_4\langle 0.8, 0.2\rangle\}$
A_2	$\{s_3\langle 0.7, 0.2\rangle\}$	$\{s_4\langle 0.8,0.2\rangle\}$	$\{s_4\langle 1\rangle\}$	$\{s_2\langle 0.5,0.3\rangle, s_3\langle 0.5, 0.2\rangle\}$	$\{s_1\langle 0.9, 0.1\rangle\}$
A_3	$\{s_4\langle 0.8, 0.2\rangle\}$	$\{s_3\langle 0.7,0.3\rangle, s_4\langle 0.3, 0.2\rangle\}$	$\{s_5\langle 0.9, 0.1\rangle\}$	$\{s_4\langle 0.9,0.2\rangle\}$	$\{s_4\langle 0.8, 0.2\rangle\}$
A_4	$\{s_2\langle 0.9, 0.1\rangle\}$	$\{s_2\langle 0.8,0.2\rangle\}$	$\{s_3\langle 0.8, 0.2\rangle\}$	$\{s_4\langle 0.8,0.2\rangle\}$	$\{s_1\langle 0.8, 0.2\rangle\}$

Table 7.
Judgment matrix $A(a_{ij})_{q\times q}$.

	t1	t2	t3
t1	1	1/3	5
t2	3	1	7
t3	1/5	1/7	1

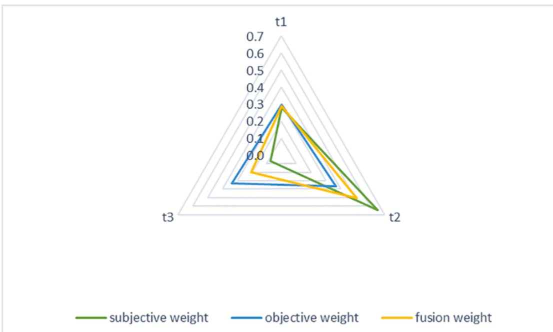

Fig. 3. . The time weight radar map of E1.

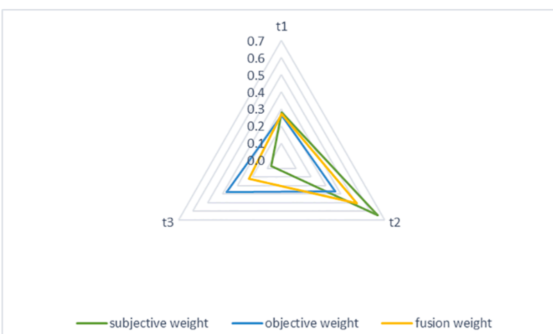

Fig. 4. The time weight radar map of E2.

$$V_{ij}^{(com-g)} = \sum_{e=1}^{p} \theta(t_e) PL_{ij}{}^{(ge)} \tag{40}$$

Where, $PL_{ij}{}^{(ge)} = [PL_{ij}{}^{(gk)}(t_e)\langle \mu_{ij}(t_e)^{(gk)}, \nu_{ij}(t_e)^{(gk)}\rangle]$, $PL_{ij}{}^{(gk)}(t_e) \in S, k = 1, 2, \ldots, \#PL_{ij}^{(ge)}$. According to formula in Definition 6,

$$V_{ij}^{(com-g)} = \sum_{e=1}^{p} \theta(t_e)\left[PL_{ij}{}^{(g)}(t_e)\langle \mu_{ij}(t_e)^{(g)}, \nu_{ij}(t_e)^{(g)}\rangle\right]$$
$$= \sum_{e=1}^{p} \cup_{PL_{ij}{}^{(gk)}(t_e)\langle \mu_{ij}(t_e)^{(gk)}, \nu_e(t_e)^{(gk)}\rangle \in PL_{ij}{}^{(ge)}}$$
$$\left\{\left(1-\left(1-PL_{ij}{}^{(gk)}(t_e)\right)^{\theta(t_e)}\right)\langle \mu_{ij}(t_e)^{(gk)}, \nu_{ij}(t_e)^{(gk)}\rangle\right\} \tag{41}$$

Step6: According to the score function in Definition 2 and Definition 7, find the positive and negative ideal solution of each DM, respectively.

Step7: Calculate the distance from each alternative to the positive and negative ideal solution, and get the positive and negative distance matrix of every DM, respectively.

Step8: Calculate the criteria weights with time-varying based on criterion recognition by Eq. (17).

Step9: Calculate the criteria weights with time-varying based on information disorder degree by Eq. (21).

Step10: Calculate the criteria weights with time-varying based on information hesitation degree by Eq. (23).

Step11: Calculate the fusion criteria weights based on the dual ideal point-vector projection method as Eq. (32). The time-varying fusion weight vector is recorded as $\omega^{[g]} = (\omega_1{}^{[g]}, \omega_2{}^{[g]}, \ldots, \omega_n{}^{[g]})^T$.

Step12: Calculate the dominance vector v_d and the indifference vectors $(v_{fj})_{n-1}$ by the fusion criteria weight $\omega^{[g]}$. Then, build the basis matrix M.

Step13: Construct a psychological matrix $H = diag(1, 1, \ldots, 1, w_{dom})$

Step14: Compute the psychological distances $d_{psy}^{(g)}(V_{ij}{}^{(com-g)}, V_j{}^{+})$ and $d_{psy}^{(g)}(V_{ij}{}^{(com-g)}, V_j{}^{-})$, $V_j{}^{+}$ and $V_j{}^{-}$are the positive ideal solution and negative ideal solution of $V_{ij}{}^{(com-g)}$respectively.

Step15: Through the weighted average method, the comprehensive psychological distance $d_{psy}^{(g)}(A_i, A^{+})$ and $d_{psy}^{(g)}(A_i, A^{-})$ are aggregated based on each DM's psychological distance in Step 14, and the DMs' weight vector is $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_g)^T$.

Step16: Calculate the closeness η_i for each alternative A_i.

Step17: Sort the alternatives according to η_i and choose the best one.

6. Case study and analysis

This section will give an example about the site selecting of the COVID-19 vaccination center to illustrate the proposed method.

6.1. Case study: site selecting of the COVID-19 vaccination center

With the control of the COVID-19 in the country, production and life are slowly recovering. China successfully eliminated COVID-19 and

Table 8
Time comprehensive decision matrix D^1.

	c_1	c_2
A_1	$\{s_5\langle 0.8, 0.056\rangle\}$	$\{s_5\langle 0.81, 0.004\rangle\}$
A_2	$\{s_2\langle 0.125, 0.018\rangle, s_{2.2335}\langle 0.1, 0.006\rangle, s_{2.3329}\langle 0.075, 0.009\rangle, s_{2.5405}\langle 0.06, 0.003\rangle, s_{2.5604}\langle 0.1, 0.006\rangle, s_{2.7503}\langle 0.08, 0.002\rangle, s_{2.8311}\langle 0.06, 0.003\rangle, s_3\langle 0.048, 0.001\rangle\}$	$\{s_{3.7773}\langle 0.336, 0.008\rangle, s_{3.9999}\langle 0.168, 0.008\rangle\}$
A_3	$\{s_{3.9999}\langle 0.294, 0.006\rangle, s_5\langle 0.435, 0.009\rangle\}$	$\{s_{3.3641}\langle 0.216, 0.027\rangle, s_{3.5758}\langle 0.144, 0.027\rangle, s_{3.8512}\langle 0.144, 0.018\rangle, s_{3.9999}\langle 0.096, 0.018\rangle, s_5\langle 0.3, 0.03\rangle\}$
A_4	$\{s_{1.546}\langle 0.441, 0.012\rangle, s_{1.7391}\langle 0.189, 0.004\rangle, s_{2.1912}\langle 0.189, 0.012\rangle, s_{2.3482}\langle 0.081, 0.004\rangle\}$	$\{s_1\langle 0.336, 0.012\rangle, s_{1.2236}\langle 0.084, 0.006\rangle, s_{1.32}\langle 0.224, 0.018\rangle, s_{1.5257}\langle 0.056, 0.009\rangle, s_{1.546}\langle 0.144, 0.012\rangle, s_{1.7391}\langle 0.036, 0.006\rangle, s_{1.8223}\langle 0.096, 0.018\rangle, s_2\langle 0.024, 0.009\rangle\}$

	c_3	c_4	c_5
A_1	$\{s_2\langle 0.3920, 0.018\rangle, s_{2.3329}\langle 0.112, 0.024\rangle, s_{2.5604}\langle 0.112, 0.024\rangle, s_{2.8311}\langle 0.032, 0.032\rangle\}$	$\{s_{3.7773}\langle 0.162, 0.02\rangle, s_{3.9999}\langle 0.567, 0.008\rangle, s_5\langle 0.171, 0.147\rangle\}$	$\{s_5\langle 0.648, 0.056\rangle\}$
A_2	$\{s_5\langle 0.576, 0.002\rangle\}$	$\{s_3\langle 0.648, 0.001\rangle\}$	$\{s_{1.546}\langle 0.567, 0.004\rangle, s_{2.1912}\langle 0.081, 0.003\rangle\}$
A_3	$\{s_5\langle 0.729, 0.001\rangle\}$	$\{s_{3.9999}\langle 0.576, 0.002\rangle\}$	$\{s_{1.9684}\langle 0.504, 0.009\rangle, s_{2.1912}\langle 0.216, 0.006\rangle\}$
A_4	$\{s_{2.7503}\langle 0.21, 0.004\rangle, s_3\langle 0.168, 0.006\rangle\}$	$\{s_3\langle 0.567, 0.02\rangle, s_{3.3641}\langle 0.243, 0.002\rangle\}$	$\{s_1\langle 0.504, 0.006\rangle, s_{1.32}\langle 0.144, 0.004\rangle\}$

Table 9
Time comprehensive decision matrix D^2.

	c_1	c_2	c_3	c_4	c_5
A_1	$\{s_5\langle 0.864,0.006\rangle\}$	$\{s_5\langle 0.81,0.004\rangle\}$	$\{s_3\langle 0.336,0.024\rangle, s_{3.3413}\langle 0.168,0.006\rangle\}$	$\{s_{3.7943}\langle 0.576,0.012\rangle, s_5\langle 0.568,0.018\rangle\}$	$\{s_5\langle 0.576,0.004\rangle\}$
A_2	$\{s_{2.7685}\langle 0.21,0.008\rangle, s_3\langle 0.126,0.004\rangle\}$	$\{s_{3.7943}\langle 0.32,0.012\rangle, s_4\langle 0.448,0.008\rangle\}$	$\{s_5\langle 0.81,0\rangle\}$	$\{s_{1.9999}\langle 0.216,0.036\rangle, s_{2.2559}\langle 0.288,0.012\rangle, s_{2.3112}\langle 0.18,0.018\rangle, s_{2.5406}\langle 0.24,0.006\rangle, s_{2.5603}\langle 0.288,0.012\rangle, s_{2.7685}\langle 0.384,0.004\rangle, s_{2.8135}\langle 0.24,0.006\rangle, s_3\langle 0.32,0.002\rangle\}$	$\{s_{3.0276}\langle 0.729,0.004\rangle\}$
A_3	$\{s_4\langle 0.512,0.002\rangle\}$	$\{s_5\langle 0.968,0.03\rangle\}$	$\{s_5\langle 0.729,0.001\rangle\}$	$\{s_4\langle 0.648,0.012\rangle, s_5\langle 0.405,0.008\rangle\}$	$\{s_1\langle 0.576,0.008\rangle\}$
A_4	$\{s_{2.3112}\langle 0.648,0.002\rangle\}$	$\{s_{1.9999}\langle 0.392,0.018\rangle, s_{2.3112}\langle 0.336,0.012\rangle\}$	$\{s_3\langle 0.336,0.012\rangle, s_{3.3413}\langle 0.28,0.008\rangle, s_{3.5956}\langle 0.24,0.006\rangle, s_{3.8352}\langle 0.2,0.004\rangle\}$	$\{s_{3.283}\langle 0.448,0.006\rangle, s_{3.7943}\langle 0.32,0.002\rangle\}$	$\{s_1\langle 0.512,0.006\rangle, s_{1.2988}\langle 0.32,0.004\rangle\}$

Table 10
The positive and negative ideal solution of each DM.

	c_1	c_2	c_3	c_4	c_5
$A^{(1)+}$	$\{s_5\langle 0.8,0.056\rangle\}$	$\{s_4\langle 0.9,0.2\rangle\}$	$\{s_2\langle 0.3920,0.018\rangle, s_{2.3329}\langle 0.112,0.024\rangle, s_{2.5604}\langle 0.112,0.024\rangle, s_{2.8311}\langle 0.032,0.032\rangle\}$	$\{s_{3.7773}\langle 0.162,0.02\rangle, s_{3.9999}\langle 0.567,0.008\rangle, s_5\langle 0.171,0.147\rangle\}$	$\{s_5\langle 0.648,0.056\rangle\}$
$A^{(1)-}$	$\{s_2\langle 0.125,0.018\rangle, s_{2.2335}\langle 0.1,0.006\rangle, s_{2.3329}\langle 0.075,0.009\rangle, s_{2.5405}\langle 0.06,0.003\rangle, s_{2.5604}\langle 0.1,0.006\rangle, s_{2.7503}\langle 0.08,0.002\rangle, s_{2.8311}\langle 0.06,0.003\rangle, s_3\langle 0.048,0.001\rangle\}$	$\{s_1\langle 0.336,0.012\rangle, s_{1.2236}\langle 0.084,0.006\rangle, s_{1.32}\langle 0.224,0.018\rangle, s_{1.5257}\langle 0.056,0.009\rangle, s_{1.546}\langle 0.144,0.012\rangle, s_{1.7391}\langle 0.036,0.006\rangle, s_{1.8223}\langle 0.096,0.018\rangle, s_2\langle 0.024,0.009\rangle\}$	$\{s_{2.7503}\langle 0.21,0.004\rangle, s_3\langle 0.168,0.006\rangle\}$	$\{s_3\langle 0.567,0.02\rangle, s_{3.3641}\langle 0.243,0.002\rangle\}$	$\{s_1\langle 0.504,0.006\rangle, s_{1.32}\langle 0.144,0.004\rangle\}$
$A^{(2)+}$	$\{s_5\langle 0.864,0.006\rangle\}$	$\{s_5\langle 0.968,0.03\rangle\}$	$\{s_5\langle 0.81,0\rangle\}$	$\{s_{3.7943}\langle 0.576,0.012\rangle, s_5\langle 0.568,0.018\rangle\}$	$\{s_5\langle 0.576,0.004\rangle\}$
$A^{(2)-}$	$\{s_{2.7685}\langle 0.21,0.008\rangle, s_3\langle 0.126,0.004\rangle\}$	$\{s_{1.9999}\langle 0.392,0.018\rangle, s_{2.3112}\langle 0.336,0.012\rangle\}$	$\{s_3\langle 0.336,0.024\rangle, s_{3.3413}\langle 0.168,0.006\rangle\}$	$\{s_{3.283}\langle 0.448,0.006\rangle, s_{3.7943}\langle 0.32,0.002\rangle\}$	$\{s_1\langle 0.576,0.008\rangle\}$

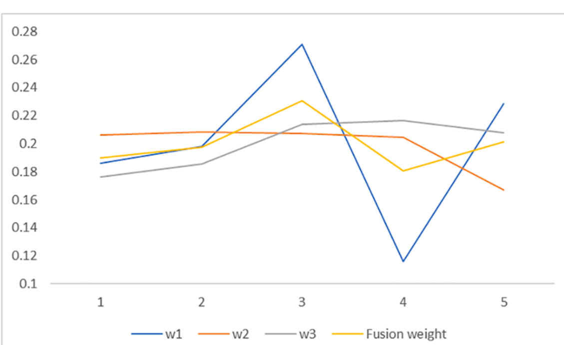

Fig. 5. The criteria weights relation of E1.

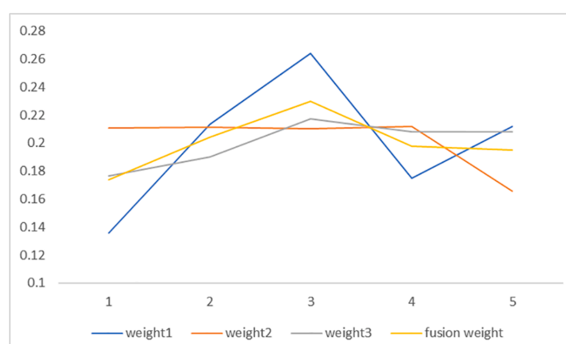

Fig. 6. The criteria weights relation chart of E2.

became a non-epidemic area. However, no country can afford the cost of isolation from the world. At most, it can only temporarily block personnel exchanges with a few countries. When many countries outside of China treat the COVID-19 as the flu, we cannot stand alone. How to adapt to this possible prospect is a major challenge and a subject worth considering.

The good news is that China has sufficient industrial and health strength to provide vaccination for a large-scale population. At the same time, the State Council promised that individuals would not bear any vaccination costs. The principle of vaccination is to make the body produce antibodies naturally by accessing the deactivated COVID-19. Vaccinating can significantly enhance the immunity of this virus. As of June 18, 2021, 31 provinces (autonomous regions and municipalities directly under the central government) and the Xinjiang Production and Construction Corps reported a total of 99.257 million doses of COVID-19 vaccination. Choosing the site of the COVID-19 vaccination center will be related to the public recognition, vaccination efficiency and vaccination success rate, and ultimately affect the essential role of the vaccine. Temporary vaccination centers are generally transformed from some public places, such as gymnasiums, conference halls. The choice of vaccination sites is an MCGDM problem. Suppose that the health administration of District A needs to establish a vaccination site. After preliminary screening, four alternatives $\{A_1,A_2,A_3,A_4\}$are formed, and two DMs $\{E_1,E_2\}$ will evaluate the alternatives from the following five criteria:

(1) Traffic conditions(c_1): Reasonable and efficient traffic condition is conducive to vaccinators to save traffic costs. In addition,

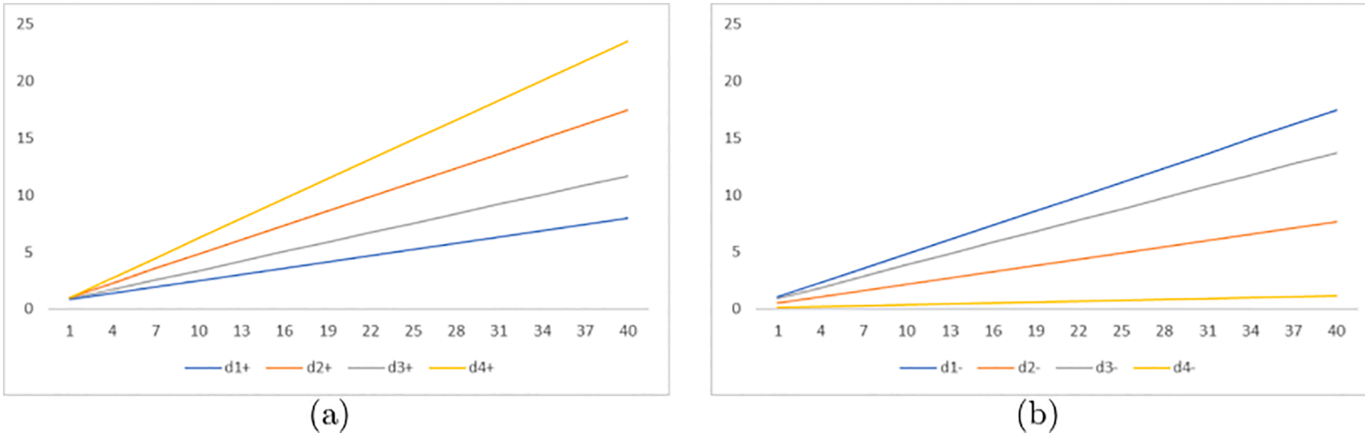

Fig. 7. The influence of w_{dom} on $d_{psy}(A_i, A^+)$ and $d_{psy}(A_i, A^-)$.

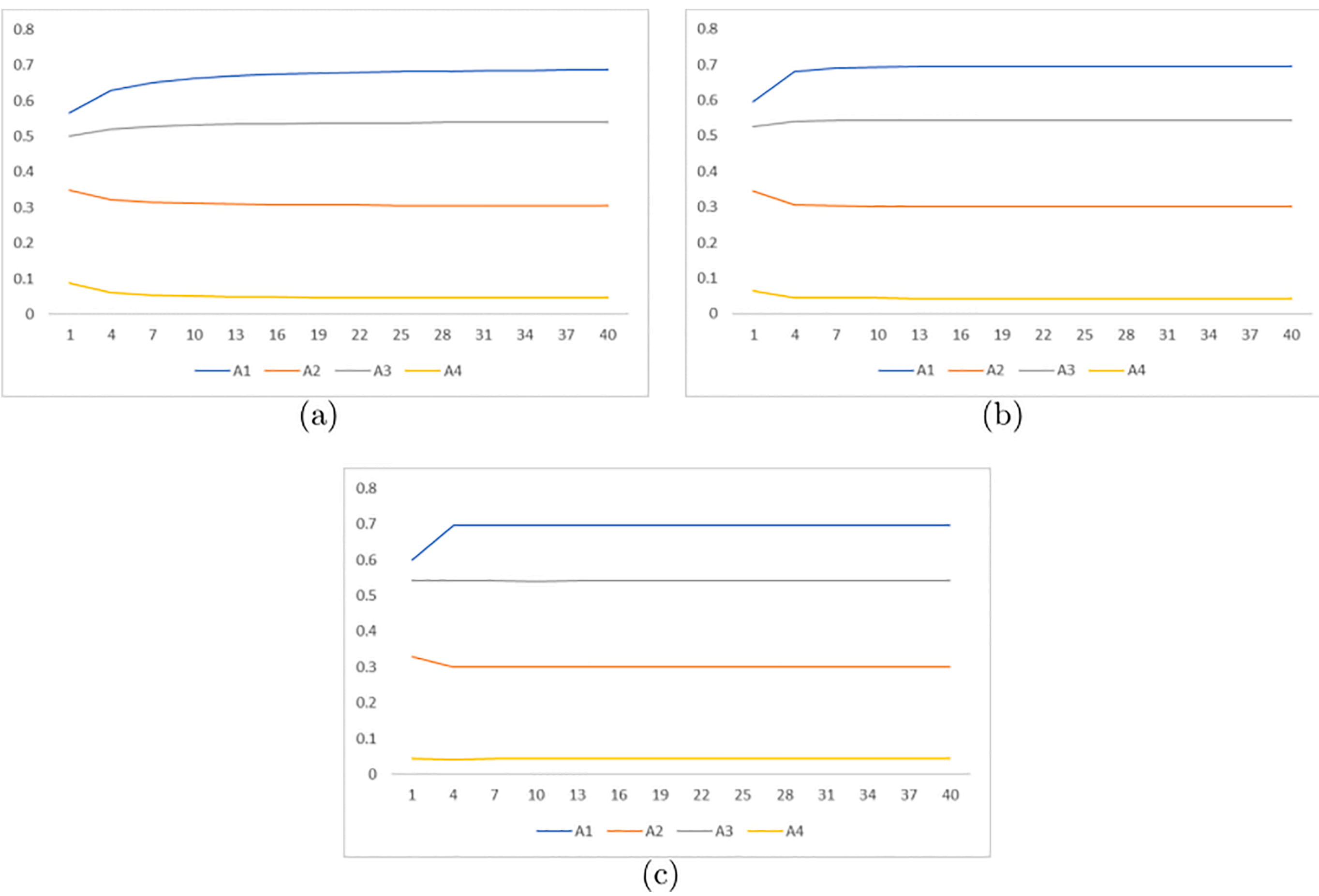

Fig. 8. The ranking result with different norms.

convenient transportation also helps increase vaccine acceptance.

(2) Flow density (c_2): There are many people waiting to be vaccinated, and most of them are vulnerable to infection. To prevent the possibility of the spread of COVID-19, the flow density of the site should not be too high.

(3) Internal facilities(c_3): There should be sufficient material storage and turnover space and good emergency equipment to prevent particular circumstances such as trampling and chaos.

(4) Surrounding supporting construction(c_4): Complete security facilities and drainage systems are needed. Moreover, it should also consider surrounding eating and resting places so that the vaccinators can eat and rest nearby.

(5) Transportation and modification costs(c_5): During the transportation of vaccines, due to factors such as the length of transportation time and temperature, the vaccine's potency is reduced or invalid. The transportation cost should be considered on the premise that the vaccine will not be damaged. At the same time, the cost and complexity of site reconstruction should not be too high. Then, the steps to solve the problem are as follows:

Step1: Collect and preprocess the evaluation information of PFPLTS

Table 11
Final ranking by different ranking aggregation methods.

Ranking aggregation method	Final ranking	Optimal location
Method in [17]	$A_1 \succ A_3 \succ A_2 \succ A_4$	A_1
Method in [2]	$A_1 \succ A_2 \succ A_3 \succ A_4$	A_1
Method in [30]	$A_1 \succ A_3 \succ A_2 \succ A_4$	A_1
Proposed method(1-norm)	$A_1 \succ A_3 \succ A_2 \succ A_4$	A_1
Proposed method(2-norm)	$A_1 \succ A_3 \succ A_2 \succ A_4$	A_1
Proposed method(infinity-norm)	$A_1 \succ A_3 \succ A_2 \succ A_4$	A_1

decision matrices $D^{(ge)} = (PL_{ij}^{(ge)})_{m\times n}$ given by each DM under the same criteria at three different stages. The decision matrices are shown in Tables 1–6, which already meet the standard and do not need to be processed.

Step2: Calculate the objective weight $\theta_1^{(g)}$ of the gth DM at different stages according to the method in Definition 15. The objective time weights of the two DMs are $\theta_1^{(1)} = (0.30, 0.37, 0.33)^T$and $\theta_1^{(2)} = (0.26, 0.37, 0.37)^T$, respectively.

Step3: Use the 1~9 scale method to construct pairwise judgment matrix $A(a_{ij})_{q\times q}$, which is shown in Table 7. Calculate the subjective weight θ_2 through the AHP method.

The objective time weight is $\theta_2 = (0.28, 0.65, 0.07)^T$.

Step4: To integrate the subjective and objective time weights, we use the nonlinear optimization model to calculate coefficients of the time combination weights by the method in Section 4.1, and then get the time combination weights $\theta^{(1)} = (0.29, 0.51, 0.20)$, $\theta^{(2)} = (0.27, 0.51, 0.22)$. The radar map of the fusion time weight, subjective and objective time weight is shown in Figs. 3 and 4.

Step5: Derive the comprehensive PFPLTS decision matrix $D^{(g)} = (V_{ij}^{(com-g)})_{m\times n}$ of each expert with time weight. The comprehensive time decision matrices are shown in Tables 8 and 9.

Step6: According to the score function in Definition 2,7, the positive and negative ideal solutions of each DM are shown in Table 10.

Step7: Calculate the distance from each alternative to the positive and negative ideal solution, get the positive and negative distance matrix of every DM, respectively.

Step8: According to Eq. (17), the criteria weights vectors with time-varying based on criterion recognition are ${\omega_1}^{(1)} = (0.19, 0.20, 0.23, 0.18, 0.20)^T$ and ${\omega_1}^{(2)} = (0.14, 0.21, 0.26, 0.17, 0.21)^T$.

Step9: According to Eq. (21), the criteria weights vectors with time-varying based on information disorder degree are ${\omega_2}^{(1)} = (0.21, 0.21, 0.21, 0.20, 0.17)^T$ and ${\omega_2}^{(2)} = (0.21, 0.21, 0.21, 0.21, 0.17)^T$.

Step10: According to Eq. (23), the criteria weights vectors with time-varying based on information hesitation degree are ${\omega_3}^{(1)} = (0.18, 0.19, 0.21, 0.22, 0.21)^T$ and ${\omega_1}^{(2)} = (0.18, 0.19, 0.22, 0.21, 0.21)^T$.

Step11: Based on the dual ideal point-vector projection method as Eq. (32). The fusion weight vectors are recorded as $\omega^{(1)} = (0.19, 0.20, 0.23, 0.18, 0.20)^T$ and $\omega^{(2)} = (0.17, 0.20, 0.23, 0.20, 0.19)^T$. And the relation between these weights can be shown in Figs. 5 and 6.

Step12: Calculate the dominance vector v_d and the indifference vectors $(v_{fj})_{n-1}$ by the fusion criteria weight $\omega^{(1)}$ and $\omega^{(2)}$. Then, the basis matrix M is obtained.

Step13: Construct a psychological matrix $H = diag(1, 1, \ldots, 1, w_{dom})$, and let $w_{dom} = 10$.

Step14: Compute the psychological distance $d_{psy}^{(g)}({V_{ij}}^{(com-g)}, V^+)$ and $d_{psy}^{(g)}({V_{ij}}^{(com-g)}, V^-)$ with the comprehensive value in Step 5.

Step15: The DMs' weight vector is $\sigma = (0.5, 0.5)^T$. Then, with the weighted average method, the comprehensive psychological distances $d_{psy}^{(g)}(A_i, A^+)$ and $d_{psy}^{(g)}(A_i, A^-)$ are aggregated based on each DM's psychological distance in step 14.

Step16: Calculate the closeness η_i for each alternative A_i. The comprehensive closeness $\eta_i = (0.6631, 0.3120, 0.5324, 0.0513)$.

Step17: In accordance with η_i, the final alternative ranking is $A_1 \succ A_3 \succ A_2 \succ A_4$.

In the case calculation, keep the dominance vector at an appropriate importance and set the psychological index to 10. According to the evaluation information given by the two experts and the criteria weights calculated according to the time weights of different stages, the recommended ranking of the COVID-19 vaccination center selection in the four alternatives is $A_1 \succ A_3 \succ A_2 \succ A_4$. Therefore, it is suggested that the health administration of District A establish a vaccination center at A_1.

6.2. Comparison and discussion

In the former section, we mention that the variable w_{dom} reflects the preference of DMs. Different distance measures and decision-making methods make the Psy-TOPSIS method more robust and flexible. Hence, to further illustrate the effectiveness of the proposed method, the analysis is conducted with different distance measures, decision-making methods and the value of w_{dom} ranges from 1 to 40.

6.2.1. Sensitivity analysis

We draw figures to show the influence of parameter w_{dom} and different distance measures on the alternative rankings. Fig. 7(a)-(b) describes the influence of varying w_{dom} on the distance from each alternative to the positive and negative ideal solutions when 1-norm is used. It is easy to find that both $d_{psy}(A_i, A^+)$ and $d_{psy}(A_i, A^-)$ increase with $w_{dom}$from Fig. 7(a)-(b), and $d_{psy}(A_i, A^+)$ increases faster than $d_{psy}(A_i, A^-)$.

DMs can express their unique personal preferences by providing diverse w_{dom}, which determines the weight between dominant and indifferent directions. Especially, with 1-norm, when $w_{dom} = 1$, which means the preferential relationship has no difference between dominant vector and indifferent vectors. From Fig. 8(a)-(c), we can conclude that the rankings of the alternatives are identical when w_{dom} or the norms are different, yet the closeness gap increases with the increase of w_{dom}, no matter which norm is used. And when w_{dom} is greater than 4, the closeness gradually stabilizes, and the ranking of the alternatives becomes stable, which shows that our method is adequately effective and robust. Hence, DMs can select the value of w_{dom} that will affect the alternative closeness and final rankings to describe their psychological preference of the dominant vector and indifferent vectors.

6.2.2. Comparison with different ranking aggregation methods

To illustrate the effectiveness of the proposed aggregation method, we compare the ranking results of several ranking aggregation methods with ours. As we can see from Table 11, the optimal location obtained by all ranking aggregation methods is A_1, but the final rankings are slightly different by these ranking aggregation methods. The results produced by all these methods are not very different or even basically the same. The most important reason is that we use the same weights and weight method in the calculation, that is, the weight determination method proposed in Section 4 above. The traditional TOPSIS method [17] can effectively avoid data subjectivity and well depict the comprehensive influence of multiple indicators. The advantage of the OWA operator [2] is that it can reflect the importance of the information itself, as well as the importance of the location of the information. The GBWM method has a broad application prospect because of its low time complexity in computation. However, these methods can only determine the only decision alternatives ranking according to the known weight and decision information, that is, time varying factors and decision maker's psychology are not considered.

In the Psy-TOPSIS method, we can fully use the changing and uncertain information to derive the time and criteria weights, which can directly influence the final ranking. At the same time, the DMs can fully reflect their psychological preference for different alternatives between dominant vector and indifferent vectors, which indicates the effectiveness and superiority of the proposed method to other aggregation methods.

7. Conclusion

PLTS is a valuable technique in linguistic evaluation. However, DMs may be uncertain and self-denying about the given linguistic terms. To reflect the uncertainty and hesitation of DMs, we have extended the traditional PLTS to a new fuzzy linguistic set named PFPLTS. Then, some corresponding basic operations and aggregation operators have been proposed. A linear programming method with minimum deviation and the vector projection method to determine the time and criteria weights are submitted, respectively, which can determine the importance of different stages in dynamic Pythagoras fuzzy probabilistic linguistic MCGDM problems and make full use of the hesitation and uncertainty of the evaluation information. Furthermore, DMs' psychological preference information has been considered. With the new time and criteria weights method, the TOPSIS method with psychological distance has been developed. Finally, the validity and feasibility are verified with a numerical example, site selecting of COVID-19 vaccination center.

In the future study, the proposed method can be applied in other MCGDM problems, such as medical diagnosis and investment decisions combined with forecasting model. In addition, the weight methods and the Psy-TOPSIS method can be further used in other fuzzy sets, such as intuitionistic fuzzy set and so on.

Declaration of Competing Interest

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service or company that could be construed as influencing.

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